Efficient encoder design of LDPC code using circulant matrix and eIRA codes

Seul-Ki Bae

sk.bae@coding.yonsei.ac.kr

November, 5, 2004

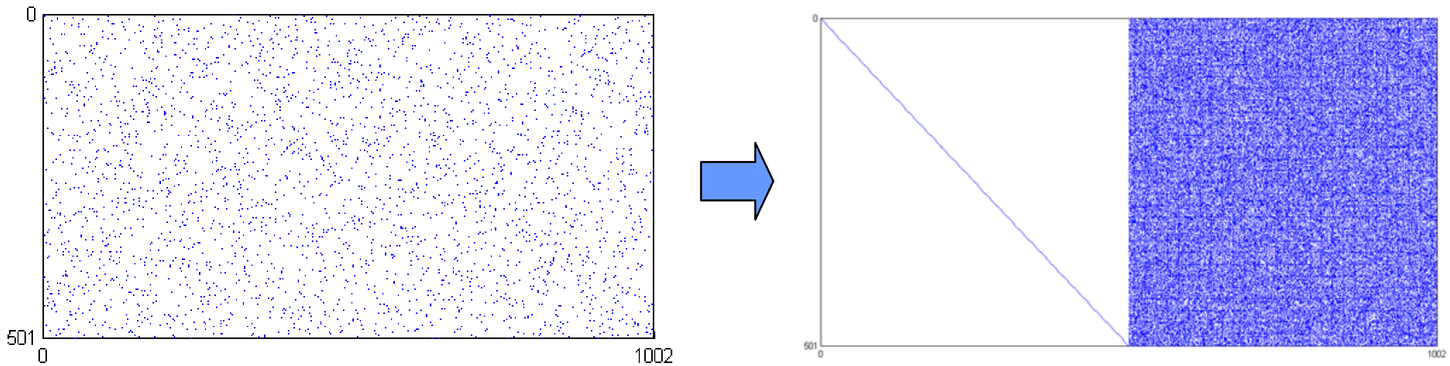
Coding & Information Theory Lab.
Department of Electrical and Electronic Engineering, Yonsei Univ.

