



# 길이 64를 갖는 almost polyphase sequence를 설계하는 다양한 방법

---

이 민 형, 송 홍 엽

연세대학교



# Contents of View

- **Introduction**

- ✓ Correlations of the sequence
- ✓ Almost-polyphase sequence

- **Application of almost-polyphase sequence of length 64**

- ✓ PSS in Downlink Synchronization of 4G
- ✓ Construction of  $z_1$

- **New constructions**

- ✓ Construction of  $z_2$
- ✓ Construction of  $z_3$
- ✓ Construction of  $z_4$

- **Conclusion**

- ✓ Comparison



# Contents of View

- **Introduction**

- ✓ Correlations of the sequence
- ✓ Almost-polyphase sequence

- **Application of almost-polyphase sequence of length 64**

- ✓ PSS in Downlink Synchronization of 4G
- ✓ Construction of  $z_1$

- **New constructions**

- ✓ Construction of  $z_2$
- ✓ Construction of  $z_3$
- ✓ Construction of  $z_4$

- **Conclusion**

- ✓ Comparison

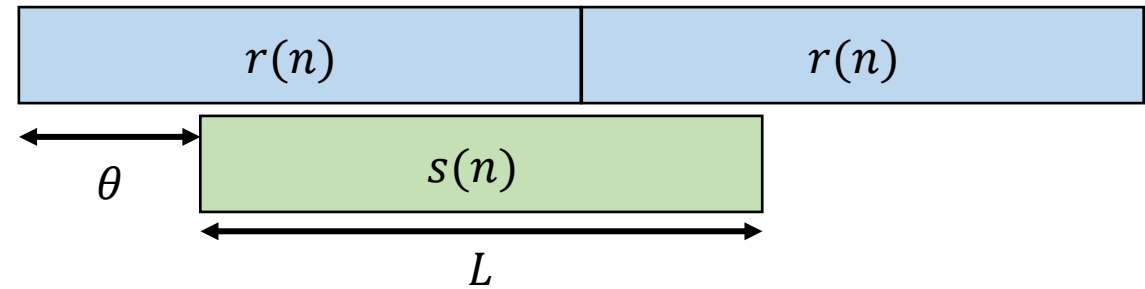
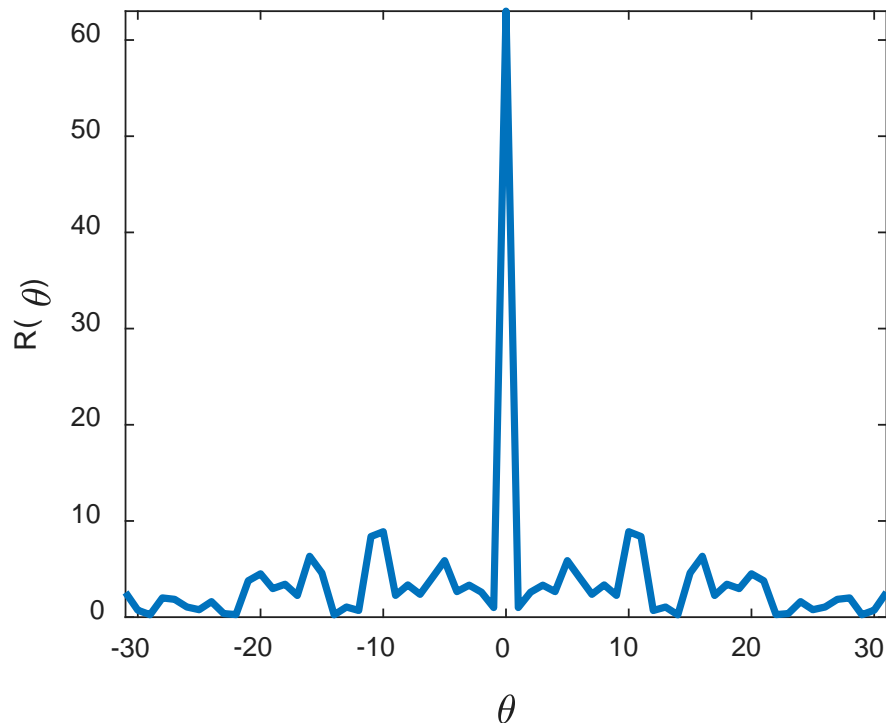
## Similarity between received sequence and reference sequence

두 수열이 같으면 Autocorrelation, 다르면 Crosscorrelation

### Even-periodic correlation

$$R(\theta) = \sum_{i=0}^{L-1} r(i + \theta) s^*(i)$$

Autocorrelation of D<sub>25</sub> of period 64



### Peak sidelobe ratio (PSLR)

$$\frac{\max |R(\theta)|}{\theta = 0 \text{ 일 때의 Autocorrelation}}$$

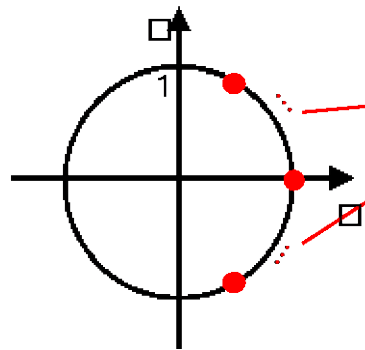


\*Autocorrelation의 경우 out-of-phase( $\theta \neq 0$ )

