New Interpretation on Gaussian Approximation for Determining the Threshold of LDPC Code Ensemble

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Coding & Crypto Lab.

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Introduction

- Bit node -> Check node

$$M = R + \sum_{j'} E_{j'}$$

$$m_{M,i}(l) = m_{M_0} + (i-1)m_E(l-1)$$

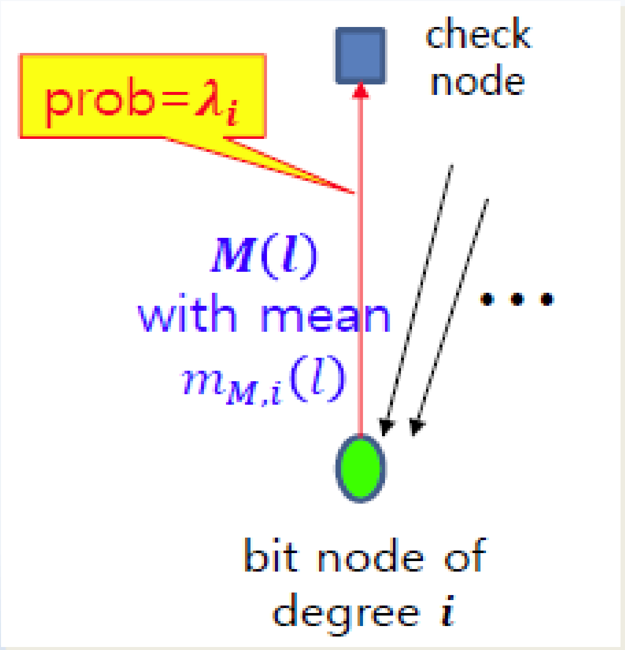
$$m_M(l) = \sum_i \lambda_i m_{M,i}(l)$$

$$m_M(l) = \sum_i \lambda_i (m_{M_0} + (i-1)m_E(l-1))$$

- Check node -> Bit node

$$\tanh\left(\frac{E}{2}\right) = \prod_{i'} \tanh\left(\frac{M_{i'}}{2}\right)$$

$$\varepsilon\left\{\tanh\left(\frac{E}{2}\right)\right\} = \prod_{i'} \varepsilon\left\{\tanh\left(\frac{M_{i'}}{2}\right)\right\}$$



$$\varepsilon\left\{\tanh\left(\frac{E}{2}\right)\right\} = \prod_{i'} \varepsilon\left\{\tanh\left(\frac{M_{i'}}{2}\right)\right\}$$

Definition

$$\phi(m) = 1 - \varepsilon\left\{\tanh\left(\frac{Z}{2}\right)\right\}$$

,where $Z \sim N(m, 2m)$

$$\varepsilon\left\{\tanh\left(\frac{M}{2}\right)\right\} = \sum_i \lambda_i \varepsilon\left\{\tanh\left(\frac{M_i}{2}\right)\right\} = \sum_i \lambda_i [1 - \phi(m_{M,i}(l))] = 1 - \sum_i \lambda_i \phi(m_{M,i}(l))$$

L.H.S

$$\varepsilon\left\{\tanh\left(\frac{E}{2}\right)\right\} = 1 - \phi(m_{E,j}(l))$$

for check node of degree j

R.H.S

$$\prod_{i'} \varepsilon\left\{\tanh\left(\frac{M_{i'}}{2}\right)\right\} = \left[1 - \sum_i \lambda_i \phi(m_{M,i}(l))\right]^{j-1}$$

for check node of degree j

$$1 - \phi(m_{E,j}(l)) = \left[1 - \sum_i \lambda_i \phi(m_{M,i}(l))\right]^{j-1}$$

$$m_{E,j}(l) = \phi^{-1}\left\{1 - \left[1 - \sum_i \lambda_i \phi(m_{M,i}(l))\right]^{j-1}\right\}$$

Introduction

$$m_{E,j}(l) = \phi^{-1}\left\{1 - \left[1 - \sum_i \lambda_i \phi\left(m_{M,i}(l)\right)\right]^{j-1}\right\}$$