Contents

Introduction

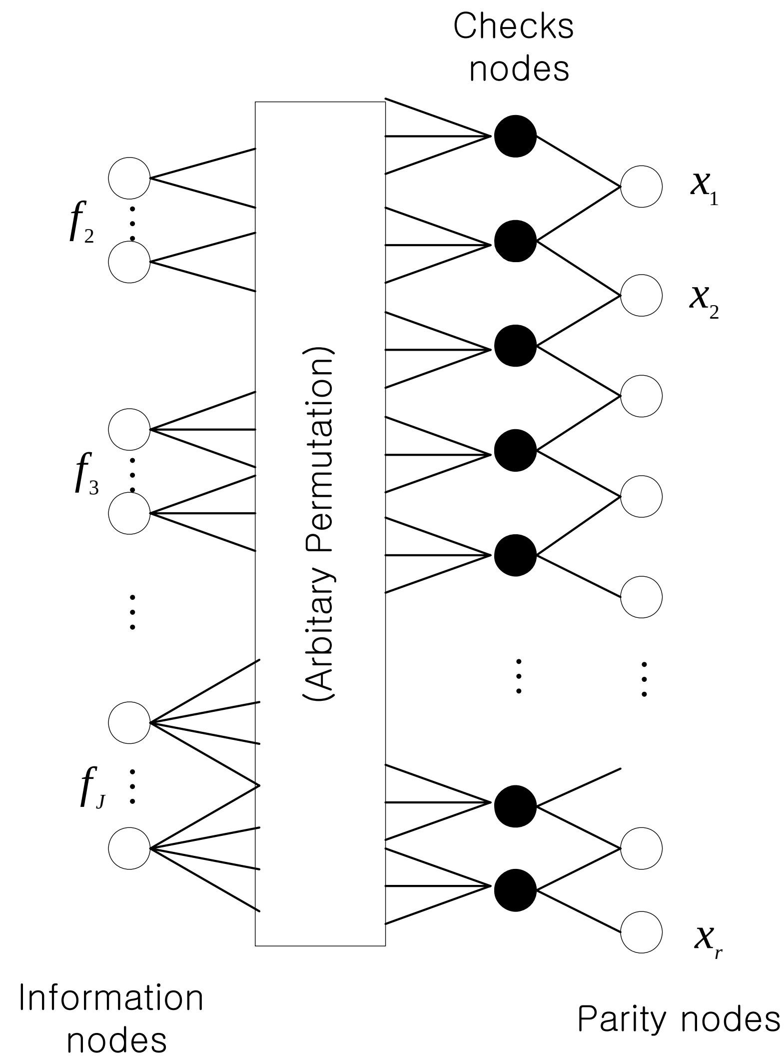
Encoding of LDPC codes

- Information bit & Parity bit are computed by matrix multiplication in linear block code.
- 1's in generator matrix are more than parity-check matrix
- Encoding complexity is much high with matrix multiplication operation.



Design of Structural Codes(1/2)

- **Irregular Repeated-Accumulate (IRA) codes**
  - k : number of variable nodes
    r : number of checks nodes
    r : number of parity nodes
  - Each check node is connected to a information nodes
  - The value of parity bit is determined uniquely by the condition that the mod-2 sum of the values of the variable nodes connected to each of the check nodes is zero.



Design of Structural Codes(2/2)

- **Extended Irregular Repeated-Accumulate (eIRA) codes**
  - Reference : Michael Yang, "Design of Efficiently Encodable Moderate-Length High-Rate Irregular LDPC Codes", *IEEE Trans. Comm*. Vol. 52. pp 564-571, April. 2004
  - Complexity is reduced using differential encoder structure of IRA codes
  - $\boldsymbol{H} = [\boldsymbol{H}_1 \mid \boldsymbol{H}_2]$

    $\boldsymbol{H}_1 :\ (n-k)\times k$ sparse matrix (using density evolution)

    $\boldsymbol{H}_2 :\ (n-k)\times(n-k)$ dual diagonal matrix

$$H_2 = \begin{pmatrix} 1 & & & & & \\ 1 & 1 & & & & \\ & 1 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & 1 & 1 & \\ & & & & 1 & 1 \end{pmatrix} \qquad H_2^{-T} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ & 1 & 1 & 1 & 1 \\ & & \cdots & \cdots & \cdots \\ & & & 1 & 1 \\ & & & & 1 \end{pmatrix}$$

Modified Encoder Structure(1/6)

- **Circulant matrix**
  - $\boldsymbol{H_1}$ matrix is constructed by $p \times q$ matrix blocks which are $l \times l$ circulant submatrix , where p is a number of row blocks, q is a number of column blocks.
  - Each weight in each circulant submatrix can be different.

$$H_1 = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1q} \\ h_{21} & h_{22} & & h_{2q} \\ \vdots & & \ddots & \vdots \\ h_{p1} & h_{p2} & \cdots & h_{pq} \end{pmatrix}$$

  - Proposed $\boldsymbol{H_1}$ matrix can be more modified by row permutation matrix.
  - Row permutation matrix $\boldsymbol{P}$ is $(n-k)\times(n-k)$ matrix and each rows and columns has only one 1's.
  - To prevent from performance degradation, modify to $\boldsymbol{H_1}' = \boldsymbol{PH_1}$ using row permutation matrix.

- **Parity-check matrix**

$$
\begin{aligned}
H &= \left[H_1^{'} \mid H_2 \right] \\
&= \left[PH_1 \mid H_2 \right]
\end{aligned}
$$

$$
= \left[\begin{array}{ccccccccccccc|cccccc}
 & & 1 & & & & h_{11} & h_{12} & \cdots & \cdots & \cdots & h_{1q} & & 1 & & & & & \\
 & 1 & & & & & h_{21} & h_{22} & \cdots & \cdots & \cdots & h_{2q} & & 1 & 1 & & & & \\
 & & & & 1 & & & & & & & & & & 1 & 1 & & & \\
 & & \ddots & & & \times & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & & & & \ddots & \ddots & & \\
1 & & & & & & & & & & & & & & & & 1 & 1 & \\
 & & & 1 & & & h_{p1} & h_{p2} & \cdots & \cdots & \cdots & h_{pq} & & & & & & 1 & 1
\end{array} \right]
$$

