Construction of Reed-Solomon Based Quasi-Cyclic LDPC Codes Based on Protograph

Inseon Kim and Hong-Yeop Song

Yonsei University, Korea

The 24th Asia-Pacific Conference on Communications(APCC2018)

Nov. 13, 2018

Contents

- Introduction
- Motivation
- 2 Constructions of QC-LDPC codes
  - Construction of RS based QC-LDPC codes
  - QC-LDPC codes based on Protograph
- Proposed construction of Protograph based RS-QC-LDPC codes
- Conclusion

Introduction

- **LDPC (Low-Density Parity Check) Codes**
  - LDPC codes are code family with parity-check matrix of $(\#\, of\ \ 1s) \ll (\#\, of\ 0s)$

$$\begin{bmatrix} 1\,0\,1\,0\,0\,1\,0\,1 \\ 1\,1\,0\,1\,1\,0\,0\,0 \\ 0\,1\,0\,1\,0\,0\,1\,1 \\ 0\,0\,1\,0\,1\,1\,1\,0 \end{bmatrix}$$

  - LDPC codes first proposed by Gallager in early 1960s
  - LDPC code family are known to approach Shannon limits (by MacKay).
  - LDPC codes have good error correcting performance with good encoding and decoding process.
  - LDPC codes are selected as error correcting codes in many communication system standards.
    e.g.) IEEE 802.11 WLAN, Broadcasting(DVB-T2, ATSC3.0), 5G communication systems…

Introduction

- **LDPC (Low-Density Parity Check) Codes**
  - Parity-check matrix can be represented as bipartite(Tanner) graph; Check nodes, Variable nodes, edges

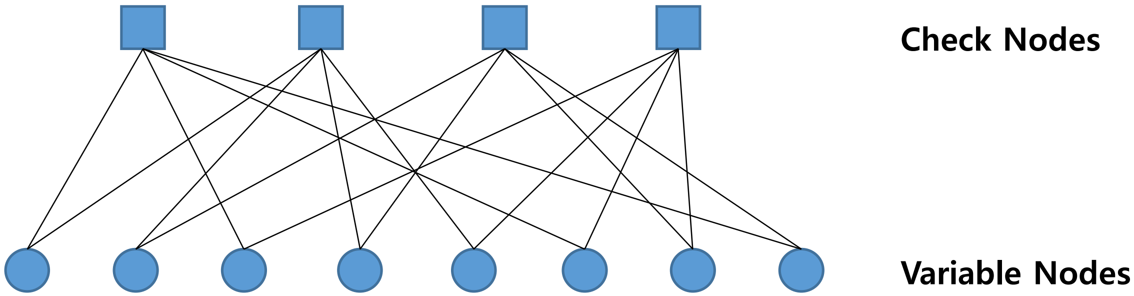


Fig. 1 Tanner graph of LDPC codes

  - Short cycles and girths play a key role to show good BER performance
  - Many researchers concerns to reduce the number of short cycles of Tanner graph

Construction of RS based QC-LDPC codes

Introduction

- Protograph based LDPC codes
  - A bipartite graph with multiple edges between variable nodes and check nodes
  - We can get Tanner graph of LDPC codes using protograph with copy-and-permute procedure
  - Recently, LDPC based on protograph gains attention because protograph has simple construction and analysis.

