SISO Decoding of Coded Bi-directional Relaying System
Using Block Turbo Codes
QingJun Jin†, Ki-Hyeon Park‡, Mi-Young Nam§, Hong-Yeop Song†, Jung-Hyun Kim‡ and Kwang-Jae Lim§
†Department of Electrical & Electronics Engineering, Yonsei University
‡Wireless System Research Department, ETRI
§{kg.kim, kh.park, my.nam, hysong}@yonsei.ac.kr {jh.kim06, kjlim}@etri.re.kr

Abstract

This paper proposes a soft transformation technique based on the same posteriori probability that could transform received bit to Soft-Input-Soft-Output (SISO) decodable bit in the multiple access relay channels (MARC) channel of Coded Bi-directional Relaying system (CBR). The simulation result states that the two-step CBR combined with SISO decoder is more attractive, because the SNR loss is only 0.8dB when compare with the three-step CBR combined with SISO decoder.

I. Introduction

Coded Bi-directional Relaying [2], [3], [8] has drawn researchers’ attention in recent years. And network coding introduced by [1], is used in such system, which is beneficial to improve the throughput. There are two categories of Coded Bi-directional Relaying Systems. The first one is a three-step scheme. As shown in the Fig.1, at the first time slot, the node A sends $X_A$ to the relay node B. Then, node C sends $X_C$ to node B. At the last time slot, node B broadcasts the network coded information to node A and node C. The other one is a two-step scheme, as shown in the Fig.2. At first, node A and node C send $X_A$ and $X_C$ to node B simultaneously. Then node B broadcasts the network coded information to both node A, and C.

![Fig.1. Three-Step CBR](image1)

![Fig.2. Two-Step CBR](image2)

When the SNR is low, conventional network coding does not exhibit good error correction capabilities. Many research works have been focusing on approach of designing and detecting the channel and network codes jointly. Hausl and Dupraz [4] consider network and channel coding jointly for improving the error performance in the case of multiple access relay channels. Joint network and channel coding with regular LDPC code has been considered using tanner graph in [5]. Recently, Guo et.al. [6] have proposed a practical method to combine non-binary channel coding with network coding together with iterative decoding scheme to jointly exploit the redundancy of the two schemes.

In this paper block turbo codes are used as error correcting codes. Block turbo codes (BTC) are extremely flexible in terms of rate, and tradeoff between error performance and complexity. The structure of block turbo codes is also suitable to overcome the burst errors in wireless communication. A fast optimal decoding algorithm of block turbo codes has been proposed in [9], which greatly decreased hardware complexity. However, for the case of multiple-access channel of two-step CBR, the optimal decoding algorithm cannot be applied directly. Fortunately, under the condition of the same posteriori probability, the receive bit could be transformed to a soft value that is appropriate for SISO decoding without performance loss. After the transformation is applied, the channel decoding and network encoding procedure ends simultaneously.

This article is organized as follows. Section II gives a brief introduction to BTC and its application in this paper. Section III shows the derivation and validity of the soft transformation. Section IV provides simulation results and conclusions.

II. Block Turbo Codes

Block turbo codes can be obtained from serial concatenation of simple linear block codes, such as BCH codes [7]. Two systematic linear block codes with parameters $(n_1, k_1, \delta_1)$ and $(n_2, k_2, \delta_2)$ are considered. The $n$, $k$ and $\delta$ stands for code length, number of information bits, and minimum distance respectively. Then the parameter of the block turbo codes is $(n_1 \times n_2, k_1 \times k_2, \delta_1 \times \delta_2)$ and the construction is shown in Fig.3.

For the situations of both three-step scheme and two-step scheme, both node A and C send BTC to the relay node B. Due to the linear property of BTC, the codeword that node B sends is also BTC. So at the last time slot of each scheme, after decoding procedure of node A and C ends, they could share their information by XOR operation of decoded codeword and their own codeword.
III. Soft Transformation of Received Bit

Assumed that the wireless channel is real Gaussian channel, such that for the two-step scheme.

At first time slot, multiple-access channel at the relay B:
\[ Y_B = X_A + X_C + Z_B \]

At next time slot, it is broadcasting channel:
\[ Y_A = X_B + Z_A \]
\[ Y_C = X_B + Z_C \]

where \( Z \) is the Gaussian noise, \( X \) is transmit symbol, \( Y \) is the received value. And it assumed that
\[ P(X_A = 1) = P(X_A = -1) = \frac{1}{2} \]
\[ P(X_C = 1) = P(X_C = -1) = \frac{1}{2} \]

For simplicity, assume the Gaussian noise is i.i.d and with zero mean and the same variance \( \sigma^2 \).

