일반화된 지연 디버시티 Space-Time Code의
트렐리스 구조 및 성능 분석

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Introduction

System beyond IMT-2000(4G)
- Information data transmission rate: 2M - 150Mbps
- BER: data $10^{-6}$, video $10^{-9}$

Space-time Code is attractive for high data transmission.
- Jointly combining multiple antenna and channel coding without sacrificing the BW.
  - Improve data rate and reliability in wireless channel
  - Diversity gain using higher temporal and spatial dimensions
  - Achieve coding gain
  - Increase channel capacity
MIMO System

\[
\begin{bmatrix}
C_1^1 & C_2^1 & \cdots & C_l^1 \\
C_1^2 & C_2^2 & \cdots & C_l^2 \\
\vdots & \vdots & \ddots & \vdots \\
C_1^n & C_2^n & \cdots & C_l^n 
\end{bmatrix}
\]

MIMO System

n개의 송신 안테나

m개의 수신 안테나

fading channel
Design Criteria

- Fundamental Bound (Tarokh et. Al)
  - Quasi-static channel (frame: \( l \) symbols)

\[
Pr(c \rightarrow e) \leq \left( \frac{\eta E_s}{4N_0} \right)^{-rm}
\]

- Rank Criterion
  - Maximize the diversity advantage

\[
r = \text{rank} \left( f(c) - f(e) \right)
\]

over all pairs of distinct codewords \( c, e \in C \)

- Determinant Criterion
  - Maximize the coding advantage \( \eta = \left( \lambda_1 \lambda_2 \cdots \lambda_r \right)^{1/r} \)

over all pairs of distinct codewords \( c, e \in C \)

Where \( \eta \) is the geometric mean of nonzero eigenvalues of

\[
A = \left( f(c) - f(e) \right) \left( f(c) - f(e) \right)^H
\]
Tarokh’s Simple Design Rule \((n = 2)\)

- Rule 1: Transitions departing from the same state differ in the second symbol
- Rule 2: Transitions merging into the same state differ in the first symbol

\[\Rightarrow \text{Satisfying full rank for the case of } n = 2\]

**Example: Delay Diversity**

\[
\begin{array}{ccccccccc}
00 & 01 & 02 & 03 \\
10 & 11 & 12 & 13 \\
20 & 21 & 22 & 23 \\
30 & 31 & 32 & 33 \\
\end{array}
\]

Input: 0 1 3 2 1 0 3 1
Tx 1: 0 0 1 3 2 1 0 3
Tx 2: 0 1 3 2 1 0 3 1
Delay Diversity

Delay Diversity = Repetition code + Delay element between multiple Tx Antennas
Delay Diversity (Baseline Performance)

FER Performance [QPSK, 2b/s/Hz]

Frame Error Rate (FER) vs. SNR per Receive Antenna [dB]

- Delay (2Tx,1Rx)
- Delay (3Tx,1Rx)
- Delay (4Tx,1Rx)
- Delay (2Tx,2Rx)
- Delay (3Tx,1Rx)
- Delay (4Tx,1Rx)
- Delay (2Tx,4Rx)
- Delay (3Tx,1Rx)
- Delay (4Tx,1Rx)
Product Distance Code

More Efficient Block Code than Repetition Code?
→ Opt. Product Distance Code
(Originally best in case of TCM, Divsalar 1988)

Definition: Product Distance

\[ D_{(p,q)} = \prod_{i=1}^{n} \left| f(a_p^{(i)}) - f(a_q^{(i)}) \right|^2 \]

\[
\begin{pmatrix}
0 & 0 & \cdots & 0 \\
1 & 1 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
M-1 & M-1 & \cdots & M-1
\end{pmatrix}
\]

\[
A = \begin{pmatrix}
A_0 \\
A_1 \\
\vdots \\
A_{M-1}
\end{pmatrix} = \begin{pmatrix}
a_0^{(1)} & a_0^{(2)} & \cdots & a_0^{(n)} \\
a_1^{(1)} & a_1^{(2)} & \cdots & a_1^{(n)} \\
\vdots & \vdots & \ddots & \vdots \\
a_{M-1}^{(1)} & a_{M-1}^{(2)} & \cdots & a_{M-1}^{(n)}
\end{pmatrix}
\]

M-ary Delay Diversity (Tx Ant: n)  M-ary Opt. PDC (Tx Ant: n)
**Product Distance Code**

**Example (QPSK, Tx Ant.: 3)**

- **Opt. Product Distance Code** {000, 112, 231, 323}

  - **Delay Diversity**
    - Code: {000, 111, 222, 333}
    - Minimum Dist. = 8
    - # of Minimum = 4
    - Avg. Dist. = 26.67

  - **Opt. PDC**
    - Code: {000, 112, 231, 323}
    - Minimum Dist. = 16
    - # of Minimum = 6
    - Avg. Dist. = 16.00
Generalized Delay Diversity Code