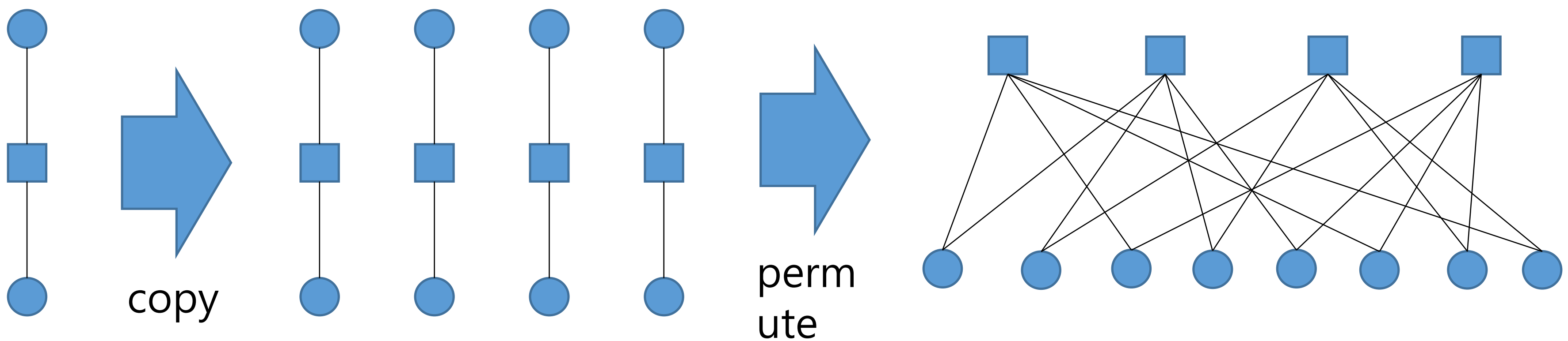
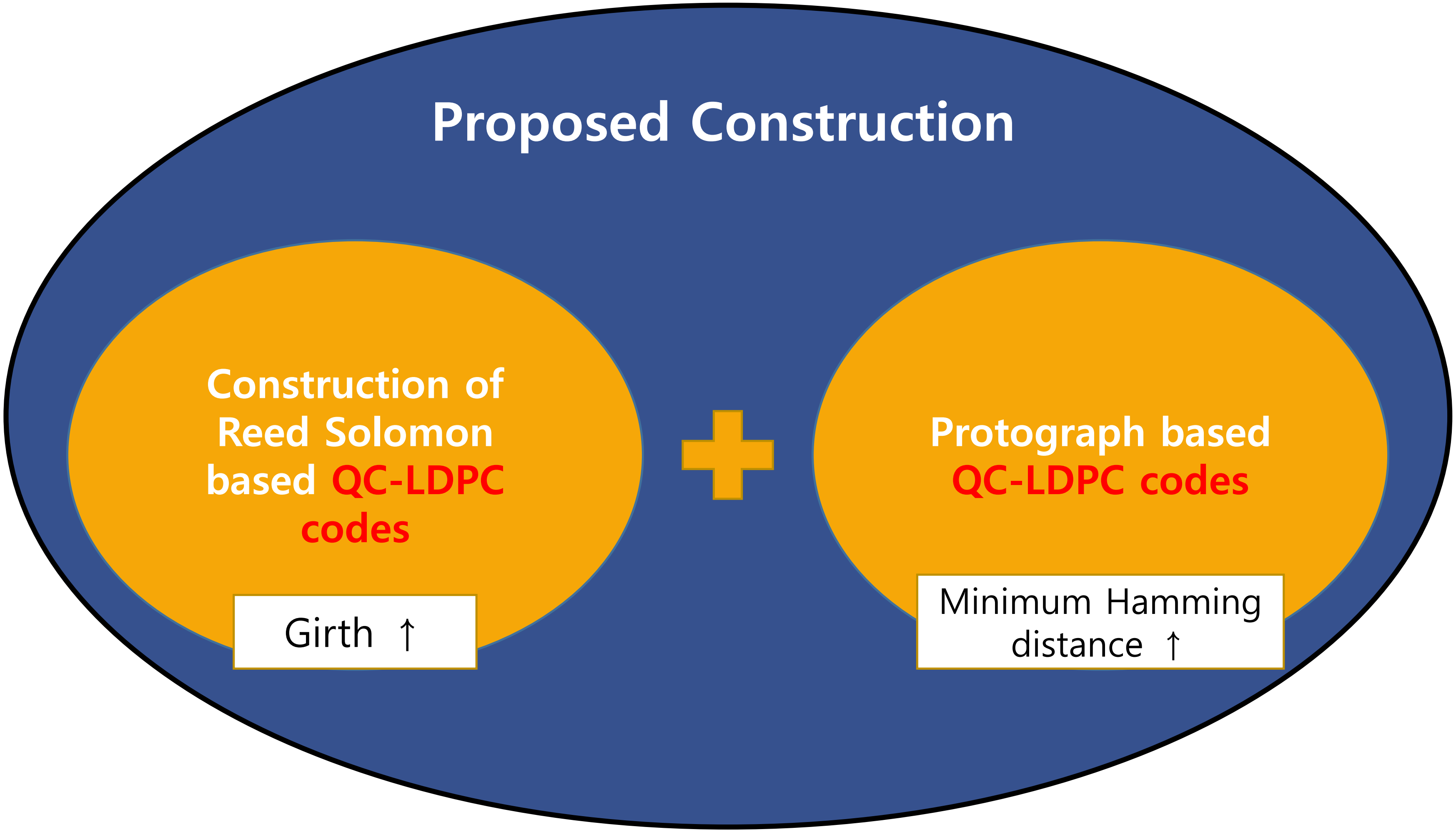


