Geometric Analysis for the Cell Coverage Extension with Wireless Relay

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Motivation

- Cell coverage problem of 4G system
  - High data rate service – high required power
  - 3.5~5GHz band

Cell coverage of 2G or 3G system

Cell coverage by 3.5~5Ghz band

Cell coverage by high required power and 3.5~5Ghz band
Geometric Model and Relaying Strategy

- Degraded Gaussian relay channel\textsuperscript{[5]}

- Simple path loss model
- Transmission power assignment:
  - proportional to the square of coverage, i.e. $P \propto d^2$ \textsuperscript{[13]}

\textsuperscript{[5]} T. Cover and A. El Gamal (1979) , \textsuperscript{[13]} A. Agarwal and P. R. Kumar (2004)
Geometric Model and Relaying Strategy

- **CRIS scheme**[6]
  - Consider the interference due to signal from source to relay
  - Cooperation ratio $0.5 < a \leq 1$: represent the dependence on relaying

$$R = \max_{0 \leq a \leq 1} \min \left\{ S \left( \frac{\alpha_{sr}^2 a P_s}{N_1} \right), S \left( \frac{\alpha_{sd}^2 P_s + \alpha_{rd}^2 P_r + 2\alpha_{sd}\alpha_{rd}(1-a)P_sP_r}{N_1 + N_2} \right) \right\}$$

where $S(x) = \frac{1}{2} \log_2 (1 + x)$.

Geometric Model and Relaying Strategy

- If \( SNR_{rd} \geq SNR_{sr} \), \( a = 1 \), no cooperation (only repetition).
  \[
  R = S \left( \frac{\alpha_{sr}^2 P_s}{N_1} \right), \quad (a = 1)
  \]

- If \( SNR_{rd} < SNR_{sr} \), \( a \) is strictly less than 1.
  - For optimal \( a^* \),
    \[
    R = S \left( \frac{\alpha_{sr}^2 a^* P_s}{N_1} \right)
    \]
Geometric Model and Relaying Strategy

- **CRIS scheme**
  - Optimal value of $a (= a^*)$ can be determined from:
    \[
    \frac{\alpha_{sr}^2 a^* P_s}{N_1} = \frac{\alpha_{sd}^2 P_s + \alpha_{rd}^2 P_r + 2\alpha_{sd} \alpha_{rd} \sqrt{1-a^*} P_s P_r}{N_1 + N_2}
    \]
  - For simplicity let $N_1 = N_2 = N$:
    \[
    a^* = \frac{2AB - 1 + \sqrt{(2AB - 1)^2 - A^2 (B^2 - 1)}}{2A^2}
    \]
    where $A = \frac{2\alpha_{sr}^2 P_s}{2\alpha_{sd} \alpha_{rd} \sqrt{P_s P_r}}$, $B = \frac{\alpha_{sd}^2 P_s + \alpha_{rd}^2 P_r}{2\alpha_{sd} \alpha_{rd} \sqrt{P_s P_r}}$.  


Geometric Model and Relaying Strategy

\[ d_{rd} = kd_{sr} \quad (0 < k \leq 1) \]
\[ d_{sd} = \sqrt{d_{sr}^2 + (kd_{sr})^2 - 2kd_{sr}^2 \cos(\pi - \theta)} \]
\[ 0 \leq \theta \leq \pi - \arccos \frac{k}{2} \]

\[ \alpha_{sr}^2 = Kd_{sr}^{-\gamma} \]
\[ \alpha_{sd}^2 = Kd_{sd}^{-\gamma} \]
\[ \alpha_{rd}^2 = Kd_{rd}^{-\gamma} \]
\[ P_r = k^2 P_s \]

\[ k: \text{distance ratio} \]
\[ K: \text{constant related to attenuation normalized to 1} \]
Capacity Theorem over Geometric Model

- Descriptions of Optimized parameters

\[ \frac{A - AB + B \pm \sqrt{AB(A - 2)(B - 2)}}{2} = 1, \]

where \( A = \sqrt{1 + k^2 - 2k \cos(\pi - \theta)^{-\gamma}} \) and \( B = k^{-\gamma}k^2 \).

<table>
<thead>
<tr>
<th></th>
<th>( \gamma = 4 )</th>
<th>( \gamma = 3 )</th>
<th>( \gamma = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( k &gt; 0.7 )</td>
<td>( k \geq 0.5 )</td>
<td>( k &gt; 0 )</td>
</tr>
<tr>
<td>( a^* )</td>
<td>0.6~1</td>
<td>1</td>
<td></td>
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<tr>
<td>strategy</td>
<td>Cooperation depending on ( k ) and ( \theta )</td>
<td>Low cooperation, repetition</td>
<td></td>
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Capacity Theorem over Geometric Model

- $\theta'$ (= maximum value of $\theta$) of relay that guarantees maximum achievable rate for the given value of $k$ and $\gamma$
  
  - Example

  $\gamma = 4$

  $k = 0.8$

  $\theta' = 1.65$

  $s$

  $r$

  $a^* = 1, \theta' = 95^\circ$
Cell Coverage Extension with Multiple Relays

- Coverage angle ($\alpha$) and coverage range ($r_2$)\[^{[4]}\]

Coverage range:

$$r_2 = \sqrt{d_{sr}^2 + (kd_{sr})^2 - 2d_{sr}(kd_{sr})\cos(\pi - \theta')}$$

Coverage angle:

$$\alpha = 2\arccos\left(\frac{d_{sr}^2 + r_2^2 - (kd_{sr})^2}{2d_{sr}r_2}\right)$$

Required number of relays:

$$N_R = \lceil 360^\circ / \alpha \rceil$$

- For large coverage:
  - Decrease $\theta'$ (lower the power level) → more relays are needed

Cell Coverage Extension with Multiple Relays

- $\gamma = 4$

$r_2$ is extended reciprocally to the increase of $k$. Extend cell coverage up to $r_2=1.6$ with $N_R \leq 15$
Cell Coverage Extension with Multiple Relays

- $\gamma = 2$

$2N_R$ relays are used to cover circular cell shape. $r_2$ is extended in proportion to the increase of $k$. 
# Cell Coverage Extension with Multiple Relays

- For maximum coverage range $r_2$

<table>
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<tr>
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<th>$\gamma = 4$</th>
<th>$\gamma = 3$</th>
<th>$\gamma = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_2 / r_1$</td>
<td>1.6</td>
<td>1.5</td>
<td>1.7</td>
</tr>
<tr>
<td>$P_r / P_s$ [dB]</td>
<td>-2.22</td>
<td>-5.23</td>
<td>0</td>
</tr>
<tr>
<td>$N_R$</td>
<td>15</td>
<td>16</td>
<td>6</td>
</tr>
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</table>
Concluding Remarks

- Geometric analysis
  - Relation of cooperation ratio $a$, distance ratio $k$, and $\theta$
  - Relation of required number of relays $N_R$, distance ratio $k$, and effective coverage angle $\theta'$ to guarantee maximum achievable rate ($a^*=1$)
  - Condition to achieve maximum coverage range for $a^*=1$
    - Low attenuation regime: increase $P_r \rightarrow$ decrease $N_R$
    - High attenuation regime: decrease $P_r \rightarrow$ increase $N_R$

- Future Directions
  - Specific deploying scheme for the case of $\gamma = 2$
  - How about $k > 1$? (rural area)
Thank you!

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