Sequence Design Example for Ultra-Wideband Impulse Radio Using Maximal Length Sequences

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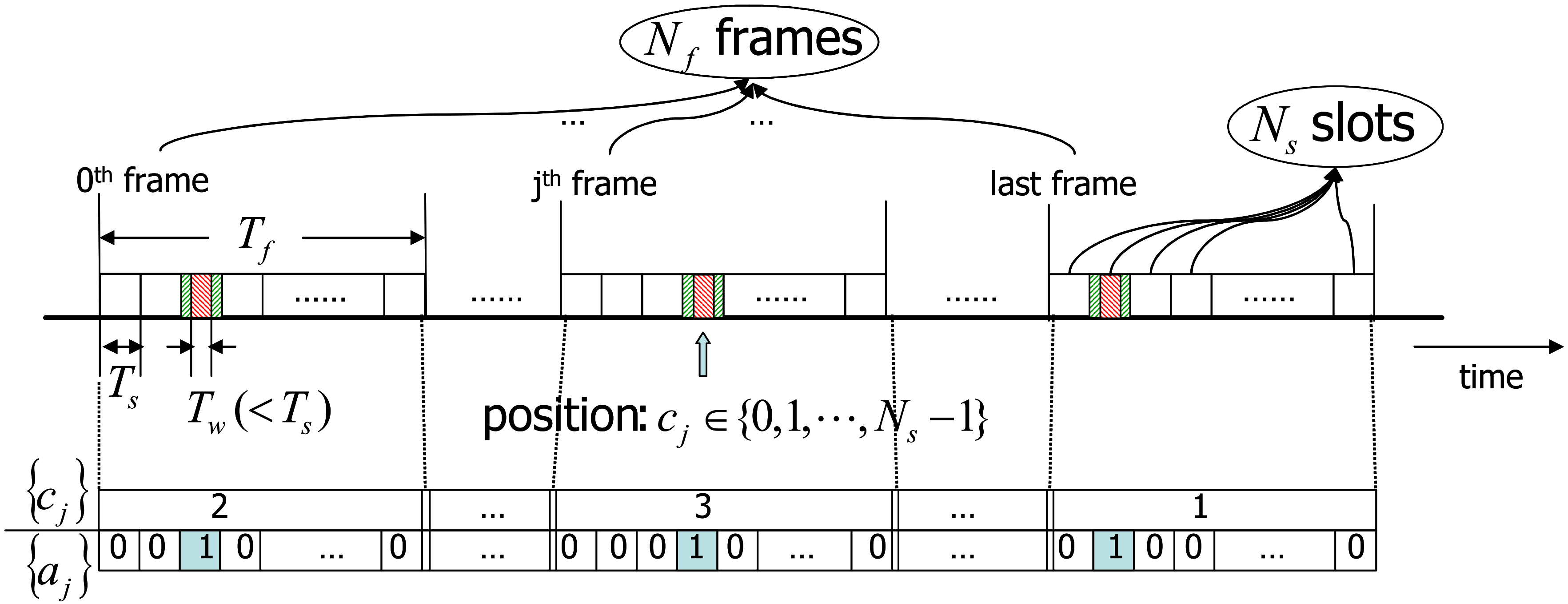
In this presentation ...

- General **signal model** for sequence designs suitable for UWB signals using TH-SSMA (Time Hopping Spread Spectrum Multiple Access)
- Sequence **construction**
- Analysis
  - **Correlation** measure
  - **Power Spectral Density** (PSD) Consideration

Signal Model

Time-Hopping Signal Models — (1)



【Fig. 1】Illustration of UWB Time-Hopping (Impluse Radio) signal models

T_s : slot width
N_s : number of slots within one frame
T_f : frame duration $(T_f = T_s \cdot N_s)$
N_f : period of integer time-hopping code
T_f : pulse width $(T_w < T_s)$

Time-Hopping Signal Models — (2)

- i th (unmodulated) TH-UWB signal

$$\boxed{s^{(i)}(t)=\sum_j p(t-jT_f-c_j^{(i)}T_s)=\sum_n a_n^{(i)}p(t-nT_s)}$$

$\{c_j^{(i)}\}_{j=0}^{N_f-1}$: N_s - ary integer TH code of period N_f

$$\{a_n^{(i)}\}_{j=0}^{N_sN_f-1}=\begin{cases}1, & \text{if } \exists j\in Z \text{ s.t } n=jN_s+c_j^{(i)}\\ 0, & O.W\end{cases}$$

: binary representation of $c_j^{(i)}$ of period N_sN_f

- Normalized periodic crosscorrelation between user i and user k

$$\boxed{\Lambda_{i,k}(n_\tau T_s)=\frac{1}{N_f}\sum_{n=0}^{N_sN_f-1}a_n^{(i)}a_{n\oplus n_\tau}^{(k)}}$$

(c.f. divided by N_f, NOT N_sN_f)

Impulse **R**adio **S**equence (**IRS**)– definition

- $\mathbb{C}$: $(N_s, N_f, \lambda_a, \lambda_c)$-IRS

$\Leftrightarrow$ $\mathbb{C}$: *a family of* $(0,1)$*-sequences of* *length* N_sN_f *and* *weight* N_f *such that*

1) **Pulse position** property: *for* $\forall x = \{x_t\}_{t=0}^{N_sN_f-1} \in \mathbb{C}$,
   - N_sN_f *slots are uniformly divided into* N_f *frames*
   - *Each frame has a unique pulse position among its* N_s *slots*
   - $supp(x) = \{c_j + jN_s \mid j = 0, 1, \cdots, N_f - 1, c_j \in \mathbb{Z}_{N_s}\}$

2) **Autocorrelation** property: for $\forall x = \{x_t\}_{t=0}^{N_sN_f-1} \in \mathbb{C}$ and $\forall \tau \not\equiv 0 \pmod{N_sN_f}$,

$$\sum_{t=0}^{N_sN_f-1} x_t x_{t \oplus \tau} \leqq \lambda_a$$

3) **Crosscorrelation** property: for $\forall x = \{x_t\}_{t=0}^{N_sN_f-1} \in \mathbb{C}$, $\forall y = \{y_t\}_{t=0}^{N_sN_f-1} \in \mathbb{C}$, $x \neq y$, $\forall \tau$,

$$\sum_{t=0}^{N_sN_f-1} x_t y_{t \oplus \tau} \leqq \lambda_c$$

Optimality in view of **Correlation**

- For given parameters $N_s, N_f, \lambda_a, \lambda_c$
  - $\Phi(N_s, N_f, \lambda_a, \lambda_c)$ = the maximum cardinality of $(N_s, N_f, \lambda_a, \lambda_c) - IRS$

- Exact value of Φ is difficult to determine

  $\Rightarrow$ Need some bound (Johnson bound)

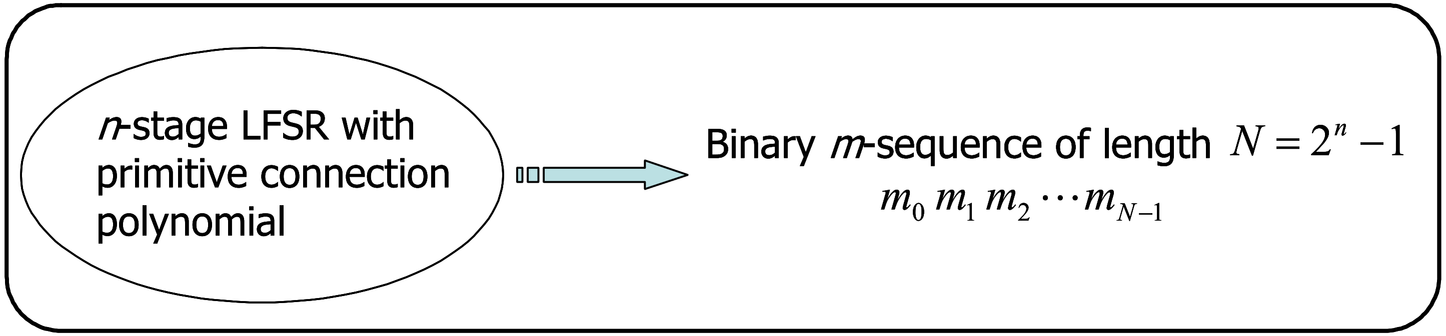
- Johnson Bound
  - **Upper** bound for $A(n, d; w)$
  - $A(n, d; w)$: *maximum # of bin. vectors of length* n, *minimum distance* $d_{\min} = d$, *weight* w
  - $A(n, 2\delta; w) \leqq \left\lfloor \frac{n}{w} \left\lfloor \frac{n-1}{w-1} \cdots \left\lfloor \frac{n-(w-\delta)}{\delta} \right\rfloor \cdots \right\rfloor \right\rfloor$
  - When $\lambda_a = \lambda_c = \lambda$:

$$\boxed{\Phi(N_s, N_f, \lambda) \leq \left\lfloor \frac{1}{N_f} \left\lfloor \frac{N_s N_f - 1}{N_f - 1} \cdots \left\lfloor \frac{N_s N_f - \lambda}{N_f - \lambda} \right\rfloor \cdots \right\rfloor \right\rfloor}$$

Sequence Construction

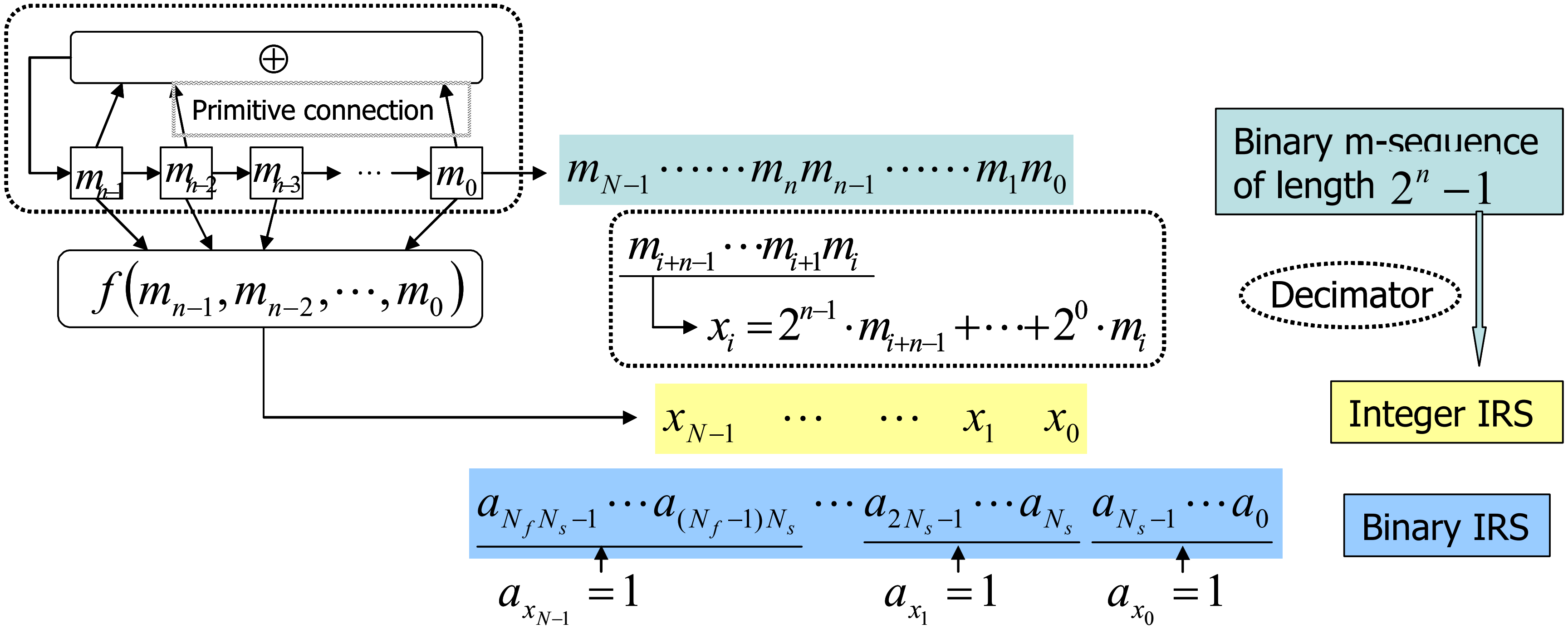
Maximal Length Sequences (*m*-sequence)

- Maximal length LFSR (Linear Feedback Shift Register) sequence (***m*-sequence**)



【Fig. 2】 Generator of binary maximal length sequences

- Properties of *m*-sequence
  - *Maximal length* (period): $N=2^n-1$
  - ***Ideal*** *(conventional)* ***autocorrelation***: $R(\tau)=\begin{cases}-1, & \tau \not\equiv 0 \pmod N \\ N, & \tau \equiv 0 \pmod N\end{cases}$
  - ***Span*** *property*: N consecutive n-bits are all distinct, $m_i \cdots m_{i+n-1} \neq m_j \cdots m_{j+n-1}$
  - ***Run*** *property*, etc.
  - Number of (cyclically) *distinct* m-sequence: $\boxed{M_n = \dfrac{\phi(2^n-1)}{n}}$ ($\phi(\cdot)$ is Euler phi-function)



【Fig. 3】 Construction of proposed impulse radio sequences (Type I)

- There are $M_n = \phi(2^n - 1)/n$ cyclically distinct m-sequences.
- $(N_s, N_f, \lambda_a, \lambda_c) = \begin{cases} (2^n, 2^n - 1, \lambda_a, \lambda_c) & , Type\ \ I \\ (2^n - 1, 2^n - 1, \lambda_a, \lambda_c) & , Type\ \ II \end{cases}$ -IRS with M_n-sequences

  (Type II; $x_i \neq 0$ for all $i = 0, 1, \cdots, N_f - 1$ by **span** property)

Analysis

Parameters and Correlation Property

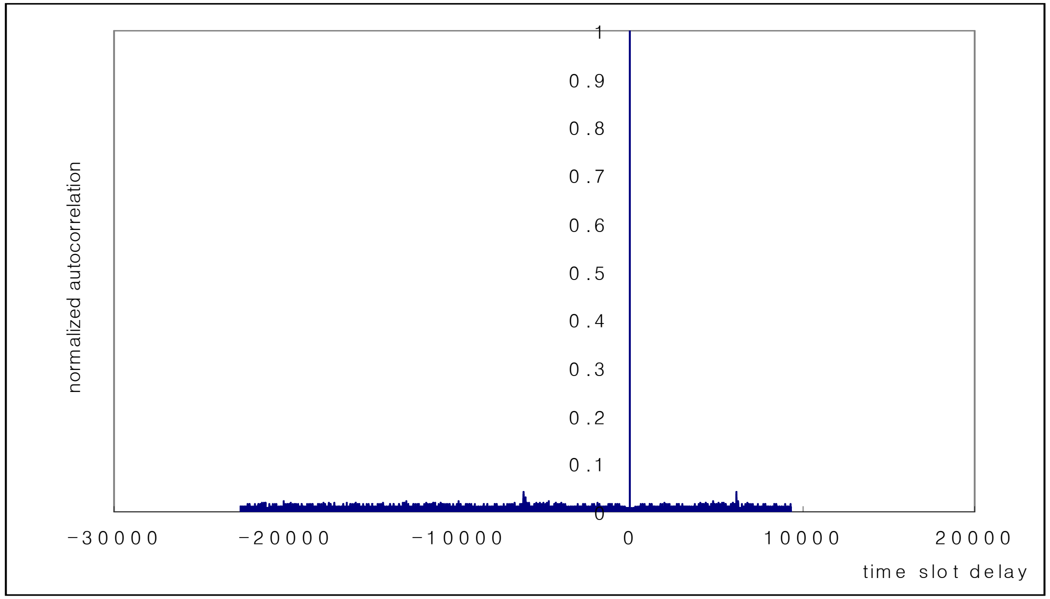
[Table 1] Parameters of proposed IRSs

n		3	4	5	6	7	8	9	10
Number of sequences		2	2	6	6	18	16	48	60
(N_s, N_f)	[3] with $l = n - 3$	N/A	N/A	(4, 15)	(8, 21)	(16, 31)	(32, 51)	(64, 85)	**(128, 146)**
	Type I	(8,7)	(16, 15)	(32, 31)	(64, 63)	**(128, 127)**	(256, 255)	(512, 511)	(1024, 1023)
	Type II	(7, 7)	(15, 15)	(31, 31)	(63, 63)	**(127, 127)**	(255, 255)	(511, 511)	(1023, 1023)
Period of Binary IRS: $(N_s \cdot N_f)$	[3]	N/A	N/A	60	168	496	1,632	5,440	18,688
	Type I	56	240	992	4032	16,256	65,280	261,632	1,047,552
	Type II	49	225	961	3969	16,129	65,025	261,121	1,046,529

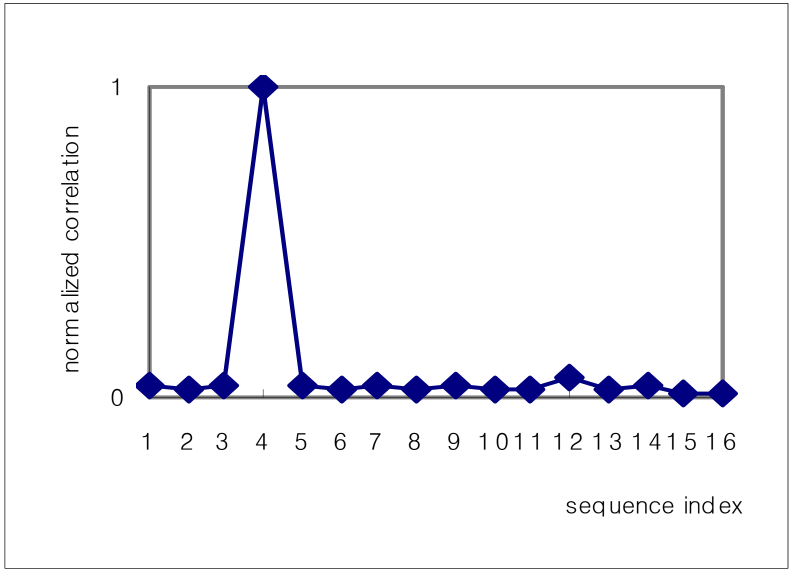
[Table 2] Correlation Properties of proposed IRS (Type I)

n		3	4	5	6	7	8	9	10
Autocorrelation (λ_a)	unnormalized $\max_i \max_{\tau \neq 0} R_{ii}(\tau)$	3	4	5	6	8	11	16	20
	normalized $\max_i \max_{\tau \neq 0} \widetilde{R_{ii}}(\tau)$	0.429	0.267	0.161	0.095	0.063	0.043	0.031	0.020
Crosscorrelation (λ_c)	unnormalized $\max_{i \neq j} \max_\tau R_{ij}(\tau)$	3	4	6	8	10	15	19	23
	normalized $\max_{i \neq j} \max_\tau \widetilde{R_{ij}}(\tau)$	0.429	0.267	0.193	0.127	0.079	0.059	0.037	0.022

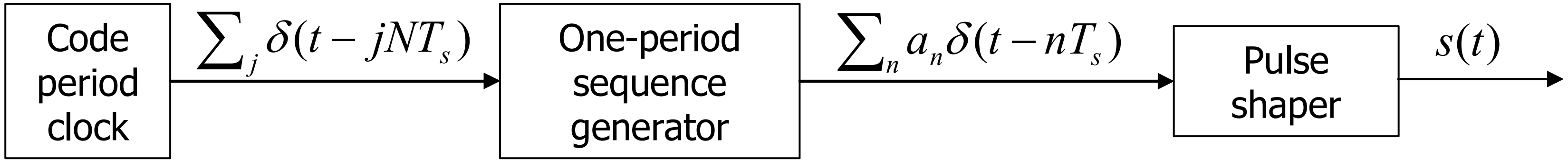
- $n = 8$: $(256, 255, 11, 15)$-IRS $\mathbb{C} = \{X^{(1)}, X^{(2)}, \cdots, X^{(16)}\}$
  - $\lambda_a = \max_i \max_{\tau \neq 0} R_{i,i}(\tau)$
  - $\lambda_c = \max_{i \neq j} \max_\tau R_{i,j}(\tau)$



【Fig. 4】 Autocorrelation profile of $X^{(4)}$ by type I construction (n=8)



【Fig. 5】 Maximum normalized crosscorrelation between $X^{(4)}$ and other sequences