# Polyphase sequence

Roots of unity로  
이루어진 수열

|                                |                                |                                |     |                                    |                                    |                                |
|--------------------------------|--------------------------------|--------------------------------|-----|------------------------------------|------------------------------------|--------------------------------|
| $\square^j \frac{2\pi a_1}{N}$ | $\square^j \frac{2\pi a_2}{N}$ | $\square^j \frac{2\pi a_3}{N}$ | ... | $\square^j \frac{2\pi a_{L-2}}{N}$ | $\square^j \frac{2\pi a_{L-1}}{N}$ | $\square^j \frac{2\pi a_L}{N}$ |
|--------------------------------|--------------------------------|--------------------------------|-----|------------------------------------|------------------------------------|--------------------------------|



□-th roots of unity

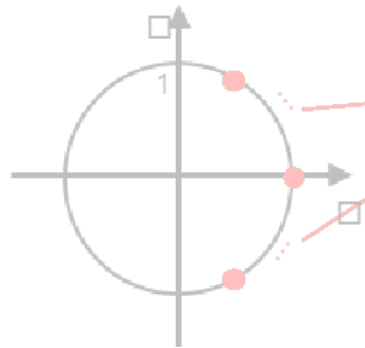
$$\square^j \frac{2\pi a_i}{N}, \quad \square_i \in \square_N$$

- Correlation 특성이 좋은 polyphase sequence는 Synchronization signal로 사용

# Almost - polyphase sequence

Roots of unity로 이루어진 수열

|                           |                           |                           |     |                               |                               |                           |
|---------------------------|---------------------------|---------------------------|-----|-------------------------------|-------------------------------|---------------------------|
| $e^{j\frac{2\pi a_1}{N}}$ | $e^{j\frac{2\pi a_2}{N}}$ | $e^{j\frac{2\pi a_3}{N}}$ | ... | $e^{j\frac{2\pi a_{L-2}}{N}}$ | $e^{j\frac{2\pi a_{L-1}}{N}}$ | $e^{j\frac{2\pi a_L}{N}}$ |
|---------------------------|---------------------------|---------------------------|-----|-------------------------------|-------------------------------|---------------------------|



□-th roots of unity

$$e^{j\frac{2\pi a_i}{N}}, \quad a_i \in \{0, 1, \dots, N-1\}$$

Roots of unity와 zero로 이루어진 수열

|                           |   |                           |     |   |                               |                           |
|---------------------------|---|---------------------------|-----|---|-------------------------------|---------------------------|
| $e^{j\frac{2\pi b_1}{N}}$ | 0 | $e^{j\frac{2\pi b_3}{N}}$ | ... | 0 | $e^{j\frac{2\pi b_{L-1}}{N}}$ | $e^{j\frac{2\pi b_L}{N}}$ |
|---------------------------|---|---------------------------|-----|---|-------------------------------|---------------------------|



# Contents of View

- **Introduction**

- ✓ Correlations of the sequence
- ✓ Polyphase sequence
- ✓ Almost-polyphase sequence

- **Application of almost-polyphase sequence of length 64**

- ✓ PSS in Downlink Synchronization of 4G
- ✓ Construction of  $z_1$

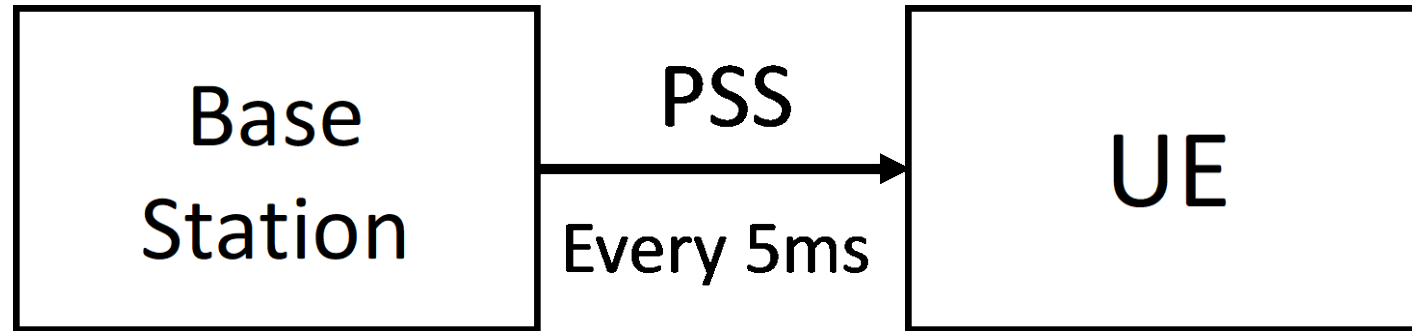
- **New constructions**

- ✓ Construction of  $z_2$
- ✓ Construction of  $z_3$
- ✓ Construction of  $z_4$

- **Conclusion**

- ✓ Comparison

# PSS in Downlink Synchronization of 4G



- Cell search
- Time & Frequency Synchronization





# PSS in Downlink Synchronization of 4G

$$d_1(n) = \begin{cases} e^{-j\frac{\pi u n(n+1)}{63}} & n = 0, 1, \dots, 30 \\ e^{-j\frac{\pi u (n+1)(n+2)}{63}} & n = 31, \dots, 61 \end{cases}$$

| $\square_{ID}^{(2)}$ | Root index $\square$ |
|----------------------|----------------------|
| 0                    | 25                   |
| 1                    | 29                   |
| 2                    | 34                   |

