The modified construction of binary locally repairable code with locality 2

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2019년도한국통신학회하계종합학술발표회

In this paper, we propose a construction for locally repairable codes (LRCs) with locality 2 using the concatenated structure of the punctured code and cyclic code. From the proposed construction, we found some new LRCs with larger dimension then those by C. Kim in 2018 at some length of n.

Introduction

- The locality r of the locally repairable codes (LRCs), which is the number of symbols needed to repair a symbol in a failed node.
- [1] proposed a construction of binary LRCs with $r=2$ using the concatenated structure of the shortened code.

Motivation

- The redundancy of code remains after the shortening, and the redundancy increases after the puncturing.
- We substitute the punctured code for the shortened code.

Main construction

- Let n be an integer with $n \geq 9$ and $3|n$;

  Let u be an odd positive integer and v be any positive integer, which satisfy $uv = \frac{2n}{3} + 1$ or $\frac{2n}{3}$, and $\gcd(u, v) = 1$;

  Let β be a primitive u^{th} root of unity in some extension field of F_2 and C_c be a binary $[uv, v(u-1) - \deg[g_1(x)]]$ cyclic code with generator polynomial $g(x) = (x^v + 1)g_1(x)$, where $g_1(x)$ is the minimal polynomial of β over F_2.
- Then, a binary $[n, v(u-1) - \deg[g_1(x)], d \geq 4, 2]$ LRC is constructed using the concatenated structure of the following outer code C_{out} and inner code C_{in}:

  ✓ C_{out}: a binary code is generated by puncturing the last $(uv \bmod 2)$ bit of C_c.

  ✓ C_{in}: a binary $[n, \frac{2n}{3}]$ cyclic code with parity check polynomial $h(x) = x^{20} + x^{10} + 1$.

Example:

- Let $n = 30$.
- We can choose $v = 3$, $u = 7$ and $g_1(x) = x^3 + x + 1$.

  And we get a binary [21,15] cyclic code C_c with generator polynomial $g(x) = (x^3+1)(x^3+x+1)$.
- The outer code C_o and inner code C_{in} are used in the concatenated structure is defined as:

  ✓ C_o : a binary $[20, 15]$ punctured code by puncturing the last bit of C_c.

  ✓ C_i : a binary $[30, 20]$ cyclic code with parity check polynomial $h(x) = x^{20} + x^{10} + 1$.

Therefore, we get a binary $[30, 15, \geq 4, 2]$ LRC.

Some constructed LRCs

Proposed construction	(9, 3)	(21, 10)	(30, 15)	(45, 25)
Construction in [1]	(9, 2)	(21, 9)	(30, 14)	(45, 24)

Conclusion

- We propose a new construction for binary LRCs and found some new LRCs with larger dimension at some length of n.
- In the future, it is necessary to find other outer code to increase the dimension at the given length of n.

REFERENCES

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