



A study on the repair set of locally repairable codes

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 KICS Fall Conference 2015

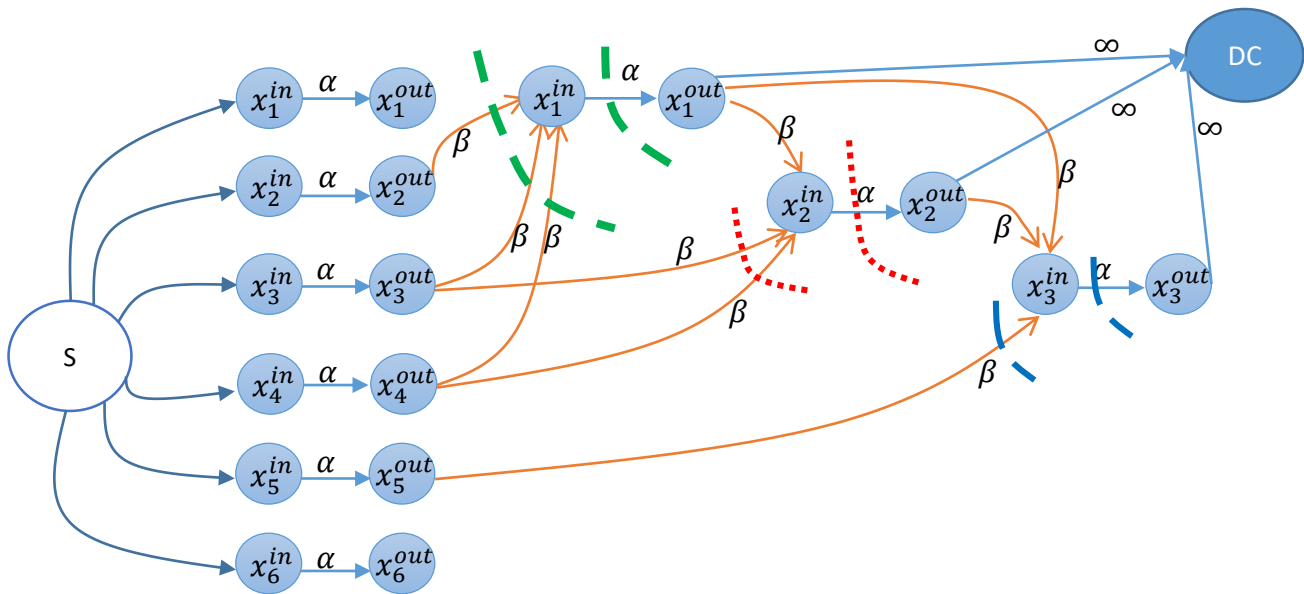


Regenerating Code

ASSUMPTIONS:

- ✓ The number of nodes r that should be contacted by a new comer for node repair
- ✓ The number of nodes K by a Data Collector for file retrieval
- ✓ $r \geq K$ is assumed
- ✓ **Any** r helper node should be able to repair a failed node

ANALYSIS using Information-Flow Graph



- ✓ The maximum file size that can be stored by using a Regenerating code is

$$B \leq \sum_{i=0}^{K-1} \min\{(r-i)\beta, \alpha\}$$

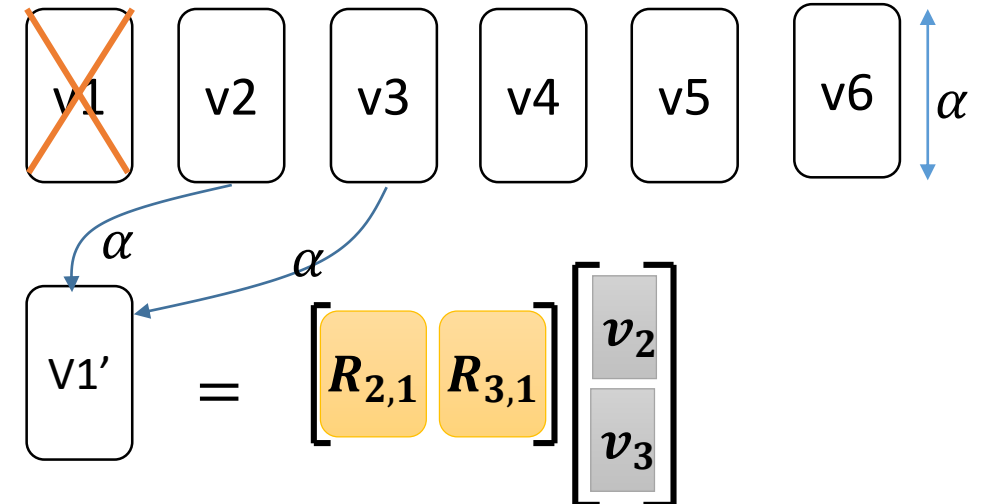
$$= \min\{3\beta, \alpha\} + \min\{2\beta, \alpha\} + \min\{\beta, \alpha\}$$