Modified Encoder Structure(3/6)

❑ Generator matrix

- $\boldsymbol{H} = [\boldsymbol{H}_1' \mid \boldsymbol{H}_2] = [\boldsymbol{H}_2^{-1}\boldsymbol{H}_1' \mid \boldsymbol{I}_{n-k}] \;\rightarrow\; \boldsymbol{G} = [\boldsymbol{I}_k \mid \boldsymbol{H'}_1^T \boldsymbol{H}_2^{-T}]$

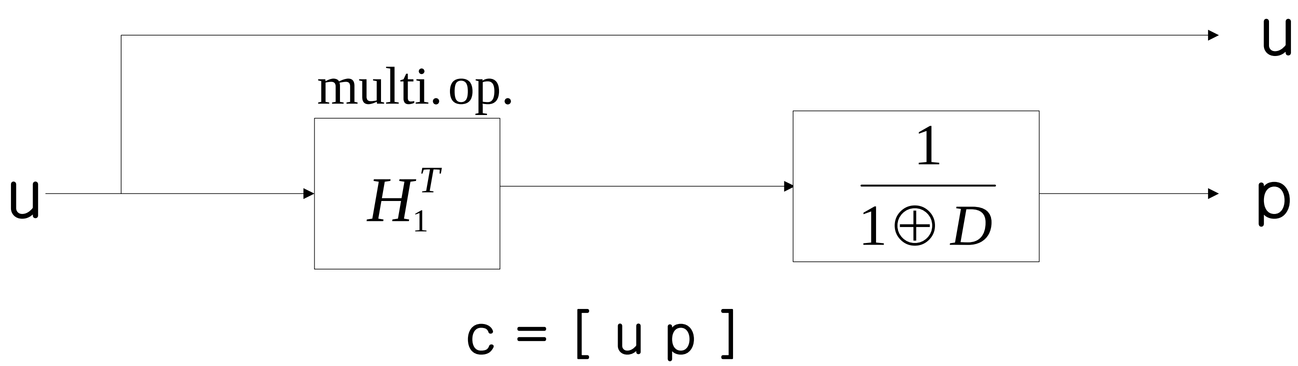
$$G = \left[I \mid H_1^T P^T H_2^{-T} \right]$$

$$= \left[\begin{matrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{matrix} \middle| \begin{matrix} h_{11}^T & h_{21}^T & \cdots & h_{p1}^T \\ h_{12}^T & h_{22}^T & \cdots & h_{p2}^T \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{1q}^T & h_{2q}^T & \cdots & h_{pq}^T \end{matrix} \times P^T \times \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ & 1 & 1 & 1 & 1 \\ & & \cdots & \cdots & \cdots \\ & & & 1 & 1 \\ & & & & 1 \end{matrix} \right]$$

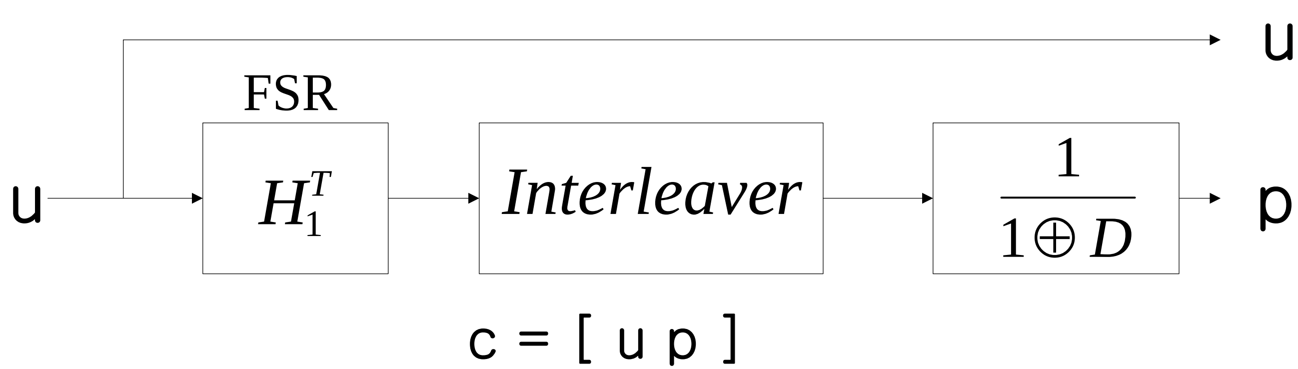
- Since $\boldsymbol{H}_1'^T$ is transposed form of circulamt matrix, it can be constructed by shift register and interleaver.
- $\boldsymbol{H}_2^{-T}$ matrix can be constructed by differential encoder.

