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**부호율  $1/L$  길쌈부호를 이용한  
 $L$  레벨 공간 디버시티를 갖는  
Space-Time Trellis Code의 성능분석**

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# Introduction

## ➔ System beyond IMT-2000(4G)

- ❑ Information data transmission rate : 2M – 150Mbps
- ❑ BER: data  $10^{-6}$  , video  $10^{-9}$

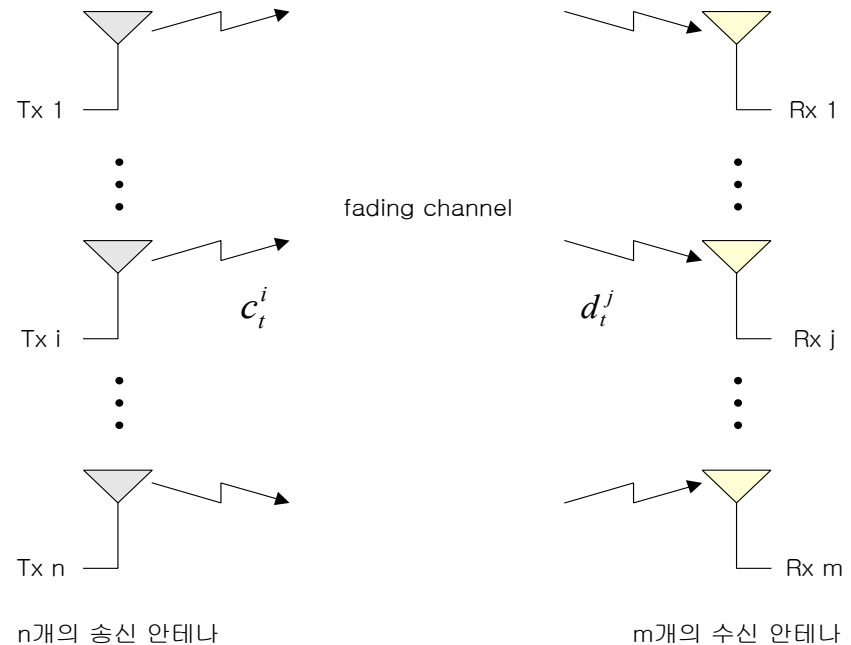
## ➔ Space-time Code is attractive for high data transmission.

- ❑ Jointly combining multiple antenna and channel coding without sacrificing the BW.
  - ⇒ improve data rate and reliability in wireless channel
  - ⇒ diversity gain using higher temporal and spatial dimensions
  - ⇒ achieve coding gain
  - ⇒ Increase channel capacity

# MIMO System

## MIMO System

$$\mathbf{c} = \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_l^1 \\ c_1^2 & c_2^2 & \cdots & c_l^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^n & c_2^n & \cdots & c_l^n \end{bmatrix}$$



## Fundamental Bound (Tarokh et. al)

– Quasi-static channel (frame:  $l$  symbols)

$$\Pr(\mathbf{c} \rightarrow \mathbf{e}) \leq \left( \frac{\eta E_s}{4N_0} \right)^{-rm}$$

$$r = \text{rank}(f(\mathbf{c}) - f(\mathbf{e}))$$

$$\eta = (\lambda_1 \lambda_2 \cdots \lambda_r)^{1/r}$$

# Design Criteria

## Rank Criterion

- Maximize the diversity advantage

$$r = \text{rank}(f(\mathbf{c}) - f(\mathbf{e}))$$

over all pairs of distinct codewords  $\mathbf{c}, \mathbf{e} \in \mathcal{C}$

## Determinant Criterion

- Maximize the coding advantage

$$\eta = (\lambda_1 \lambda_2 \cdots \lambda_r)^{1/r}$$

over all pairs of distinct codewords  $\mathbf{c}, \mathbf{e} \in \mathcal{C}$

Where  $\eta$  is the geometric mean of nonzero eigenvalues of

$$\mathbf{A} = (f(\mathbf{c}) - f(\mathbf{e}))(f(\mathbf{c}) - f(\mathbf{e}))^H$$