**3 different sequences by Cell ID**



# Construction of $z_1$

$$(u = 29)$$
$$z_1(n) = \begin{cases} \exp\left(-j\frac{un(n+1)}{63}\right) & \text{if } 0 \leq n \leq 30 \\ 0 & \text{if } n = 31 \\ \exp\left(-j\frac{u(n+1)(n+2)}{63}\right) & \text{if } 32 \leq n \leq 62 \\ 0 & \text{if } n = 63 \end{cases}$$

# Zadoff-Chu sequence

$$L \text{이 홀수일 때, } ZC(n) = \exp\left(-j \frac{un(n+1)}{L}\right)$$
$$L \text{이 짝수일 때, } ZC(n) = \exp\left(-j \frac{un(n+2)}{L}\right)$$

( $u$ 는  $L$ 과 서로소,  $0 \leq n \leq L-1$ )

길이가  $L$ 인 Zadoff-Chu sequence의 생성 방법

**Out-of-phase Autocorrelation이 항상 0**

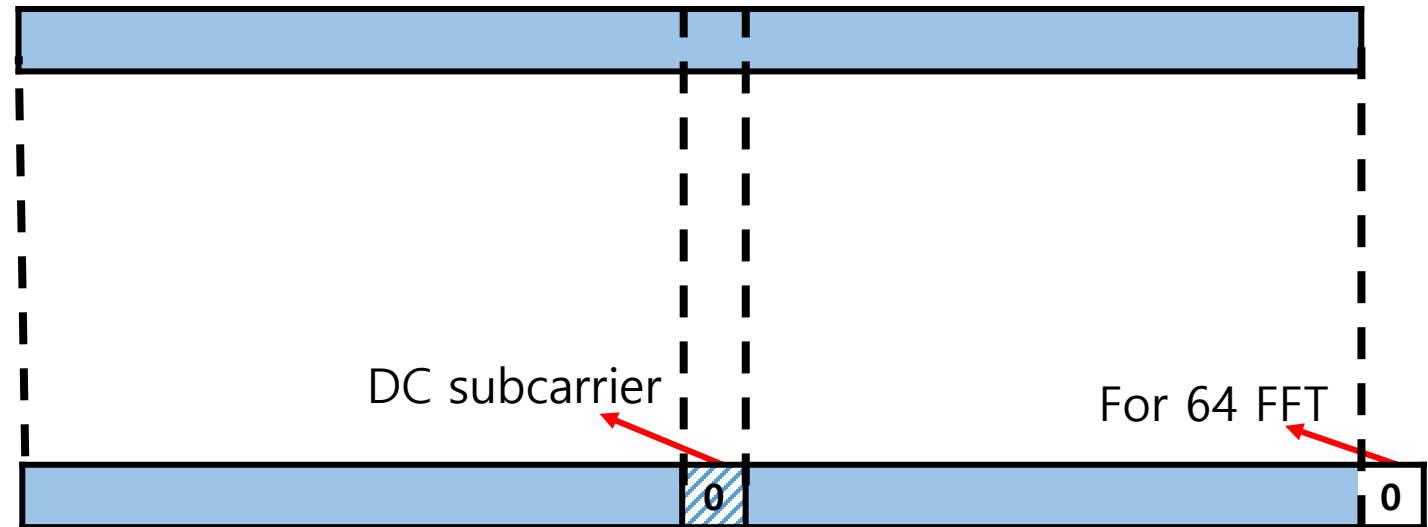
# Construction of $z_1$

Zadoff-Chu sequence

$ZC_u(n)$  of period 63

$$ZC_u(n) = e^{-j\frac{\pi un(n+1)}{63}} \quad n = 0, 1, \dots, 62$$