Encoder Block Diagram

a) Original Encoder

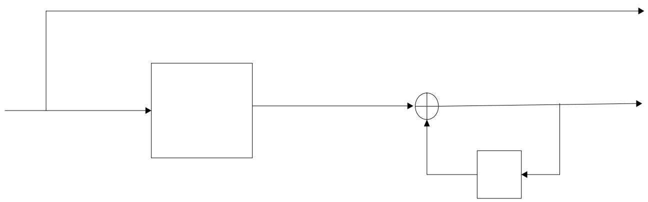


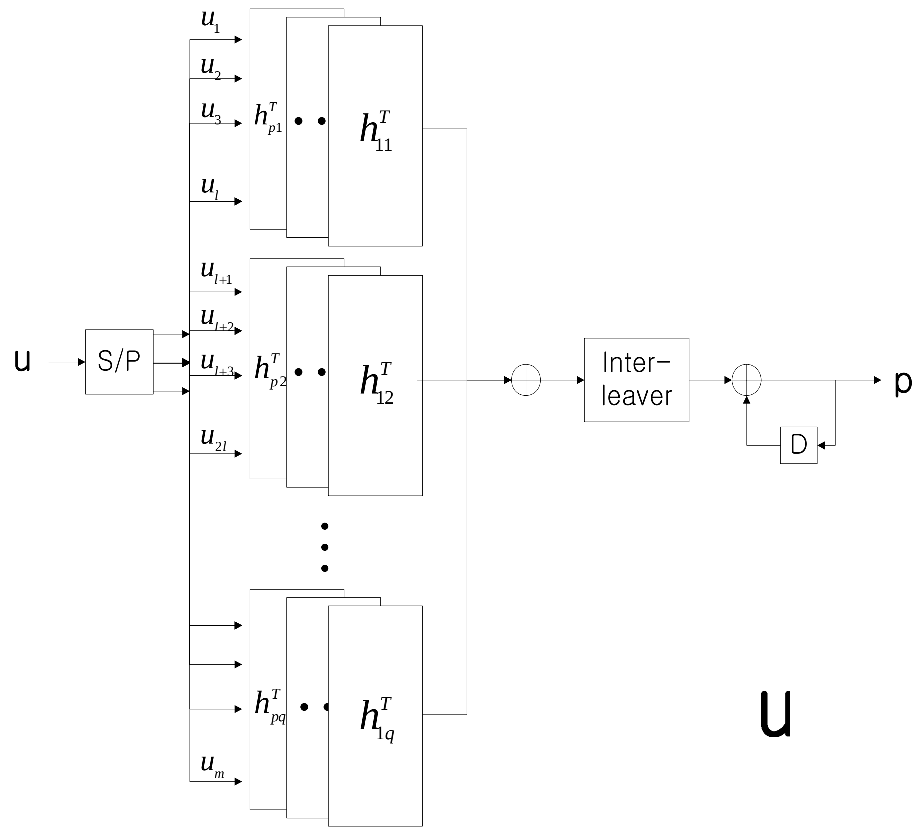
b) Modified Encoder



❑ Encoder Structure

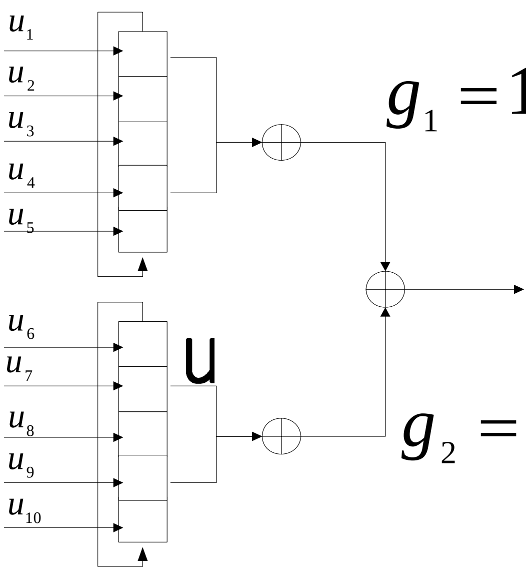
- Modified encoder structure





u

- Example of Shift Register



$$g_1 = 1 + x^3$$

u

$$H_1^T$$

$$g_2 = x + x^3$$

p

D

c = [u p]

Modified Encoder Structure(6/6)

Complexity Comparison

	δ (general)	Total Computation	Total XOR	Total Memory	Computation comparision	XOR comparison (n=1024, k=768 p=1)