# Design Rules

## ↳ Tarokh's Simple Design Rule ( $n = 2$ )

- Rule1: Transitions departing from the same state differ in the second symbol
- Rule2: Transitions merging into the same state differ in the first symbol

⇒ Satisfying full rank for the case of  $n = 2$

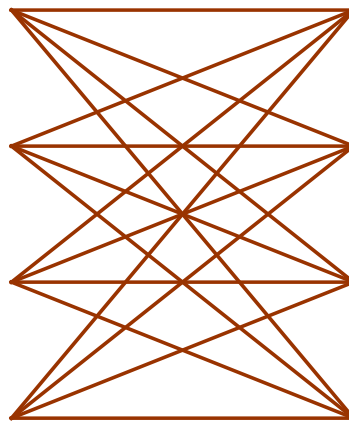
### ▶ Example : Delay Diversity

00 01 02 03

10 11 12 13

20 21 22 23

30 31 32 33



Input	0	1	3	2	1	0	3	1
Tx 1	0	0	1	3	2	1	0	3
Tx 2	0	1	3	2	1	0	3	1

## ➔ Grimm et.al : Zero Symmetry domain for full rank

- Extend the Tarokh's simple design rule to any n

$$\mathbf{G}_i = \begin{bmatrix} y_i(1) & x_i(2) & \cdots & x_i(5) & x_i(8) & 0 & 0 \\ 0 & y_i(2) & x_i(3) & \cdots & x_i(6) & y_i(3) & 0 \\ 0 & 0 & x_i(1) & x_i(4) & \cdots & x_i(7) & y_i(4) \end{bmatrix}$$

## ➔ Hammons' Design Rule

- Generally applicable to any number of transmit antenna
- BPSK criterion (for 1/L convolutional codes)
  - ➔ Example (3 transmit antenna, 8state)

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

# Generator Matrix

$L$	$v$	Grimm	$v$	Optimum $d_{free}$
2	1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	2	$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
	3	$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$	3	$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$
	6	$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$	6	$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$
3	2	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	3	$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$
	4	$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$	4	$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$
	6	$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$	6	$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$

$L$	$v$	Grimm	$v$	Optimum $d_{free}$
4	3	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	4	$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$
	6	$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$	5	$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$
5	4	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	5	$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$
	6	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$	7	$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$



# System Description

## Channel Model

- ❑ Quasi-static flat fading channel
- ❑ 1 frame = 130 symbols ( $l=130$ )
- ❑ No delay spread (No multipath)
- ❑ Complex multiplicative channel