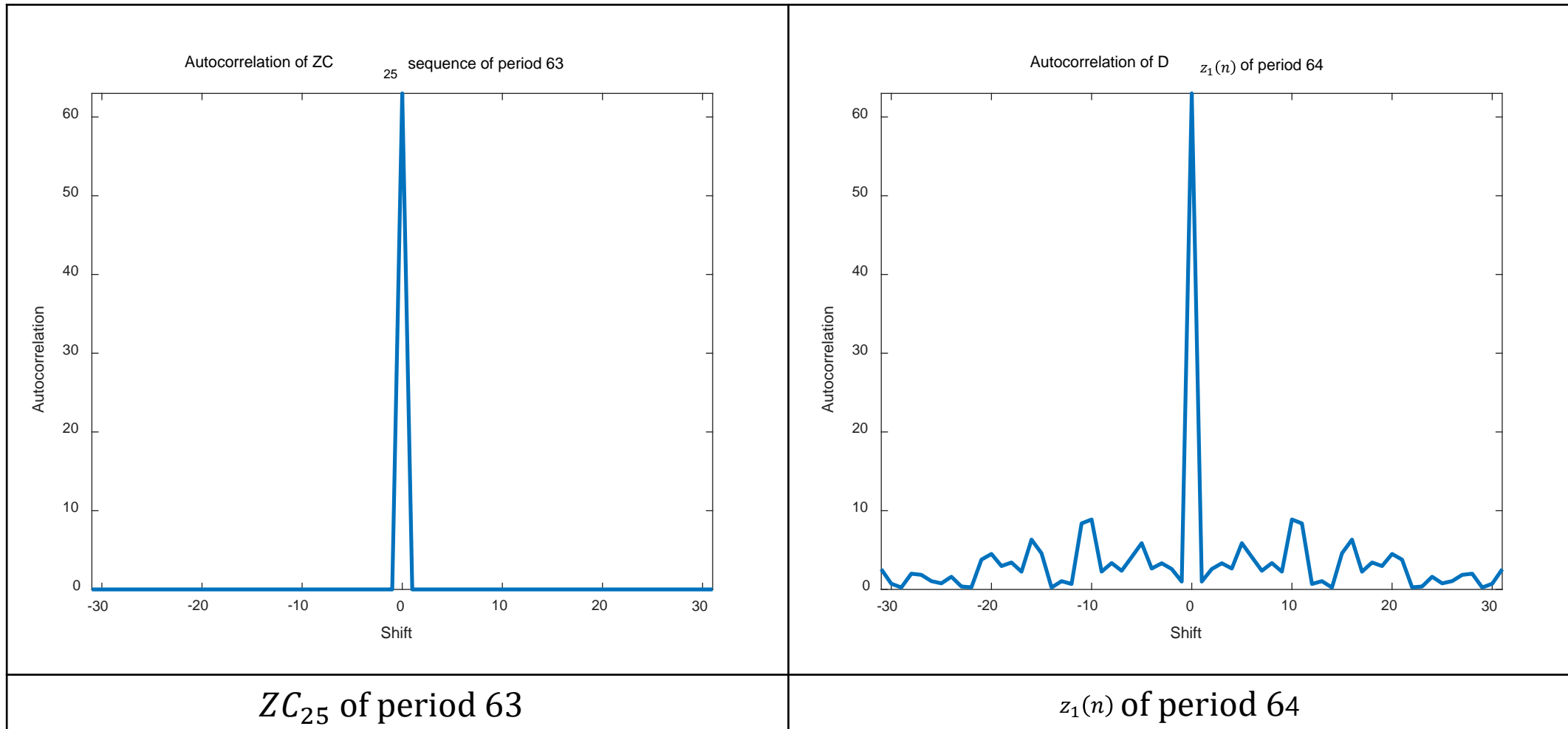
$z_1(n)$  of period 64



32번째를 0으로 대체

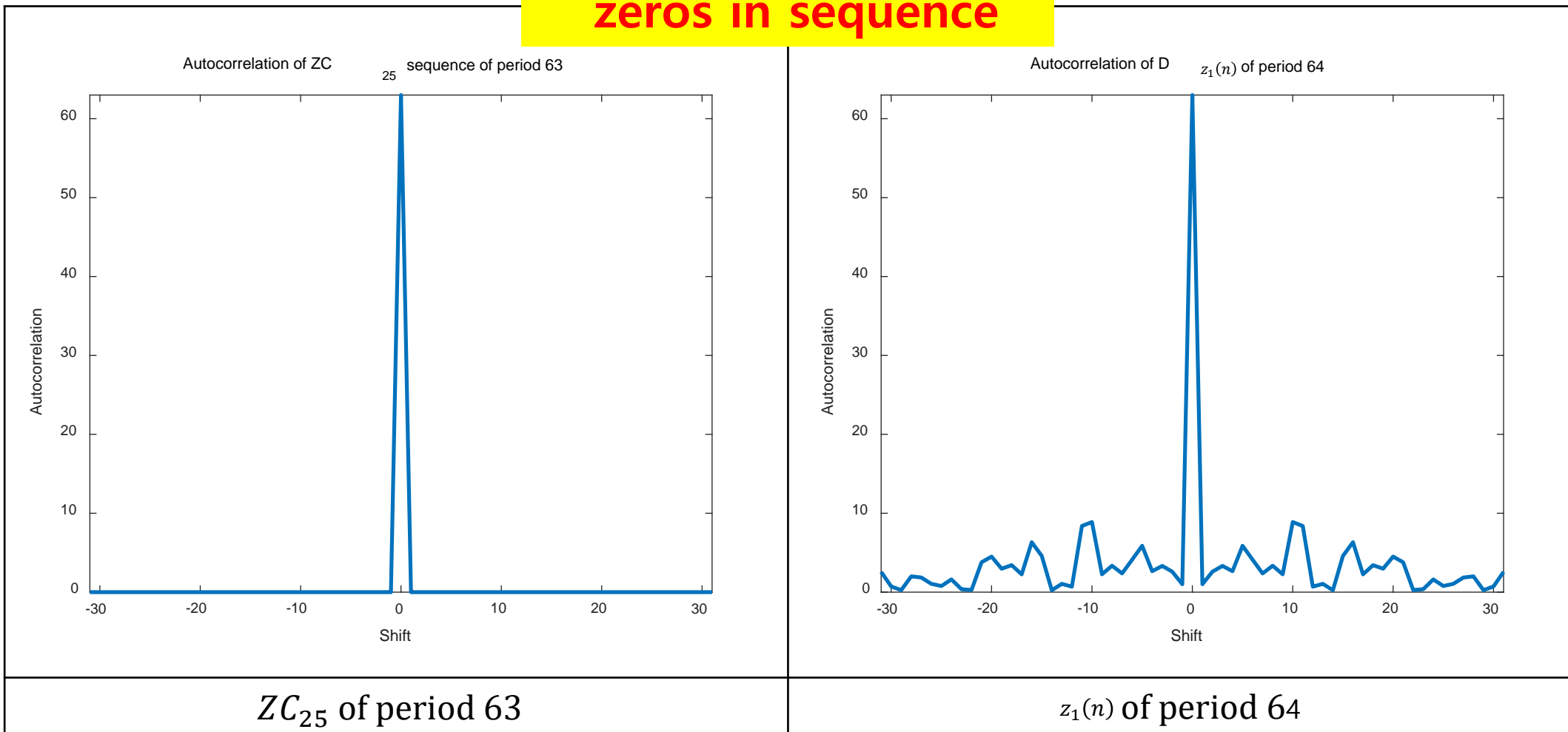
64번째에 0을 삽입

# Construction of $z_1$



# Construction of $z_1$

Because of the some zeros in sequence





# Properties of $z_1$ (4G PSS)



- 길이가 63인 Zadoff-Chu sequence를 사용
- 길이가 64 → 64 - FFT
- 32번째에 0위치 → DC subcarrier
- 대칭성 → 빠른 Decoding을 위해서

$$z_1(n) = z_1(62 - n) \quad \text{for } 0 \leq n \leq 30$$



# Properties of $z_1$ (4G PSS)



- 길이가 63인 Zadoff-Chu sequence를 사용
- 길이가 64 → 64 - FFT
- 32번째에 0위치 → DC subcarrier
- 대칭성 → 빠른 Decoding을 위해서

$$z_1(n) = z_1(62 - n) \quad \text{for } 0 \leq n \leq 30$$





# Contents of View

- **Introduction**

- ✓ Correlations of the sequence
- ✓ Polyphase sequence
- ✓ Almost-polyphase sequence