Fig. 2 Copy-and–permute procedure

Motivation


Proposed Construction
Construction of Reed Solomon based QC-LDPC codes
Girth ↑
Protograph based QC-LDPC codes
Minimum Hamming distance ↑

Construction of RS-QC-LDPC codes

- **Reed-Solomon based QC-LDPC codes**
  - Parity-check matrix of RS codes over $GF(2^s)$

$$B_{RS}(d,n) = \begin{bmatrix} 1 & \beta & \cdots & \beta^{n-1} \\ 1 & \beta^2 & \cdots & (\beta^2)^{n-1} \\ \vdots & \vdots & \ddots & \cdots \\ 1 & \beta^d & \cdots & (\beta^d)^{n-1} \end{bmatrix}$$

β is an element of order n, where n is a factor of $2^s - 1$

  - Parity-check matrix of RS based QC-LDPC codes

$$H_{RS}(d,n) = \begin{bmatrix} H_{0,0} & H_{0,1} & \cdots & H_{0,n-1} \\ H_{1,0} & H_{1,1} & \cdots & H_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ H_{d-1,0} & H_{d-1,1} & \cdots & H_{d-1,n-1} \end{bmatrix}$$

$H_{i,j}$ is $r \times r$ Identity matrix cyclically shifted by the elements of $B_{RS}(i,j)$

  - This RS based QC-LDPC codes has girth at least 6[1].

[1] X. Xiao, W.E.Ryan, B.Vasic, S.Lin and K.Abdel-Ghaffar, " Reed-Solomon-Based Quasi-Cyclic LDPC Codes: Designs, Cycle Structure and Erasure Correction," Information Theory and its Application(ITA2018), Catamaran Resort, San Diego, Feb. 21-26, 2018.

Construction of RS-QC-LDPC codes

- **Construction of RS based QC-LDPC codes with girth 8[1]**
  - Make $B_{RS}(d,t)$ by choosing t columns of $B_{RS}(d,n)$
  - $\Lambda_t = \{l_1, l_2, \cdots, l_t\}$: Index set of selected t columns and satisfying following equations

$$l_{i_3} \neq 2l_{i_2} - l_{i_1},\ l_{i_3} \neq 3l_{i_2} - 2l_{i_1},\ l_{i_3} \neq \frac{3l_{i_3} - l_{i_1}}{2},$$
$$n \nmid l_{i_2} + l_{i_3} - 2l_{i_1},\ n \nmid l_{i_2} + 2l_{i_3} - 3l_{i_1},\ n \nmid 2l_{i_2} + l_{i_3} - 3l_{i_1},$$
$$n \nmid 2l_{i_3} - l_{i_1} - l_{i_2},\ n \nmid 3l_{i_3} - 2l_{i_1} - l_{i_2},\ n \nmid 3l_{i_3} - l_{i_1} - 2l_{i_2},$$