for $r = K = 3$ in the above example

Traditional LRCs

ASSUMPTIONS:

- ✓ Every helper node sends its whole stored data to a newcomer node
- $\beta = \alpha$
- ✓ Therefore, the repair set is disjoint
- ✓ Then the worst case cannot be avoided



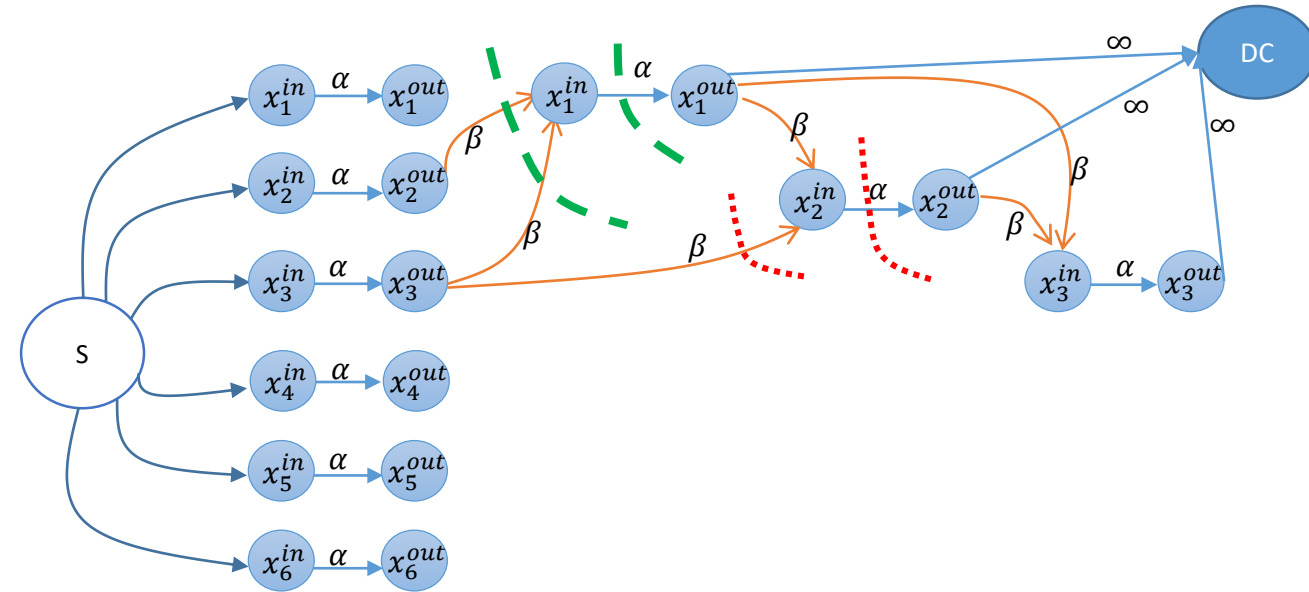
- $R_{2,1}$ is the repair matrix of size $(\alpha \times \alpha)$ that transforms a data v_2 into some useful data in a node V1
- $R_{3,1}$ is the repair matrix of size $(\alpha \times \alpha)$ that transforms a data v_3 into some useful data in a node V1
- Both $R_{2,1}$ and $R_{3,1}$ are full-rank otherwise $\beta < \alpha$ is possible
- Then node V2 can be repaired from node V1 and V3 and a node V3 also can be repaired from node V1 and V2.