Matrix multiplication	≈0.5	δk(n-k)	δk(n-k)	n-k	1	•
Michael Yang	≈0.01	δ(k+1)(n-k)	δk(n-k)	n-k	1/50	1
Shift Register	≈0.01	δ(k+1)(n-k)	δkp	n-k	1/50	1/256
Shift Register Interleaver	≈0.01	δ(k+1)(n-k)	δkp	≤ 2(n-k)	1/50	1/256

- δ : density of $\boldsymbol{H}_1$ Matrix
- Complexity is reduced compared with matrix multiplication operation.
- In our simulation, (n, k) = (256, 192), (512, 384), (1024, 768), (2924, 2193), p=1, q=3.

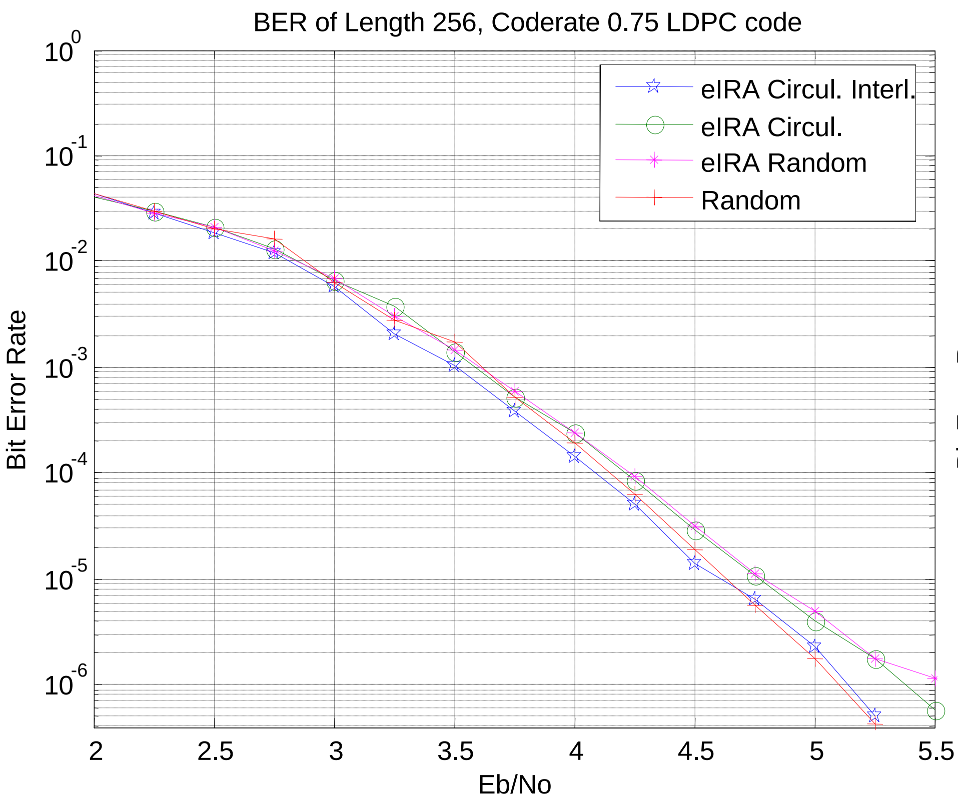
Simulation Results(1/4)