  - Column selection is based on extending cycles of length 6 → length 8
  - RS based QC-LDPC codes with parity-check $B_{RS}(d,t)$ has girth at least 8

[1] X. Xiao, W.E.Ryan, B.Vasic, S.Lin and K.Abdel-Ghaffar, " Reed-Solomon-Based Quasi-Cyclic LDPC Codes: Designs, Cycle Structure and Erasure Correction," Information Theory and its Application(ITA2018), Catamaran Resort, San Diego, Feb. 21-26, 2018.

QC-LDPC codes based protograph

- **QC-LDPC codes based on protograph**
  - Protograph: allow more than 2 edges b/w check and variable nodes
  - Copy small Tanner graph and lift the edges to copied graph

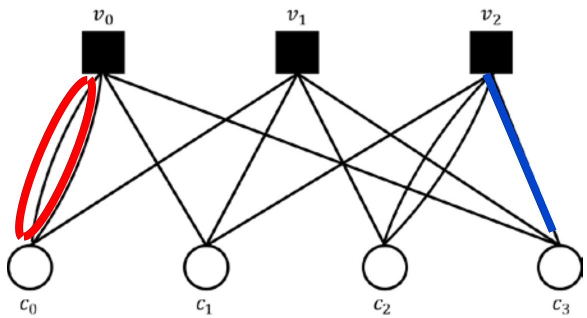


Fig. 3 Example of protograph of QC-LDPC codes

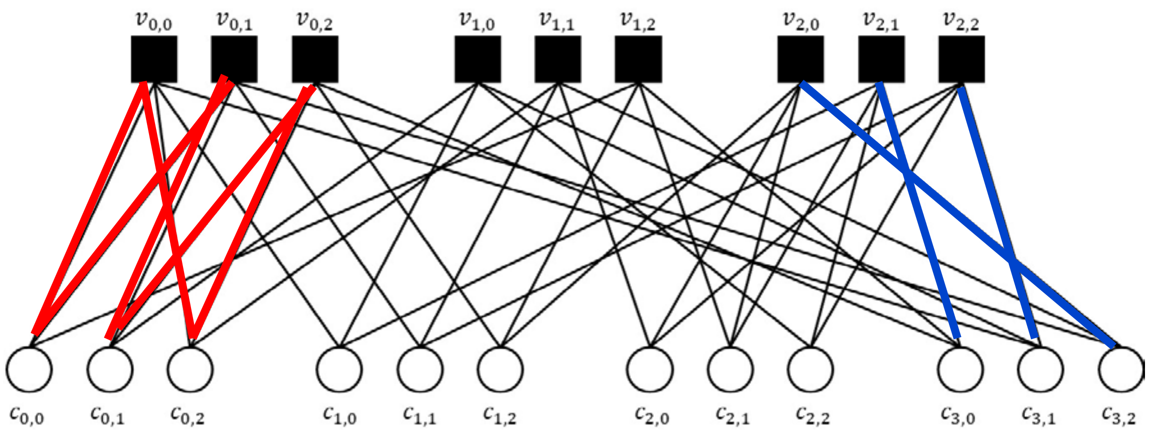


Fig. 4 Extended Tanner graph of protograph of QC-LDPC codes

QC-LDPC codes based protograph

- **QC-LDPC codes based on protograph**
  - Calculate upper bound of minimum Hamming distance of QC-LDPC codes based on protograph[2]
  - Using a function of weight matrix of protograph, they explicit the upper bound of minimum Hamming distance of protograph.
  - They found out that well constructed protograph with multi-edge have better upper bound of minimum Hamming distance than protograph with single edge

$$H = \begin{bmatrix} I_0 + I_1 & I_1 & 0 & I_2 \\ I_2 & I_0 & I_1 & I_2 \\ 0 & I_1 & I_0 + I_1 & I_1 \end{bmatrix}$$