- **Application of almost-polyphase sequence of length 64**

- ✓ PSS in Downlink Synchronization of 4G
- ✓ Construction of  $z_1$

- **New constructions**

- ✓ Construction of  $z_2$
- ✓ Construction of  $z_3$
- ✓ Construction of  $z_4$

- **Conclusion**

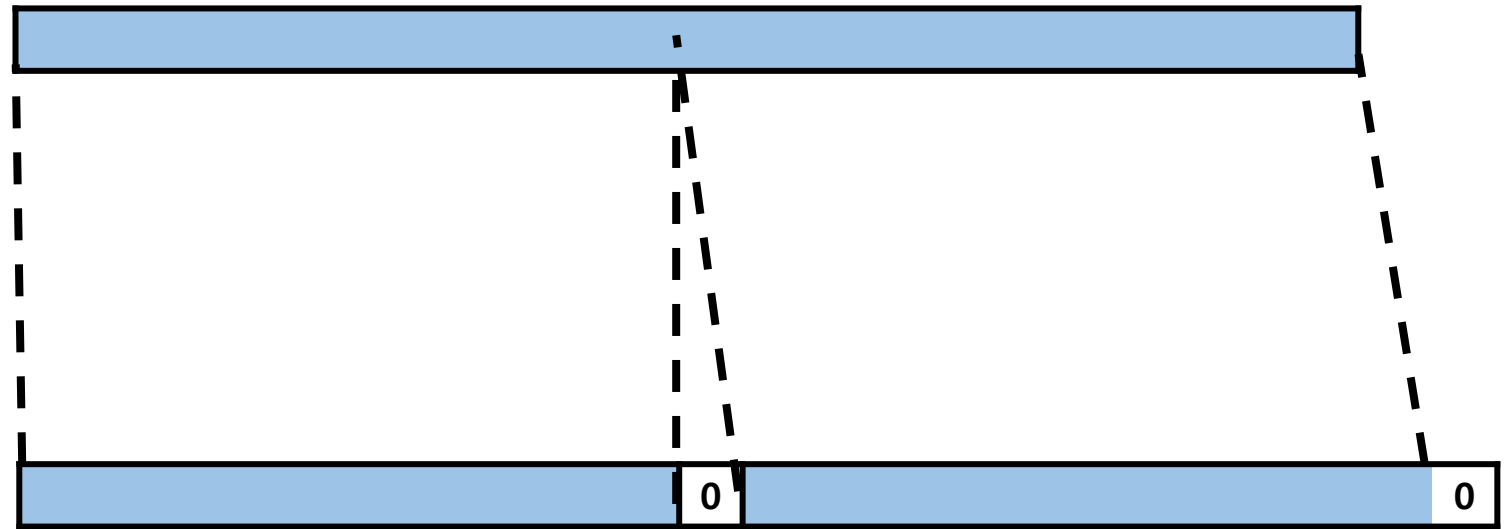