Transmitted Signal with Opt. PDC

Information Source → Optimum Product Distance code → D → D^{n-1} → Constellation Mapper

Signal 1 → Signal n

Constellation Mapper

Signal n-1
### Generalized Delay Diversity Code

#### Example: QPSK, Tx Ant. = 3

<table>
<thead>
<tr>
<th>Block Code</th>
<th>Delay Diversity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>Input 0 1 3 2 3 0 2 1 1 2</td>
</tr>
<tr>
<td>1 1 1</td>
<td>Tx 1 0 0 0 1 3 2 3 0 2 1</td>
</tr>
<tr>
<td>2 2 2</td>
<td>Tx 2 0 0 1 3 2 3 0 2 1 1</td>
</tr>
<tr>
<td>3 3 3</td>
<td>Tx 3 0 1 3 2 3 0 2 1 1 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Block Code</th>
<th>Delay Diversity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>Input 0 1 3 2 3 0 2 1 1 2</td>
</tr>
<tr>
<td>1 1 2</td>
<td>Tx 1 0 0 0 2 3 1 3 0 1 2</td>
</tr>
<tr>
<td>2 3 1</td>
<td>Tx 2 0 0 1 2 3 2 0 3 1 1</td>
</tr>
<tr>
<td>3 2 3</td>
<td>Tx 3 0 1 3 2 3 0 2 1 1 2</td>
</tr>
</tbody>
</table>
Work Procedure

1. STTC Parameters
2. Find Opt. PDC
3. Set Up Trellis Structure
4. Performance Simulation

- Tx Ant: 2, 3, 4, …
- Modulation: PSK, QAM
- Search Algorithm
- General Rule
- 130 symbol frame (IS-54)
- Delay Diversity vs Opt. PDC
Given Opt. PDC (M-ary, n Tx Ant)

\[
\begin{pmatrix}
0 & 0 & \cdots & 0 \\
1 & 1 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
M-1 & M-1 & \cdots & M-1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
a_0^{(1)} & a_0^{(2)} & \cdots & a_0^{(n)} \\
a_1^{(1)} & a_1^{(2)} & \cdots & a_1^{(n)} \\
\vdots & \vdots & \ddots & \vdots \\
a_{M-1}^{(1)} & a_{M-1}^{(2)} & \cdots & a_{M-1}^{(n)}
\end{pmatrix}
\]

Trellis Structure

Current Input: \( I \)

Transition Output

\[
\begin{pmatrix}
f(a_{S_1}^{(n)}) \\
f(a_{S_2}^{(n-1)}) \\
\vdots \\
f(a_{S_{M-1}}^{(1)})
\end{pmatrix}
\]
**Definition**: $N_{pd}$

- The number of minimum product distance of a given product distance code.

**Example (QPSK, Tx Ant.: 2)**

- $3!$ Cases: all such codes have $D_{min} = 4$.

<table>
<thead>
<tr>
<th></th>
<th>0 0</th>
<th>0 0</th>
<th>0 0</th>
<th>0 0</th>
<th>0 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1</td>
<td>1 1</td>
<td>1 1</td>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
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<tr>
<td>2 2</td>
<td>2 3</td>
<td>2 1</td>
<td>2 3</td>
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<td>3 3</td>
<td>3 2</td>
<td>3 3</td>
<td>3 1</td>
<td>3 1</td>
<td>3 1</td>
</tr>
</tbody>
</table>

\[ N_{pd} = 4 \quad \text{and} \quad N_{pd} = 2 \]
Minimum Product Distance of Delay Diversity

<table>
<thead>
<tr>
<th></th>
<th>QPSK</th>
<th>8PSK</th>
<th>16QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_{\min}$</td>
<td>$N_{pd}$</td>
<td>$D_{\min}$</td>
</tr>
<tr>
<td>Tx</td>
<td></td>
<td></td>
<td>Tx</td>
</tr>
<tr>
<td>2</td>
<td>$2^2$</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$2^3$</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>$2^4$</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>$2^5$</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
### Search Results (QPSK Opt. PDC)

#### QPSK

<table>
<thead>
<tr>
<th>Tx</th>
<th>PDC</th>
<th>$D_{opt}$</th>
<th>$N_{pd}$</th>
<th>$D_{avg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0 1 2 3, 0 1 3 2</td>
<td>4</td>
<td>2</td>
<td>6.667</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3, 0 1 2 3</td>
<td></td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>0 1 2 3, 0 1 3 2, 0 2 1 3</td>
<td>16</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>0 1 2 3, 0 1 2 3, 0 1 3 2, 0 2 1 3</td>
<td>32</td>
<td>4</td>
<td>42.667</td>
</tr>
<tr>
<td>5</td>
<td>0 1 2 3, 0 1 2 3, 0 1 3 2, 0 2 1 3</td>
<td>64</td>
<td>2</td>
<td>106.667</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3, 0 1 2 3, 0 1 3 2, 0 2 1 3</td>
<td></td>
<td>4</td>
<td>128</td>
</tr>
</tbody>
</table>
### Search Results (8PSK, 16QAM)