Take the average across check node degrees

$$m_E(l) = \sum_j \rho_j m_{E,j}(l)$$

$$m_E(l) = \sum_j \rho_j \phi^{-1}\left\{1 - \left[1 - \sum_i \lambda_i \phi\left(m_{M,i}(l)\right)\right]^{j-1}\right\}$$

- Checking for which values of m_{M_0}, we have $m_E \rightarrow \infty \; as \;\; l \rightarrow \infty$ that gives **threshold** on the channel

New Interpretation on Gaussian Approximation

Definition

$$\boldsymbol{\phi(m) = 1 - \mathcal{E}\left\{\tanh\left(\frac{Z}{2}\right)\right\}}$$

,where $Z \sim N(m, 2m)$

$$\mathcal{E}\left\{\tanh\left(\frac{E}{2}\right)\right\} = \prod_{i'} \mathcal{E}\left\{\tanh\left(\frac{M_{i'}}{2}\right)\right\}$$

$$\mathcal{E}\left\{\tanh\left(\frac{M}{2}\right)\right\} = \sum_i \lambda_i \mathcal{E}\left\{\tanh\left(\frac{M_i}{2}\right)\right\} = 1 - \sum_i \lambda_i \phi(m_{M,i}(l))$$

$$\mathcal{E}\left\{\tanh\left(\frac{M}{2}\right)\right\} = 1 - \phi(m_M(l)) = 1 - \phi\left(\sum_i \lambda_i m_{M,i}(l)\right)$$

$$m_E(l) = \sum_j \rho_j \phi^{-1}\left\{1 - \left[1 - \sum_i \lambda_i \phi(m_{M,i}(l))\right]^{j-1}\right\}$$

Original Updating Rule

$$m_E(l) = \sum_j \rho_j \phi^{-1}\left\{1 - \left[1 - \phi\left(\sum_i \lambda_i m_{M,i}(l)\right)\right]^{j-1}\right\}$$

New Updating Rule

New Interpretation on Gaussian Approximation

- For regular LDPC codes, two different updating rules are same
  - New updating rule

$$m_E(l) = \sum_j \rho_j \phi^{-1}\left\{1 - \left[1 - \phi\left(\sum_i \lambda_i m_{M,i}(l)\right)\right]^{j-1}\right\} \quad \text{irregular}$$

$$m_E(l) = \phi^{-1}\left\{1 - \left[1 - \phi\left(m_{M,i}(l)\right)\right]^{j-1}\right\} \quad (\lambda_i = 1, \rho_i = 1) \quad \text{regular}$$