- ✓ Comparison



# Construction of $z_2$

Zadoff-Chu sequence  
of period 62

$z_2$  of period 64



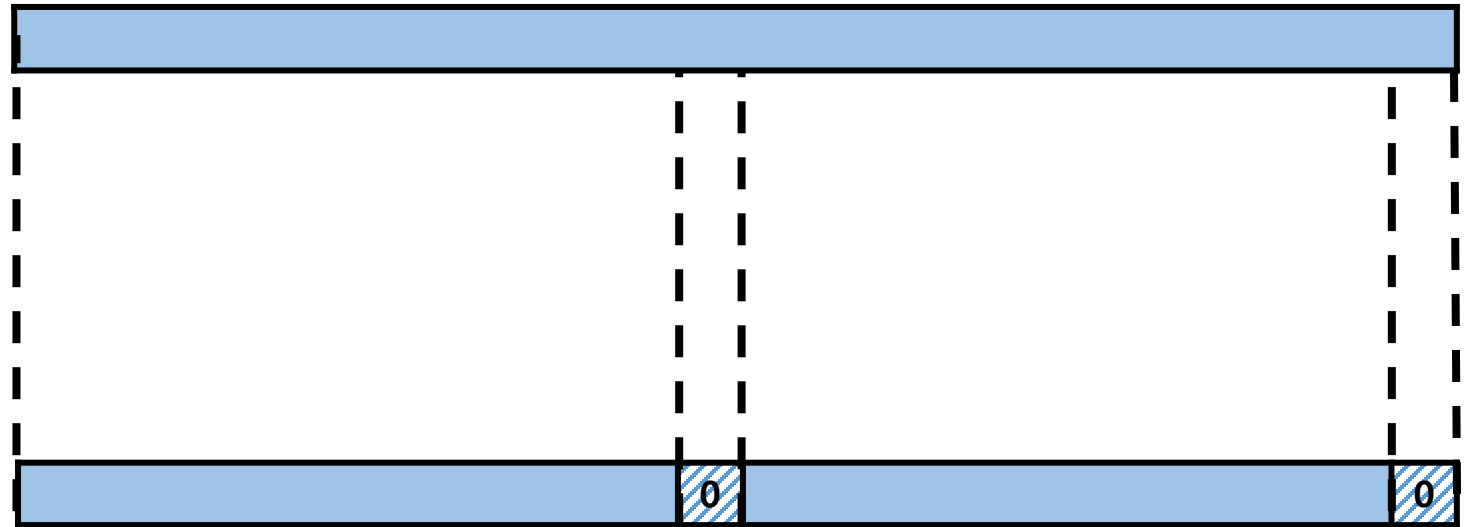
32번째에 0을 삽입

64번째에 0을 삽입

# Construction of $z_3$

Zadoff-Chu sequence  
of period 64

$z_3$  of period 64



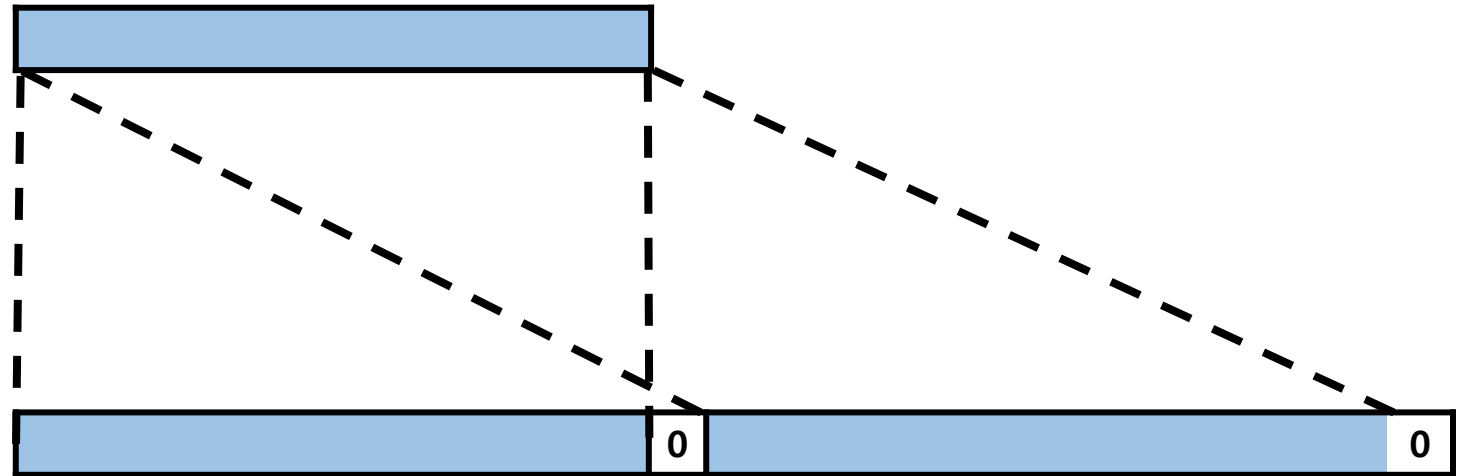