BPSK modulation is used in the CBR system and the symbols are mapped as follows: \( 0 \rightarrow 1, 1 \rightarrow -1 \). Then, without Gaussian noise, \( Y_B \) belongs to the set \( \{-2, 0, 2\} \). In this case, the iterative SISO decoding algorithm of BTC cannot be applied directly. So, conventionally hard decision is made before it is fed into hard-input-hard-output (HIHO) decoder as shown in Fig.4.

![Diagram of conventional decoder](image)

![Diagram of proposed decoder](image)

As mentioned above, \( Y_B \) is not suitable for SISO decoder. And the relay node sends network coded information, so it is reasonable to transform \( Y_B \) to corresponding network coded soft bit \( Y_B' \), which is SISO decodable, as shown in Fig.5. The conditional probability \( P(X_B = 1|Y_B) \) and \( P(X_B = 1|Y_B') \) describes the possibility that \( X_B \) is 1, given \( Y_B \) and \( Y_B' \) respectively. SISO decoding performance is influenced by the log-likelihood ratio. When the two posteriori probability is the same, the log-likelihood ratio is also identical. So it could be concluded that the SISO decoding performance that \( Y_B' \) could be achieved, if it were possible, is the same as SISO decoding performance of \( Y_B' \). Therefore, it is reasonable to calculate \( Y_B' \) based on the following condition:

\[ P(X_B = 1|Y_B) = P(X_B = 1|Y_B') \]

When \( P(X_B = 1|Y_B) = P(X_B = 1|Y_B') \), the constraint that \( P(X_B = -1|Y_B) = P(X_B = -1|Y_B') \) is also satisfied.

The posteriori probability \( P(X_B = 1|Y_B) \) can be computed as

\[ P(X_B = 1|Y_B) = \frac{1}{4} \times \left[ \frac{1}{2} + \frac{1}{2} \right] \]

Then,
\[ P(X_B = 1|Y_B') = \frac{1}{1 + e^{-(Y_B + Y_B')}} \]

where \( Y_B' \) could be viewed as it is sent from the “Virtual” node V with BPSK modulation, which encodes \( X_A \) and \( X_C \) in advance as shown in Fig.6.

The posteriori probability of \( Y_B' \) could be computed as

\[ P(X_B = 1|Y_B') = \frac{P(Y_B|X_B=1)P(X_B=1)}{P(Y_B|X_B=1)P(X_B=1) + P(Y_B|X_B=-1)P(X_B=-1)} \]

Then the result is

\[ P(X_B = 1|Y_B') = \frac{1}{1 + e^{-(2Y_B' + 2Y_B')}} \]

There are only one variable in the formulation of both \( P(X_B = 1|Y_B) \) and \( P(X_B = 1|Y_B') \). So they could be represented as \( P(X_B = 1|Y_B) = f(Y_B) \) and \( P(X_B = 1|Y_B') = g(Y_B') \). Under the condition of the same posteriori probability \( P(X_B = 1|Y_B) = P(X_B = 1|Y_B') \), the SISO decodable soft bit \( Y_B' \) is represented as

\[ Y_B' = \left( \frac{\sigma^2}{2} \right) \times \ln \left( \frac{1}{2} + \frac{|Y_B|}{2} \right) \]

where \( | \cdot | \) represents the absolute value.

After the transformation procedure is end, the obtained soft value can be fed into the SISO decoder directly.
IV. Simulation Results and Conclusions

Both node A and C send BTC to node B. The BER performance and FER performance are shown in Fig.7 and Fig.8 respectively. The extended BCH product code $\left(64,51,6\right)^2$ is used in this simulation. The number of test pattern and maximum iteration number are set to 8 and 4, respectively. The simulation results exhibit that SNR loss of SISO two-step scheme is less than 0.8dB, when compare with the three-step scheme. Compare to the algebraic decoding, the coding gain at BER ($10^{-4}$) is 2.5dB. When take the time slot saving and decoding complexity into account, we could conclude that the SISO two-step scheme outperforms the other two schemes.

The obtained results show that soft transformation (5) based on the same posteriori probability is a promising solution to enhance both error correction capabilities and throughput in CBR. And it will be interesting to consider soft information relaying with the proposed technique, which is a subject of future work.

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V. REFERENCES