$$v_k = R_{k,1}^{-1} \left(v_1 - \sum_{\substack{j=1 \\ j \neq k}}^{r=3} R_{j,1} v_j \right), \text{ for } i = 2, 3$$

Locally Repairable Code

DIFFERENCE FROM REGENERATING CODE:

✓ A code which satisfies $r < K$ is a Locally Repairable Code



✓ If we assume a random selection of helper nodes, then we cannot avoid the worst case above

$$B \leq \sum_{i=0}^{r-1} \min\{(r-i)\beta, \alpha\}$$

$$= \min\{2\beta, \alpha\} + \min\{\beta, \alpha\} + 0$$

DIFFERENCE FROM REGENERATING CODE:

- ✓ For every node, a specific set of r helper nodes, that participate in the repair process of that node, are pre-determined
That is, fixed repair set is given
- ✓ Careful construction of repair set makes the code avoid the worst case performance above

Repair set

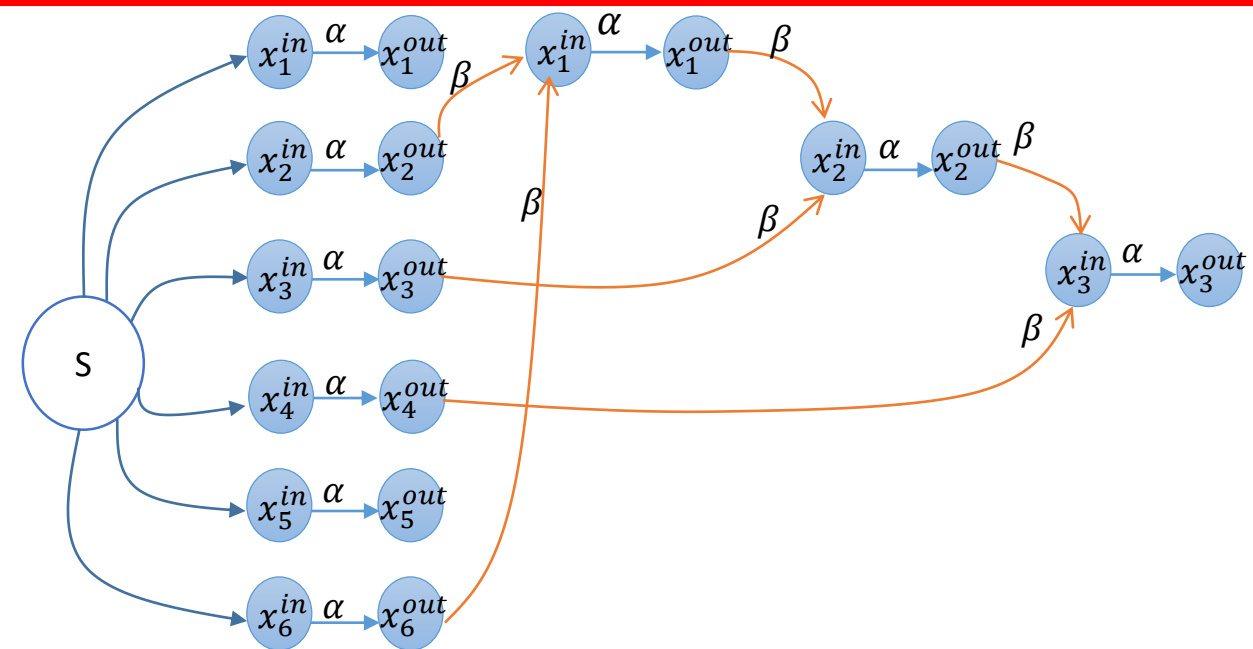
✓ If we want to avoid the worst case in LRC the followings should be satisfied:

- i) $\beta < \alpha$,
- ii) $R(i) \not\subseteq \cup_{j \in S} R(j)$,
for all $i \in [n]$ and for all $S \subseteq [n] \setminus \{i\}, |S| \leq K - 1$.

✓ Repair set construction for codes with locality 2

$$R(i) = \{(i-1), (i+1) \bmod n\}, \text{ for all } i \in [n]$$

(* $R(i)$ is the repair set of node i)



Concluding Remarks

- A structure of a repair set for LRC based on IFG
- Explicit repair-set for LRC with locality $r = 2$
- No explicit repair-set for LRC with locality $r > 2$