  - Original updating rule

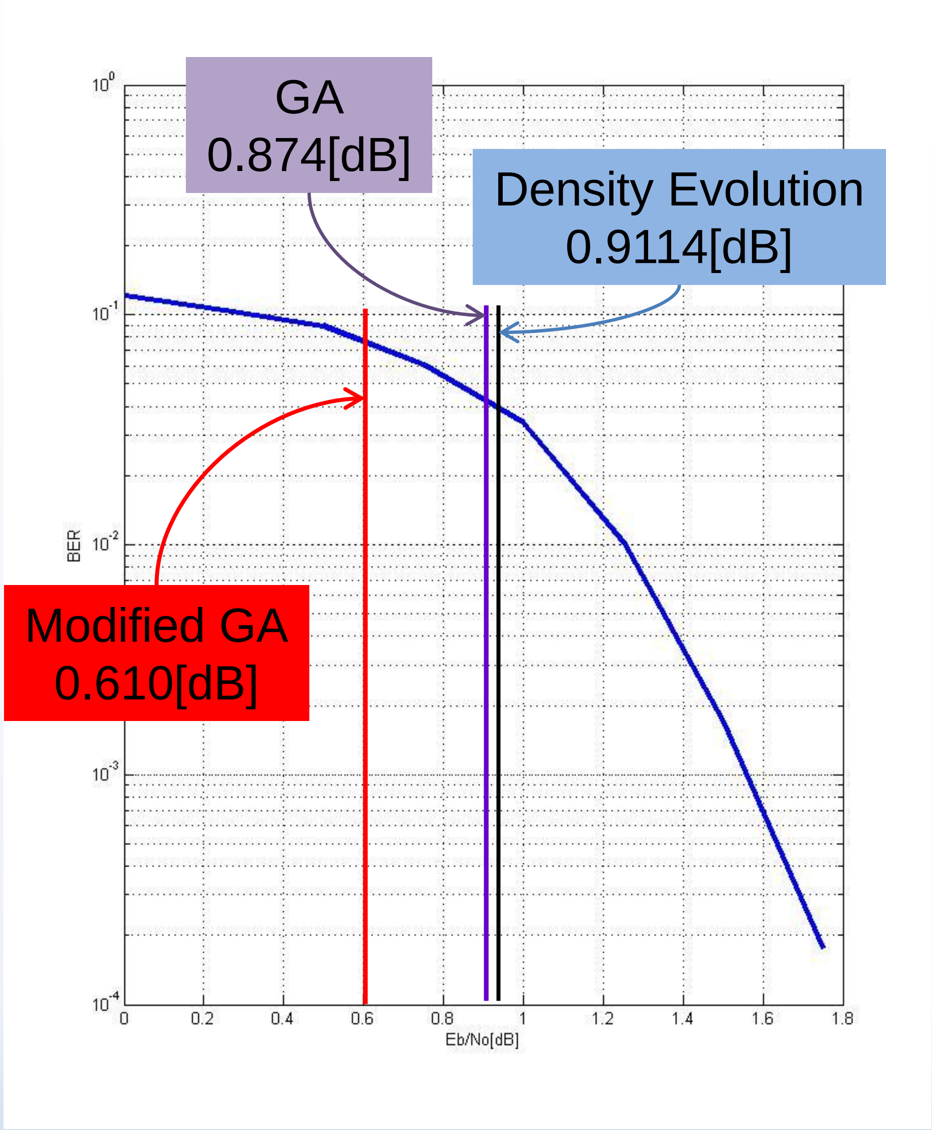
$$m_E(l) = \sum_j \rho_j \phi^{-1}\left\{1 - \left[1 - \sum_{\boldsymbol{i}} \boldsymbol{\lambda_i \phi\left(m_{M,i}(l)\right)}\right]^{j-1}\right\} \quad \text{irregular}$$

$$m_E(l) = \phi^{-1}\left\{1 - \left[1 - \phi\left(m_{M,i}(l)\right)\right]^{j-1}\right\} \quad \text{regular}$$

Numerical Results

d_v		4	5	6	8	802.11n	802.16
λ	λ_2	**0.38354**	0.3266	0.33241	0.30013	0.2558	0.4583
	λ_3	**0.04237**	0.1196	0.24632	0.28395	0.314	0.3333
	λ_4	**0.57409**	0.18393	0.11014		0.0465	0.2083
	λ_5		0.36988				
	λ_6			0.31112			
	λ_8				0.41592		
	λ_{11}					0.3837	
ρ	ρ_5	**0.24123**					
	ρ_6	**0.75877**	0.78555	0.76611	0.22919		0.6667
	ρ_7		0.21445	0.23389	0.77081	0.814	0.3333
	ρ_8					0.186	
Threshold Eb/No	DE[dB]	**0.9114**	0.9194	0.6266	0.4483		
	GA[dB]	**0.874**	0.832	0.8	0.5778	0.6446	1.533
	mod_GA[dB]	**0.61**	0.587	0.412	0.165	0.011	1.134

Numerical Results



Length : 2000 Code rate : 0.5

v_node
- deg 2 : 0.38354
- deg 3 : 0.04237
- deg 4 : 0.57409

c_node
- deg 5 : 0.24123
- deg 6 : 0.75877

Conclusions

- **Which equations are wrong?**

$$\varepsilon\left\{\tanh\left(\frac{M}{2}\right)\right\} \overset{?}{=} \sum_i \lambda_i \mathcal{E}\left\{\tanh\left(\frac{M_i}{2}\right)\right\} = 1 - \sum_i \lambda_i \phi\left(m_{M,i}(l)\right)$$

$$? \parallel \qquad\qquad\qquad\qquad \nparallel$$

$$1 - \phi\left(m_M(l)\right) = 1 - \phi\left(\sum_i \lambda_i m_{M,i}(l)\right)$$

- It seems that the original updating rule is **a linear approximation** of the new updating rule about ϕ.
- The updating speed of the new updating rule is about **10 times faster** than that of the original rule, in the simulation.