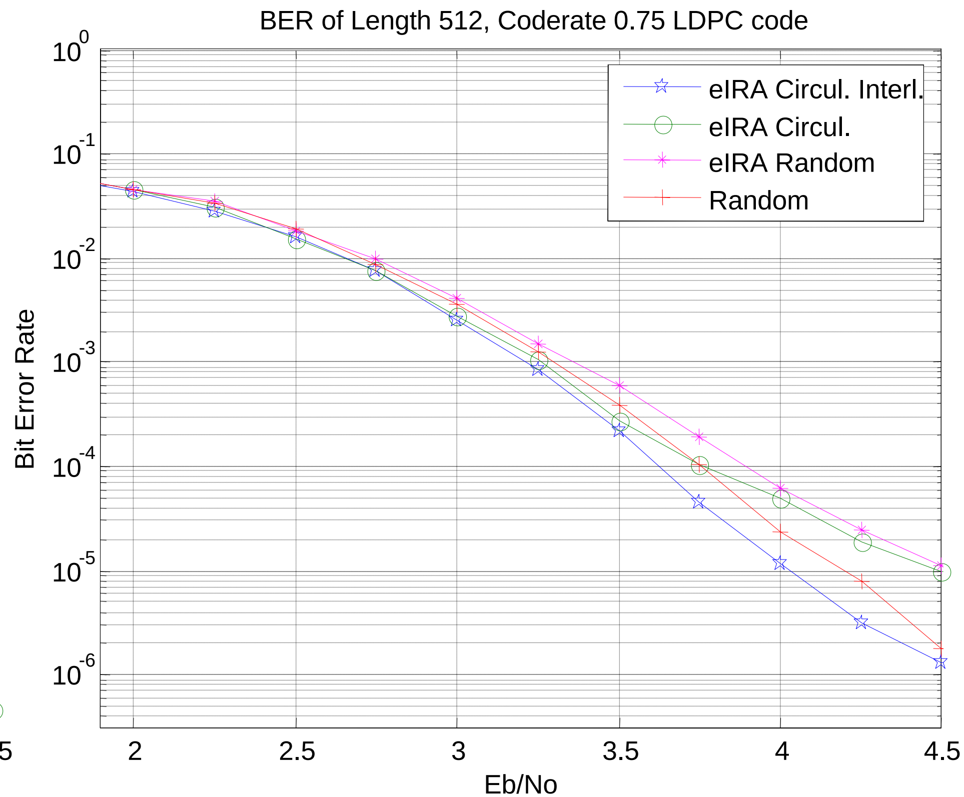
Simulation Environments

Decoding Algorithm	Sum-Product Algorithm
Maximum Iteration	50
Codeword Length	256, 512, 1024, 2924
Coderate	0.75
Modulation	BPSK
Channel Model	AWGN

Simulation Results(2/4)

