Construction of Reed-Solomon Based Quasi-Cyclic LDPC Codes Based on Protograph

Inseon Kim and Hong-Yeop Song

Yonsei University, Korea

The 24th Asia-Pacific Conference on Communications (APCC2018)

Nov. 13, 2018
Contents

- Introduction
- Motivation

- 2 Constructions of QC-LDPC codes
  - Construction of RS based QC-LDPC codes
  - QC-LDPC codes based on Protograph

- Proposed construction of Protograph based RS-QC-LDPC codes

- Conclusion
Introduction

- **LDPC (Low-Density Parity Check) Codes**
  - LDPC codes are code family with parity-check matrix of \((\# \text{ of } 1s) \ll (\# \text{ of } 0s)\)
  - LDPC codes first proposed by Gallager in early 1960s
  - LDPC code family are known to approach Shannon limits (by MacKay).
  - LDPC codes have good error correcting performance with good encoding and decoding process.
  - LDPC codes are selected as error correcting codes in many communication system standards.
    e.g.) IEEE 802.11 WLAN, Broadcasting(DVB-T2, ATSC3.0), 5G communication systems...
Introduction

- **LDPC (Low-Density Parity Check) Codes**
  - Parity-check matrix can be represented as bipartite (Tanner) graph; Check nodes, Variable nodes, edges
  - Short cycles and girths play a key role to show good BER performance
  - Many researchers concerns to reduce the number of short cycles of Tanner graph

![Fig. 1 Tanner graph of LDPC codes](image)

**Construction of RS based QC-LDPC codes**
Introduction

- **Protograph based LDPC codes**
  - A bipartite graph with multiple edges between variable nodes and check nodes
  - We can get Tanner graph of LDPC codes using protograph with copy-and-permute procedure
  - Recently, LDPC based on protograph gains attention because protograph has simple construction and analysis.

![Copy-and-permute procedure](image)

*Fig. 2 Copy-and-permute procedure*
Motivation

Proposed Construction

Construction of Reed Solomon based QC-LDPC codes

Girth ↑

Protograph based QC-LDPC codes

Minimum Hamming distance ↑
Construction of RS-QC-LDPC codes

- **Reed-Solomon based QC-LDPC codes**
  - Parity-check matrix of RS codes over $GF(2^s)$
    \[
    B_{RS}(d, n) = \begin{bmatrix}
    1 & \beta & \cdots & \beta^{n-1} \\
    1 & \beta^2 & \cdots & (\beta^2)^{n-1} \\
    \vdots & \vdots & \ddots & \vdots \\
    1 & \beta^d & \cdots & (\beta^d)^{n-1}
    \end{bmatrix}
    \]
    
    $\beta$ is an element of order $n$, where $n$ is a factor of $2^s - 1$
  - Parity-check matrix of RS based QC-LDPC codes
    \[
    H_{RS}(d, n) = \begin{bmatrix}
    H_{0,0} & H_{0,1} & \cdots & H_{0,n-1} \\
    H_{1,0} & H_{1,1} & \cdots & H_{1,n-1} \\
    \vdots & \vdots & \ddots & \vdots \\
    H_{d-1,0} & H_{d-1,1} & \cdots & H_{d-1,n-1}
    \end{bmatrix}
    \]
    
    $H_{i,j}$ is $r \times r$ Identity matrix cyclically shifted by the elements of $B_{RS}(i,j)$
  - This RS based QC-LDPC codes has girth at least 6[1].

Construction of RS-QC-LDPC codes

- Construction of RS based QC-LDPC codes with girth 8[1]
  - