Efficient encoder design of LDPC code using circulant matrix and eIRA codes

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Introduction

- **Encoding of LDPC codes**
  - Information bit & Parity bit are computed by matrix multiplication in linear block code.
  - 1’s in generator matrix are more than parity-check matrix.
  - Encoding complexity is much high with matrix multiplication operation.
Design of Structural Codes (1/2)

- **Irregular Repeated-Accumulate (IRA) codes**
  - $k$: number of variable nodes
  - $r$: number of checks nodes
  - $r$: number of parity nodes
  - Each check node is connected to an information node.
  - The value of parity bit is determined uniquely by the condition that the mod-2 sum of the values of the variable nodes connected to each of the check nodes is zero.

![Diagram of IRA codes](image)
Extended Irregular Repeated-Accumulate (eIRA) codes

- Complexity is reduced using differential encoder structure of IRA codes
- \( H = [H_1 | H_2] \)
  - \( H_1: (n-k) \times k \) sparse matrix (using density evolution)
  - \( H_2: (n-k) \times (n-k) \) dual diagonal matrix

\[
H_2 = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\]

\[
H_2^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\]
Circulant matrix

- \( H_1 \) matrix is constructed by \( p \times q \) matrix blocks which are \( l \times l \) circulant submatrix, where \( p \) is a number of row blocks, \( q \) is a number of column blocks.

- Each weight in each circulant submatrix can be different.

\[
H_1 = \begin{pmatrix}
h_{11} & h_{12} & \cdots & h_{1q} \\
h_{21} & h_{22} & \cdots & h_{2q} \\
\vdots & \vdots & \ddots & \vdots \\
h_{p1} & h_{p2} & \cdots & h_{pq}
\end{pmatrix}
\]

- Proposed \( H_1 \) matrix can be more modified by row permutation matrix.

- Row permutation matrix \( P \) is \( (n - k) \times (n - k) \) matrix and each rows and columns has only one 1’s.

- To prevent from performance degradation, modify to \( H_1' = PH_1 \) using row permutation matrix.
Parity-check matrix

\[ H = \begin{bmatrix} H_1' & H_2 \end{bmatrix} \]
\[ = \begin{bmatrix} PH_1 & H_2 \end{bmatrix} \]
\[ = \begin{bmatrix}
1 & h_{11} & h_{12} & \cdots & \cdots & h_{1q} & 1 \\
1 & h_{21} & h_{22} & \cdots & \cdots & h_{2q} & 1 & 1 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\
1 & h_{p1} & h_{p2} & \cdots & \cdots & h_{pq} & 1 & 1 \\
\end{bmatrix} \]
Generator matrix

- \( H = [H_1' \mid H_2] = [H_2^{-1}H_1' \mid I_{n-k} ] \rightarrow G= [ I_k \mid H_1^{T} H_2^{-T}] \)

\[
G = \begin{bmatrix}
I \mid H_1^{T} P^{T} H_2^{-T} \\
\end{bmatrix}
\]

- Since \( H_1^{T} \) is transposed form of circulant matrix, it can be constructed by shift register and interleaver.
- \( H_2^{-T} \) matrix can be constructed by differential encoder.
Encoder Block Diagram

a) Original Encoder

\[ c = \begin{bmatrix} u \\ p \end{bmatrix} \]

\[ u \xrightarrow{\text{multi. op.}} H_1^T \xrightarrow{\frac{1}{1 \oplus D}} p \]

b) Modified Encoder

\[ c = \begin{bmatrix} u \\ p \end{bmatrix} \]

\[ u \xrightarrow{\text{FSR}} H_1^T \xrightarrow{\text{Interleaver}} \xrightarrow{\frac{1}{1 \oplus D}} p \]
Modified Encoder Structure (5/6)

- Encoder Structure
  - Modified encoder structure
  - Example of Shift Register

\[ g_1 = 1 + x^3 \]
\[ H_1^T \]
\[ c = [u \ p] \]
## Complexity Comparison

<table>
<thead>
<tr>
<th></th>
<th>δ (general)</th>
<th>Total Computation</th>
<th>Total XOR</th>
<th>Total Memory</th>
<th>Computation comparison</th>
<th>XOR comparison (n=1024, k=768, p=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix multiplication</td>
<td>≈0.5</td>
<td>δk(n-k)</td>
<td>δk(n-k)</td>
<td>n-k</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Michael Yang</td>
<td>≈0.01</td>
<td>δ(k+1)(n-k)</td>
<td>δk(n-k)</td>
<td>n-k</td>
<td>1/50</td>
<td>1</td>
</tr>
<tr>
<td>Shift Register</td>
<td>≈0.01</td>
<td>δ(k+1)(n-k)</td>
<td>δkp</td>
<td>n-k</td>
<td>1/50</td>
<td>1/256</td>
</tr>
<tr>
<td>Shift Register Interleaver</td>
<td>≈0.01</td>
<td>δ(k+1)(n-k)</td>
<td>δkp</td>
<td>≤ 2(n-k)</td>
<td>1/50</td>
<td>1/256</td>
</tr>
</tbody>
</table>

- δ: density of $H_I$ Matrix
- Complexity is reduced compared with matrix multiplication operation.
- In our simulation, (n, k) = (256, 192), (512, 384), (1024, 768), (2924, 2193), p=1, q=3.
## Simulation Environments

<table>
<thead>
<tr>
<th>Decoding Algorithm</th>
<th>Sum-Product Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Iteration</td>
<td>50</td>
</tr>
<tr>
<td>Codeword Length</td>
<td>256, 512, 1024, 2924</td>
</tr>
<tr>
<td>Coderate</td>
<td>0.75</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>Channel Model</td>
<td>AWGN</td>
</tr>
</tbody>
</table>
Simulation Results(2/4)

- Code rate 0.75, length 256, 512

*BER of Length 256, Coderate 0.75 LDPC code*

*BER of Length 512, Coderate 0.75 LDPC code*
Simulation Results (3/4)

- Code rate 0.75, length 1024, 2924

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BER of Length 1024, Coderate 0.75 LDPC code

- eIRA Circul. Interl.
- eIRA Circul.
- eIRA Random
- Random

---

BER of Length 2924, Coderate 0.75 LDPC code

- eIRA Circul. Interl.
- eIRA Circul.
- eIRA Random
- Random
Comparison according to lengths

BER of various Length, Coderate 0.75 LDPC code

- len 256
- len 512
- len 1024
- len 2592

Bit Error Rate vs. Eb/No
Concluding Remarks

Conclusion

- Encoder complexity is lower than Yang’s method.
- Applicable to short length, high rate LDPC code

Future Works

- Research on generator polynomial structure to make large girth.
- Research on permutation matrix structure not to make 4-cycle after permutation.