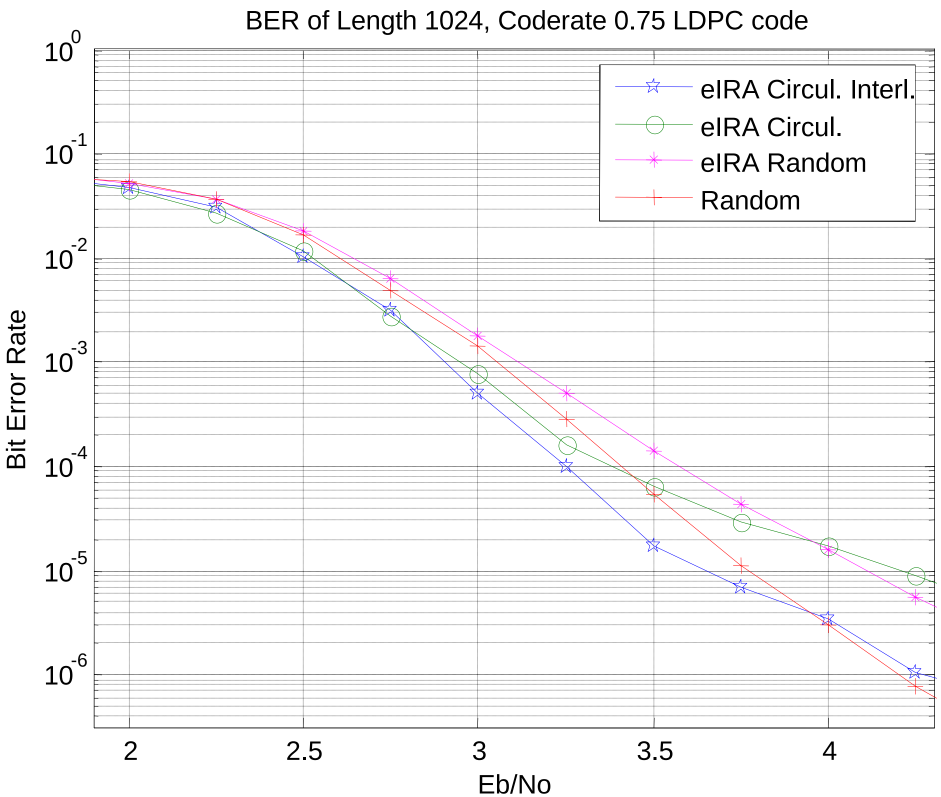
- **Code rate 0.75, length 256, 512**

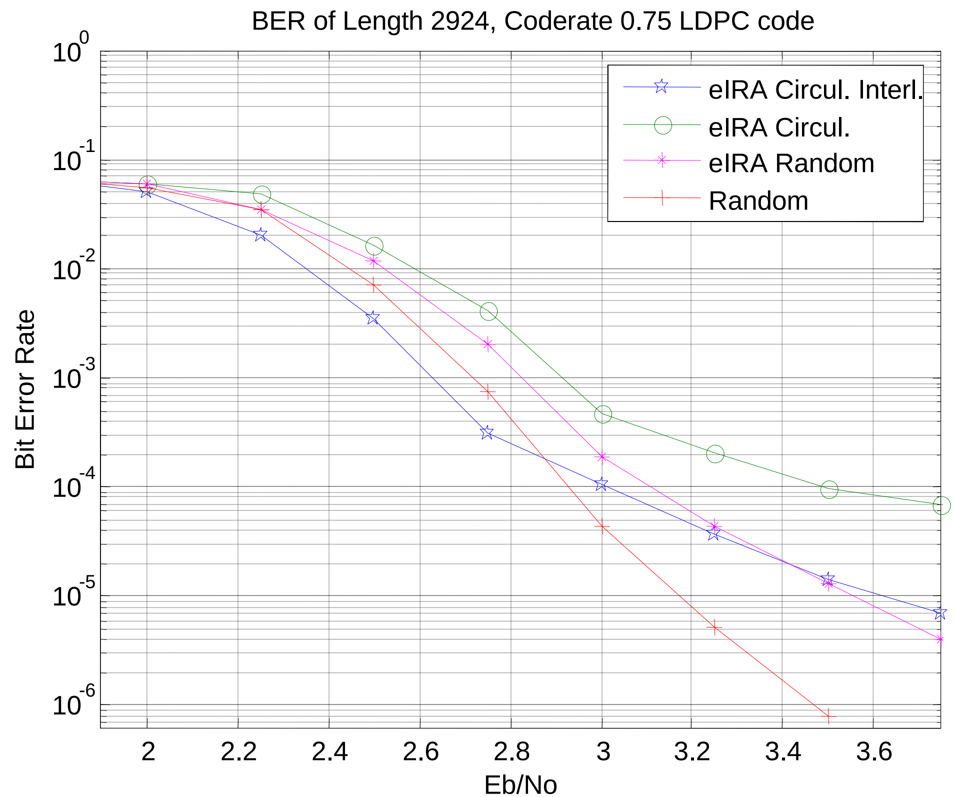




Simulation Results(3/4)