$$\mathbf{r} = \mathbf{H}\mathbf{c} + \mathbf{n}$$

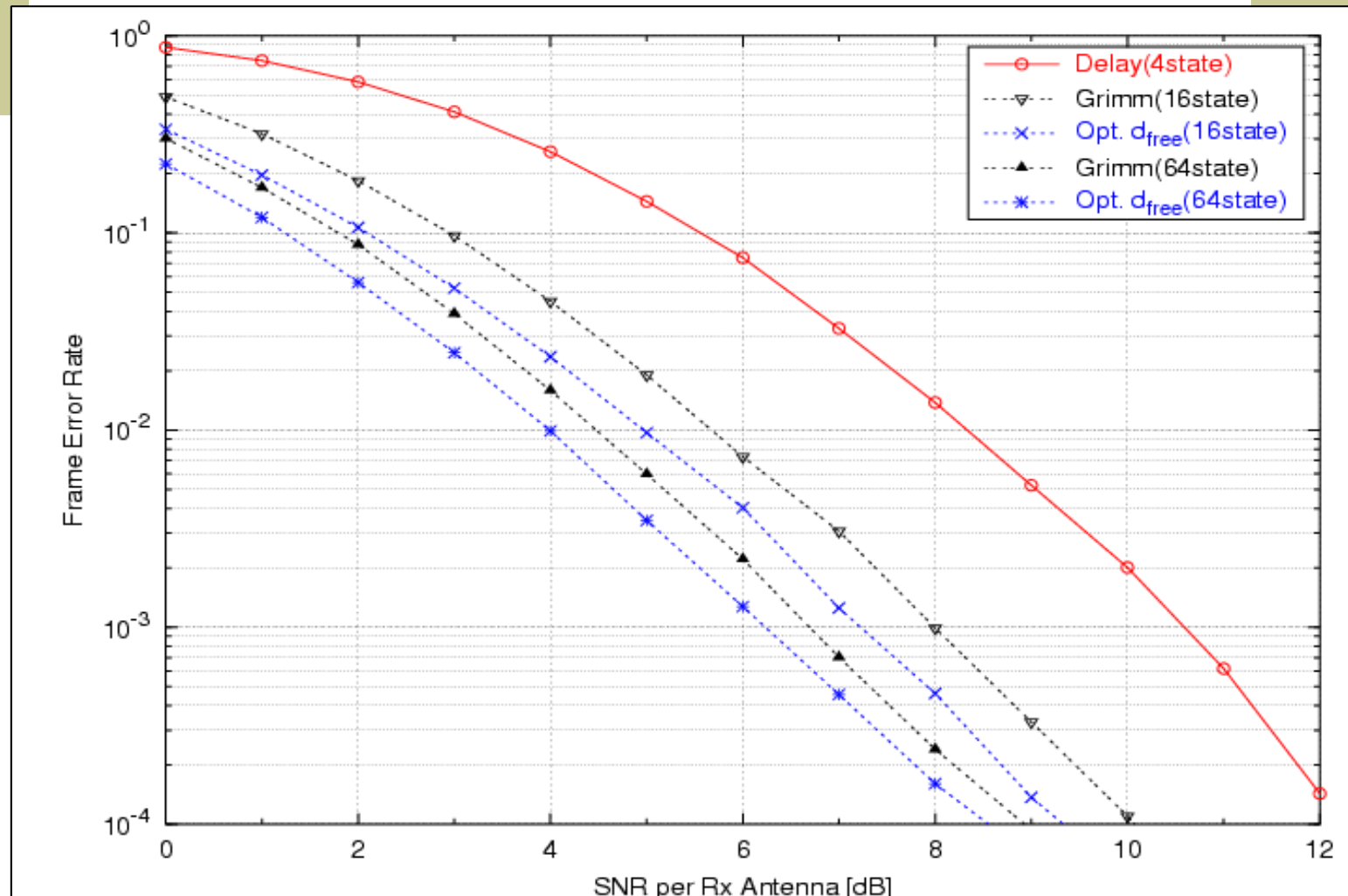
$$\mathbf{H} = [\alpha_{ij}] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix}$$