32번째를 0으로 대체

64번째를 0으로 대체

# Construction of $z_4$

Zadoff-Chu sequences  
of period 31

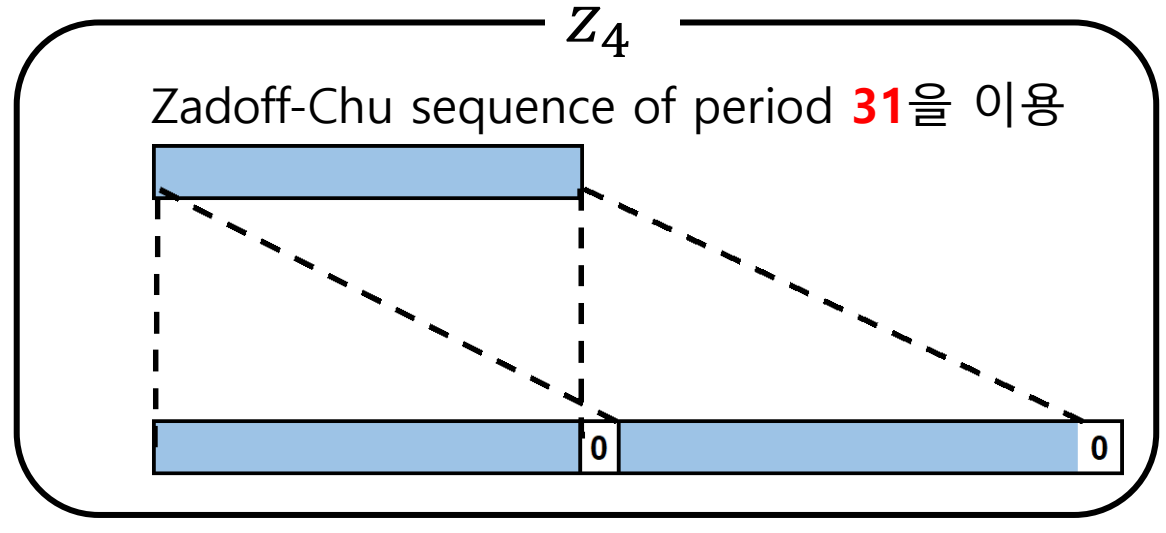
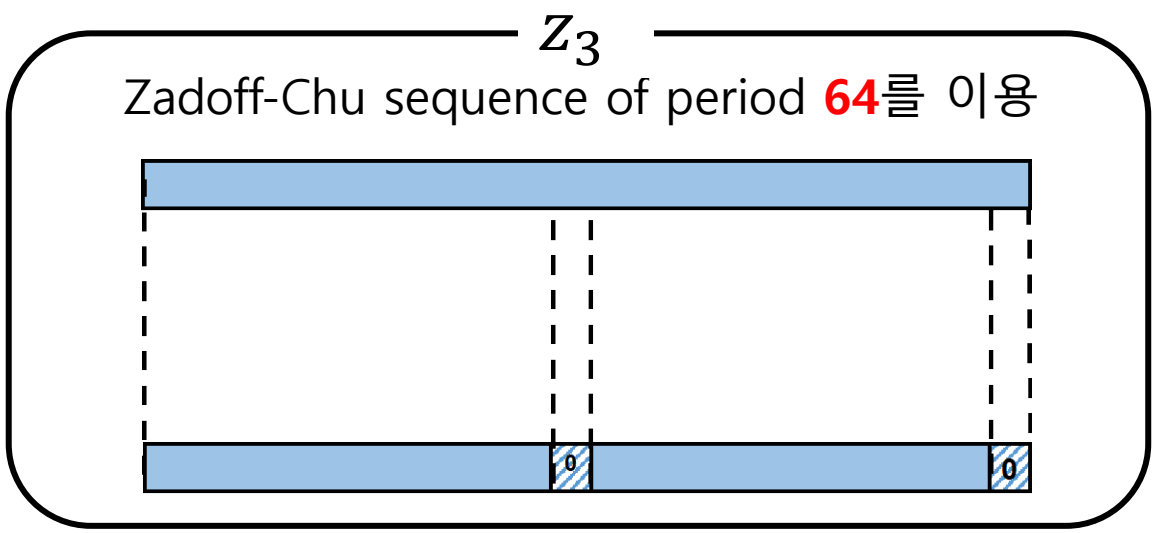
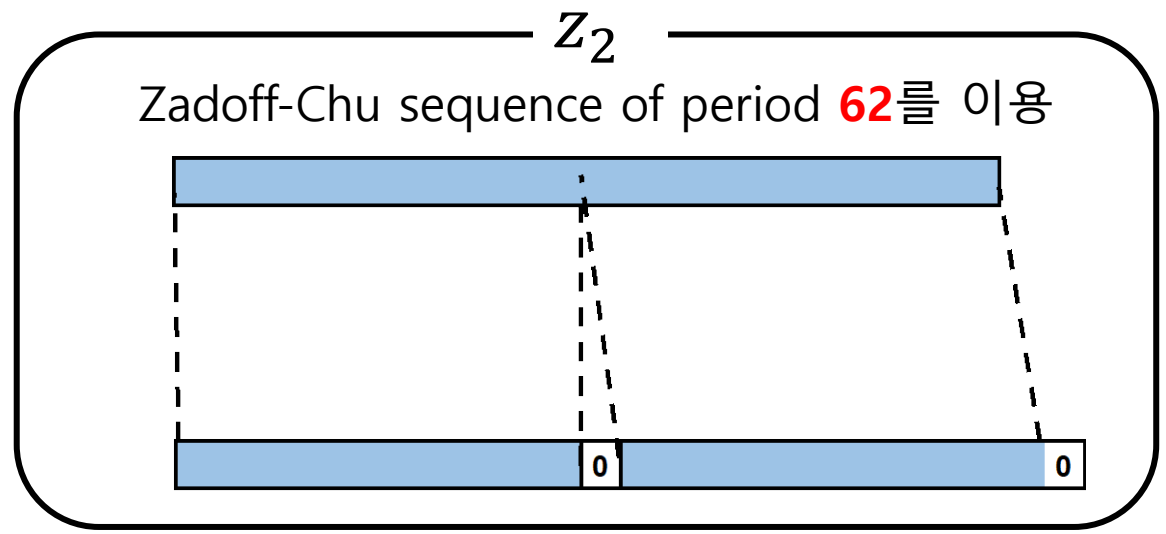
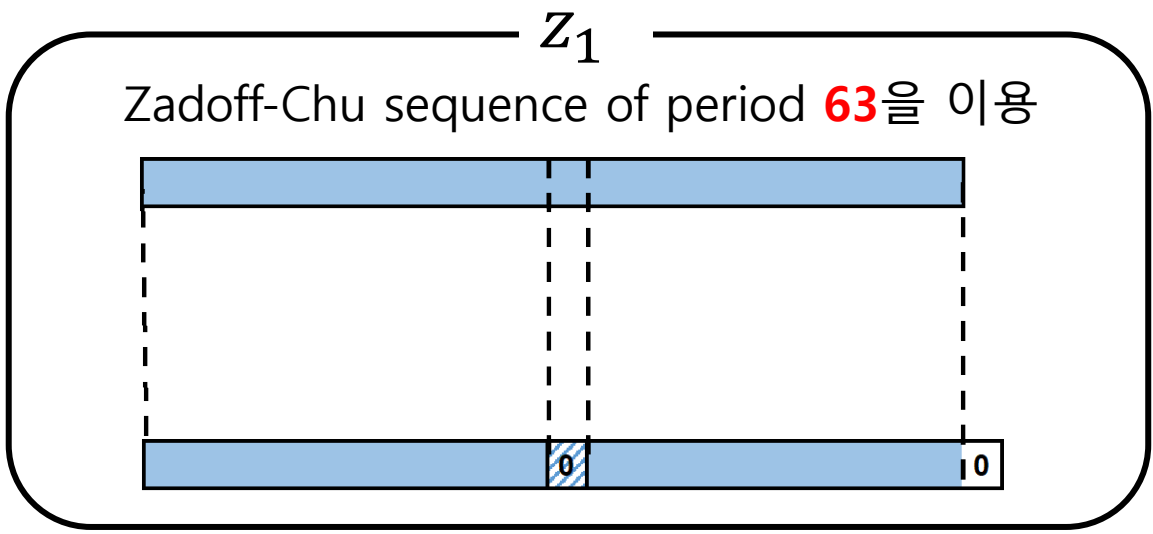
$z_4$  of period 64