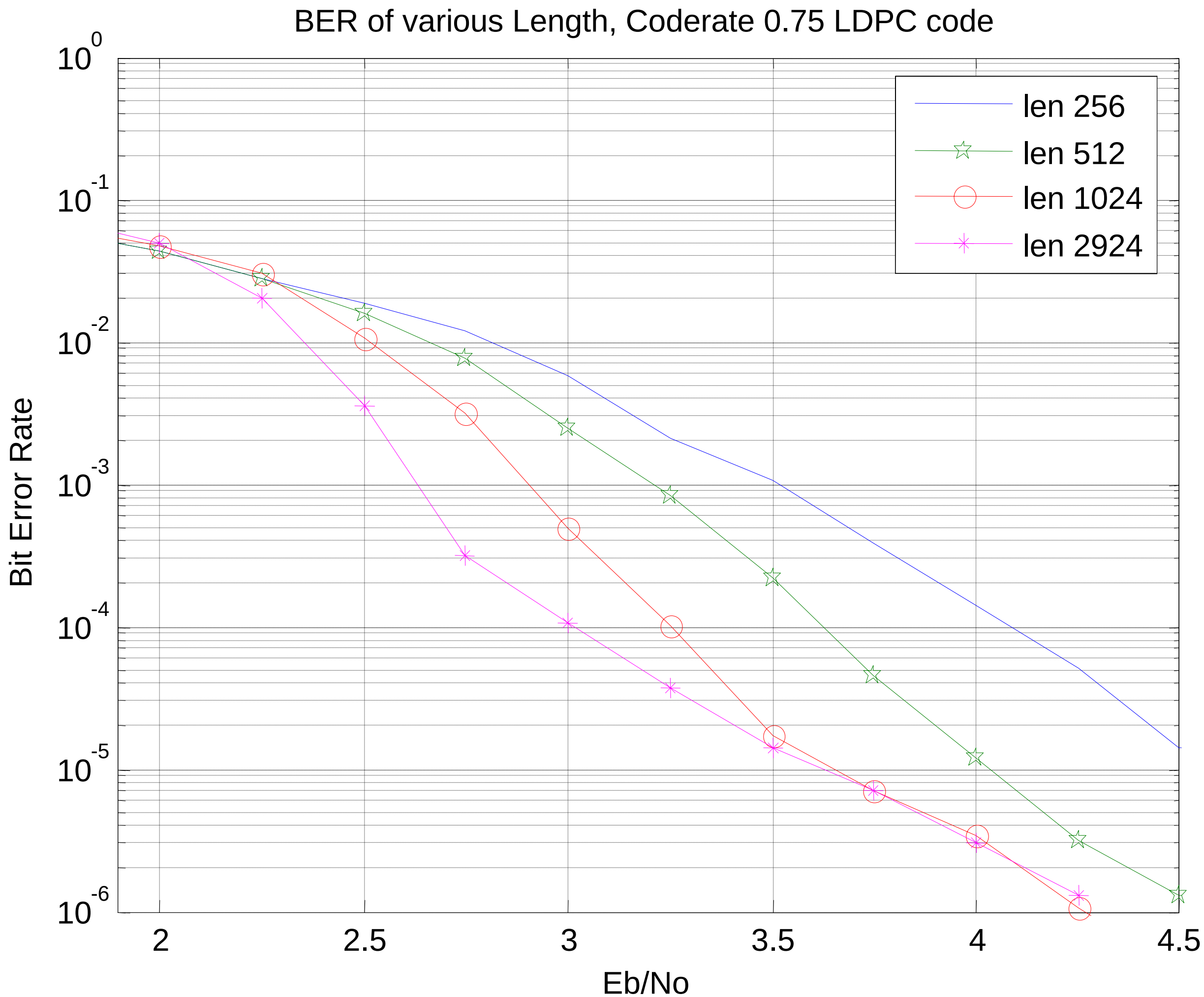
- **Code rate 0.75, length 1024, 2924**





Simulation Results(4/4)

❑ Comparison according to lengths



Concluding Remarks

Conclusion

- Encoder complexity is lower than Yang's method.
- Applicable to short length, high rate LDPC code

Future Works

- Research on generator polynomial structure to make large girth.
- Research on permutation matrix structure not to make 4-cycle after permutation.