Parity matrix of Protograph of QC-LDPC codes(Fig.4)

$$wt(H) = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

Weight matrix

[2] R. Smarandache and P. O. Vontobel, "Quasi-Cyclic LDPC Codes: Influence of Proto- and Tanner-Graph Structure on Minimum Hamming Distance Upper Bounds," IEEE Trans. Info. Theory, vol. 58, no.2, Feb. 2012.

Protograph based RS-QC-LDPC codes

- **Procedures of proposed constructions**

Consider $B_{RS}(d,t)$ satisfying the equations as mentioned before

↓

Construct the weight matrix of protograph by following conditions: make the degree of check nodes or variable nodes irregular.

↓

With constructed the weight matrix of protograph, extend the graph as parity-check matrix of LDPC codes

Protograph based RS-QC-LDPC codes

- **Propose the construction of QC-LDPC codes combining RS based QC-LDPC codes with girth 8 and protograph with increasing minimum Hamming distance.**

- Protograph based RS-QC-LDPC codes still holds the property that the girth is at least 8.
- Protograph based RS-QC-LDPC codes would have better error-correcting performance than RS based QC-LDPC codes because it has increased upper bound of minimum Hamming distance.

Protograph based RS-QC-LDPC codes

- **Simulation**
  - Assume AWGN channel and BPSK modulation
  - Attain BER performance under sum-product decoding with maximum 50 iterations
  - Use the parity-check matrix of RS codes over $GF(2^8)$
  - Construct 2 weight matrix of Protograph based RS-QC-LDPC codes with rate 1/2 and 2/3.

$$\Lambda_8 = \{2, 5, 7, 13, 20, 32, 54, 60\}$$

$$wt\left(H_{PRS}(4,8)\right) = \begin{bmatrix} 2 & 2 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 2 & 2 \end{bmatrix}$$

Rate 1/2

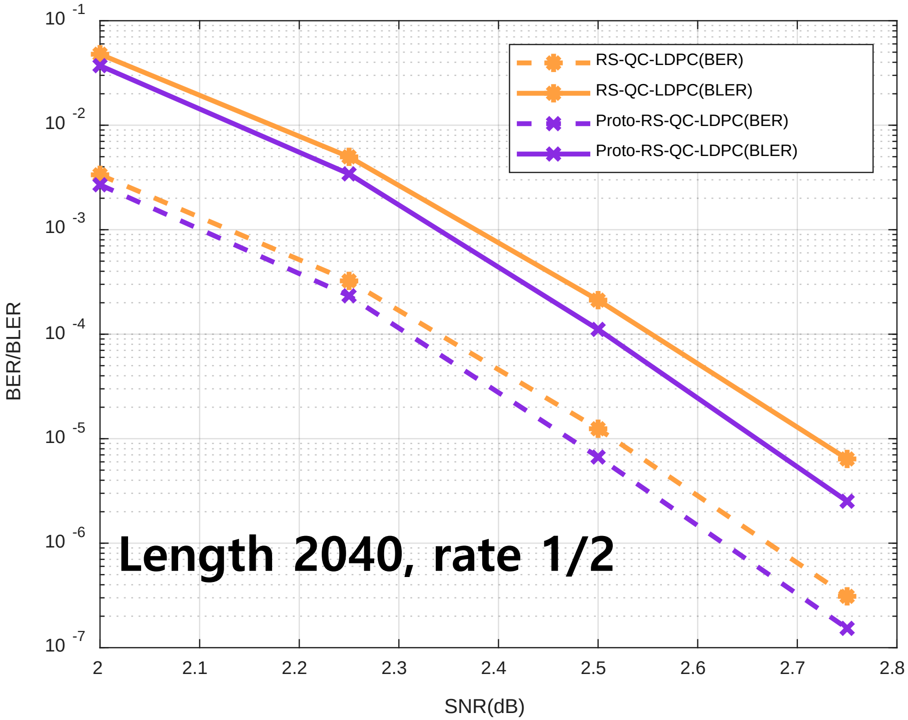
$$\Lambda_{12} = \{0, 1, 4, 9, 11, 20, 24, 35, 41, 49, 90, 225\}$$

