Joint locality (r_1, r_2) of MacDonald codes

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In this paper, we calculate the locality r_2 of the MacDonald codes: $r_2 = 3$ if $t < k-1$, and $r_2 = 4$ if $t = k-1$. The locality r_1 of MacDonald codes is known: $r_1 = 2$ if $t < k-1$ and $r_1 = 3$ if $t = k-1$. Therefore, the MacDonald codes have joint locality $(r_1, r_2) = (2,3)$ if $t < k-1$, and $(r_1, r_2) = (3,4)$ if $t = k-1$.

Introduction

- The locality r_l of the locally repairable codes (LRCs), which is the number of symbols needed to repair l failed symbols.
- [2] proposed a new concept: *joint locality*, which considers several values of r_l for multiple values of l instead of a single value of l.
- The binary simplex codes are r_1-optimal [1] and have good joint locality $(r_1, r_2) = (2,3)$ [2]. But the problem is its extremely low code rate.

Some results of the Simplex codes and MacDonald codes

- Let S_k be a $k \times (2^k - 1)$ matrix for any $k \in Z^+$, which is obtained by the following recursion. First, initialize $S_1 = (1)$, and then:

$$S_k = \begin{pmatrix} S_{k-1} & 0_{k-1}^T & S_{k-1} \\ 0_{2^{k-1}-1} & 1 & 1_{2^{k-1}-1} \end{pmatrix}, \text{ for } k = 2,3,\cdots.$$

- Definition [4]: The code C_s generated by S_k is called a Simplex code. Let $1 \le m \le k-1$. Delete the first $2^m - 1$ columns from S_k and denote the result as $G_k(m)$, and. The code $M_k(m)$ generated by $G_k(m)$ is called a MacDonald code.

  Note: For the convenience, let $G_k(m) = (A\ B)$, where B be the last 2^{k-1} columns of $G_k(m)$ and A be the rest part.
- Lemma 1 [3]: The $[n, k]$ simplex code C_s has the locality $r_1 = 2$ and the availability $t = \frac{n-1}{2}$.
- Lemma 2 [4]: The MacDonald code $M_k(m)$ has the locality $r_1 = 2$ if $m < k-1$, and $r_1 = 3$ if $m = k-1$.

Rules of the construction of the repair sets of the Simplex codes:

For any symbol c_i, the t repair sets are:

$R_1 = \{c_{\alpha^1}, c_{\beta^1}\}$, $R_2 = \{c_{\alpha^2}, c_{\beta^2}\}$, ... , $R_t = \{c_{\alpha^t}, c_{\beta^t}\}$.

1) If $i \in [1, 2^{k-1} - 1]$
   - $g_{\alpha^j}, g_{\beta^j} \in A$, for $j \in [1, \lfloor \frac{t}{2} \rfloor]$
   - $g_{\alpha^j}, g_{\beta^j} \in B$, for $j \in [\lceil \frac{t}{2} \rceil, t]$
2) If $i \in [2^{k-1}, 2^k - 1]$
   - $g_{\alpha^j} \in A$ and $g_{\beta^j} \in B$, for $j \in [1, t]$

Theorem (r_2 of the MacDonald codes):

The MacDonald code $M_k(m)$ has locality $r_2 = 3$ if $m < k-1$, and $r_2 = 4$ if $m = k-1$.

Conclusion

- From Lemma 2 and Theorem, the MacDonald code $M_k(m)$ has joint locality $(r_1, r_2) = (2,3)$ if $m < k-1$, and $(r_1, r_2) = (3,4)$ if $m = k-1$.
- In the future, we will verify the availability of the MacDonald codes, and design some LRCs with good joint locality and availability based on the MacDonald codes.

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