32번째에 0을 삽입 후

두 번 반복

# Constructions of $z_1, z_2, z_3, z_4$





# Contents of View

- **Introduction**

- ✓ Correlations of the sequence
- ✓ Polyphase sequence
- ✓ Almost-polyphase sequence

- **Application of almost-polyphase sequence of length 64**

- ✓ PSS in Downlink Synchronization of 4G
- ✓ Construction of  $z_1$

- **New constructions**

- ✓ Construction of  $z_2$
- ✓ Construction of  $z_3$
- ✓ Construction of  $z_4$

- **Conclusion**

- ✓ Comparison

# Comparison

| Almost-polyphase sequence | PSLR(dB) |
|---------------------------|----------|
| $z_1$                     | -14.874  |
| $z_2$                     | -23.806  |
| $z_3$                     | -21.390  |
| $z_4$                     | -11.423  |

Comparison of PSLR of out-of-phase autocorrelation almost-polyphase sequences

# Comparison

| Almost-polyphase sequence | PSLR(dB)       |
|---------------------------|----------------|
| $z_1$                     | <b>-14.874</b> |
| $z_2$                     | -23.806        |
| $z_3$                     | -21.390        |
| $z_4$                     | -11.423        |

Comparison of PSLR of out-of-phase autocorrelation almost-polyphase sequences



| Almost-polyphase sequence | PSLR(dB)       |
|---------------------------|----------------|
| $z_1$                     | <b>-14.874</b> |
| $z_2$                     | -23.806        |
| $z_3$                     | -21.390        |
| $z_4$                     | -11.423        |

Comparison of PSLR of out-of-phase autocorrelation almost-polyphase sequences