- ❑ Ideal channel estimation

## System model

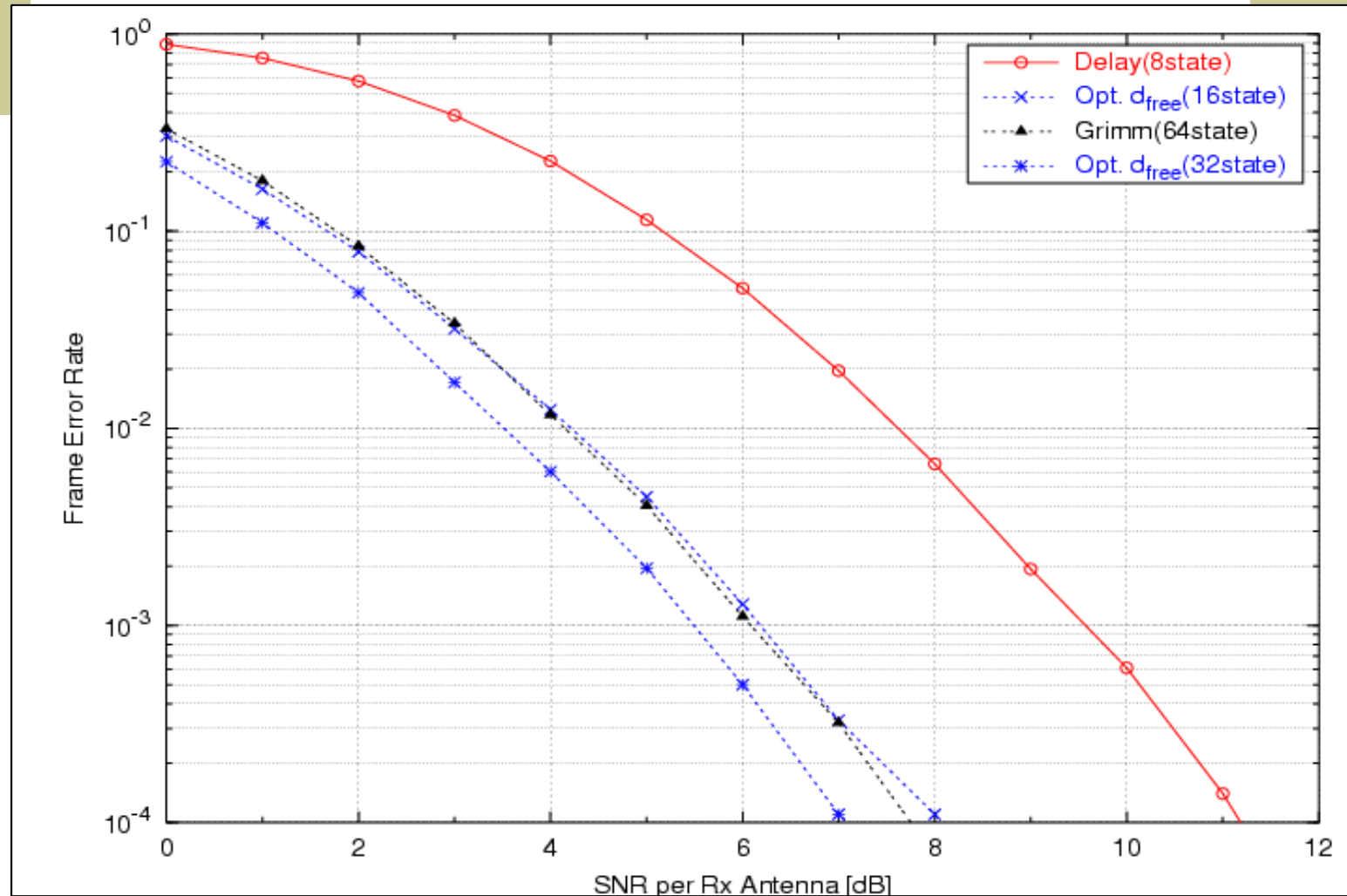
- ❑ BPSK modulation
- ❑ Tx=2-5, Rx=2
- ❑ Viterbi Decoder (ML)
- ❑ Unquantized Soft Decision

# Performance (Tx=3)



- 송신 안테나 3인 경우 송신 지연 디버시티, Grimm의 STTC, 최적의  $d_{free}$ 를 갖는 길쌈부호로부터 설계된 STTC의 성능비교

# Performance (Tx=4)



- 송신 안테나 4인 경우 송신 지연 디버시티, Grimm의 STTC, 최적의  $d_{free}$ 를 갖는 길쌈부호로부터 설계된 STTC의 성능비교

# Conclusion

## ➔ Tarokh et.al

- ❑ Geometrically Uniform Code
  - ➔ Reduced complexity in computing the coding gain
  - ➔ Generally not producing optimal codes
- ❑ 4PSK : 4,8,16,32 state, 8PSK : 8,16,32 state

## ➔ Grimm et.al

- ❑ Exhaustive Search for Zero Symmetry code
- ❑ BPSK : 2Tx, 3Tx, 4Tx, QPSK : 2Tx (4, 8 state)

## ➔ Hammons

- ❑ Use the Conventional Convolutional Codes of Rate  $1/L$
- ❑ Better Performance than Grimm's