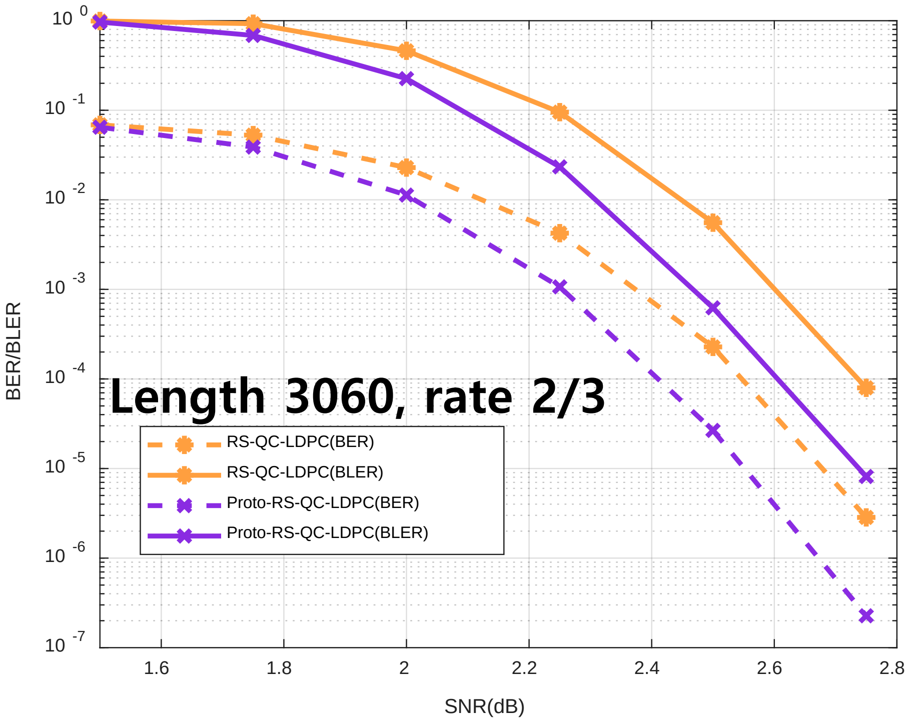
$$wt\left(H_{PRS}(4,12)\right) = \begin{bmatrix} 2 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 0 & 0 \end{bmatrix}$$

Rate 2/3

Protograph based RS-QC-LDPC codes

- **Simulation**





- **observe that BER gain is about 0.15 dB at BER 10^{-5} and BLER gain is about 0.12dB at BLER 10^{-4}.**
- **Error correcting performance of Proto-RS-QC-LDPC have more gain when the codes have high rate.**

Conclusion

- **Propose the construction of Protograph based RS-QC-LDPC codes combining two existing QC-LDPC codes construction**
  - **RS based QC-LDPC codes with girth 8**
  - **Protograph which increase the upper bounds of minimum Hamming distance**
- We observe that Protograph based RS-QC-LDPC codes have better error-correcting performance than existing RS based QC-LDPC codes
  - We found out that the high rate codes have more coding gain.

- **Future work**
  - Can we propose protograph construction which will increase the minimum Hamming distance itself?
  - Can we propose the performance analysis of Protograph based RS-QC-LDPC codes with enumerator function or EXIT chart?

Thank you for listening

Any Questions? Or comments?