#### 8PSK

<table>
<thead>
<tr>
<th>Tx</th>
<th>PDC</th>
<th>( D_{opt} )</th>
<th>( N_{pd} )</th>
<th>( D_{avg} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0 1 2 3 4 5 6 7</td>
<td>0 3 6 1 4 7 2 5</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td>0 2 5 7 3 1 6 4</td>
<td>12</td>
<td>6.28571</td>
</tr>
<tr>
<td></td>
<td>0 3 7 4 1 6 2 5</td>
<td>0 3 4 7 1 6 3 5</td>
<td>18</td>
<td>8.28571</td>
</tr>
<tr>
<td>3</td>
<td>0 1 2 3 4 5 6 7</td>
<td>0 3 6 2 5 7 1 4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td>0 3 2 5 1 7 6 4</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>0 1 2 3 4 5 6 7</td>
<td>0 3 7 4 1 6 2 5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td>0 3 2 5 1 7 6 4</td>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

#### 16QAM

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15</td>
<td>0 6 13 11 9 15 4 2 7 11 0 12 14 8 3 5</td>
<td>64</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15</td>
<td>0 2 4 6 8 10 12 14 1 3 5 7 9 11 13 15</td>
<td>33</td>
<td>494.933</td>
</tr>
</tbody>
</table>
**System Description**

**Channel Model**
- Quasi-static flat fading channel
- 1 frame = 130 symbols ($l=130$)
- No delay spread (No multipath)
- Complex multiplicative channel

\[ r = Hc + n \]

- Ideal channel estimation

**System Model**
- Modulation: QPSK, 8PSK, 16QAM
- Viterbi Decoder (ML)
- Unquantized Soft Decision
Delay VS Opt. PDC [QPSK, 2Tx]

FER Performance [QPSK, 2Tx, 4State, 2b/s/Hz]

- Frame Error Rate (FER)
- SNR per Receive Antenna [dB]
- Delay (1Rx)
- Pdt opt (1Rx)
- Delay (2Rx)
- Pdt opt (2Rx)
- Delay (4Rx)
- Pdt opt (4Rx)
Delay VS Opt. PDC [QPSK, 3Tx] 

FER Performance [QPSK, 3Tx, 16State, 2b/s/Hz] 

Frame Error Rate (FER) 

SNR per Receive Antenna [dB] 

Delay (1Rx) 
Pdt_{opt} (1Rx) 
Delay (2Rx) 
Pdt_{opt} (2Rx) 
Delay (4Rx) 
Pdt_{opt} (4Rx) 

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Delay VS Opt. PDC [QPSK, 4Tx]

Frame Error Rate (FER) Performance [QPSK, 4Tx, 64State, 2b/s/Hz]

- Delay (1Rx)
- $P_{dt_{opt}}$ (1Rx)
- Delay (2Rx)
- $P_{dt_{opt}}$ (2Rx)
- Delay (4Rx)
- $P_{dt_{opt}}$ (4Rx)

SNR per Receive Antenna [dB]

0 2 4 6 8 10 12 14 16 18 20

10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4}
FER Performance [8PSK, 2Tx, 8State, 3b/s/Hz]

Frame Error Rate (FER) vs. SNR per Receive Antenna (dB)

- Delay (1Rx)
- Pdt_{opt} (1Rx)
- Delay (2Rx)
- Pdt_{opt} (2Rx)
- Delay (4Rx)
- Pdt_{opt} (4Rx)
Delay VS Opt. PDC [8PSK, 3Tx]

Frame Error Rate (FER) vs SNR per Receive Antenna [dB]

FER Performance [8PSK, 3Tx, 64State, 3b/s/Hz]

- Delay(1RX)
- Pdt.N_{max} (1RX)
- Pdt.N_{min} (1RX)
- Delay(2RX)
- Pdt.N_{max} (2RX)
- Pdt.N_{min} (2RX)
- Delay(4RX)
- Pdt.N_{max} (4RX)
- Pdt.N_{min} (4RX)
Delay VS Opt. PDC [16QAM, 2Tx]

FER Performance [16QAM, 2Tx, 16State, 4b/s/Hz]

Frame Error Rate (FER)

SNR per Receive Antenna [dB]
Systematic Design (Construction)
- From the Optimum Product Distance Codes
  - Originally known to best in case of TCM
- Applicable to the case of the number of Tx Ant. is 2,3,4…

Search Optimal Product Distance Code
- 4PSK, 8PSK, 16QAM

Set up Trellis structure from Opt. PDC
- For ML decoding

Performance Simulation
- Comparison with Delay Diversity (←Baseline Performance)
- Analysis

Conclusion