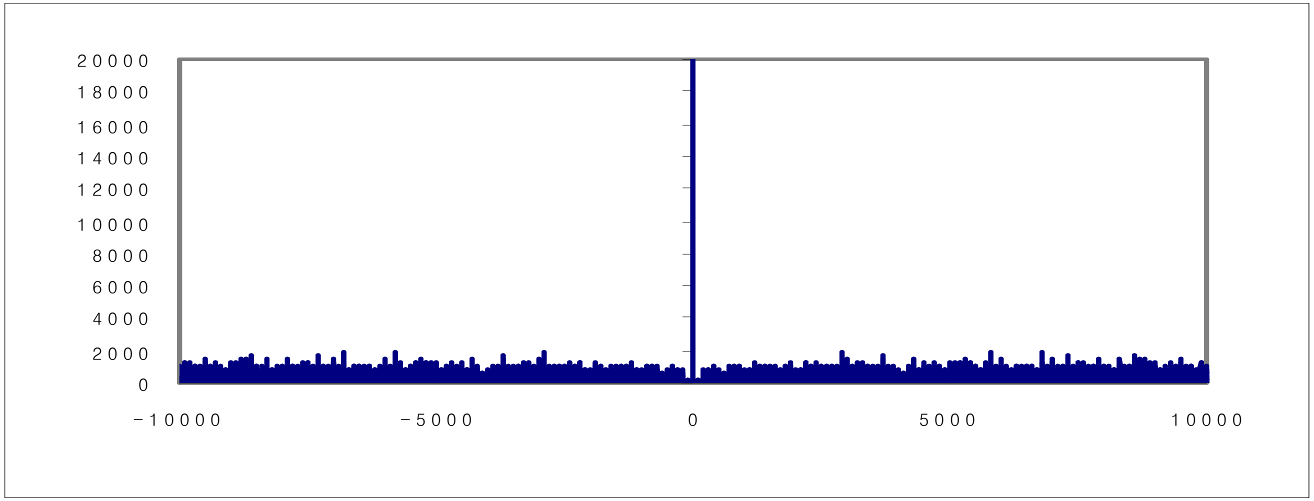


【Fig. 6】 Mathematical Model of a UWB signal generator without data modulation

- PSD calculation
  - UWB carrier in simple form: $s(t) = \sum_n a_n p(t - nT_s)$ (N=the period of $\{a_n\}$, $N = N_s N_f$)
  - $h_{op}(t) = \sum_{n=0}^{N-1} a_n \delta(t - nT_s)$, $H_{op}(f) = FT\{h_{op}(t)\} = \sum_{n=0}^{N-1} a_n e^{-j2\pi f n T_s}$
  - $PSD_{cpc}(f) = \frac{1}{(NT_s)^2} \sum_k \delta\left(f - \frac{k}{NT_s}\right)$
  - $PSD_s(f) = |P(f)H_{op}(f)|^2 PSD_{cpc}(f) = \frac{|P(f)|^2}{(NT_s)^2} \sum_k C_k \delta\left(f - \frac{k}{NT_s}\right)$
  - $\boxed{C_k = \left| \sum_{n=0}^{N-1} a_n e^{-j2\pi kn/N} \right|^2}$: **the effect of code design**

Power **S**pectral **D**ensity (**PSD**) – (2)

- Spectral flatness (*Cancelation of* ***Energy Spark***)
  - The flatter the PSD of the transmission, the larger the amount of power that can be radiated while still satisfying PSD bounds imposed by regulatory agencies.
  - Code for perfectly flat PSD
    - Codes from cyclic difference set
    - Not applicable to IRS because of "pulse position property"
- PSD of proposed IRS



【Fig. 7】 $C_k^{(4)}$ of type I construction (with peak value 65280 at $k=0$)

Conclusion

- Time-hopping signal model and correlation consideration
  - NOT conventional BUT OOC (Optical Orthogonal Code) like correlation
  - Crosscorrelation with arbitrary time delay in slot time units as well as frame duration units

- **Design goal** of time-hopping code for UWB Impulse Radio
  : to construct a signal set with
  - **Low correlation** for better performance of multiple access
  - **Spectral flatness** for efficient radiation of power
  - **Long period** for low PSD (energy spark cancellation)
  - **Easy generation** for system implementation
  - **Large number of sequences** to support as many users as possible

- Proposed design example has the features:
  - Very easy generation
  - For given parameter N_s, relatively long period
  - Improvement in correlation measure (compared to existing designs)
  - Usually flat PSD, but room for analysis and development

Future Directions

- Code design
  - Improvement of proposed design
  - New design using q-ary integer codes having good hamming correlation

- Consideration of known IRSs with optimal correlation
  - PSD analysis
  - Simulation with various modulation schemes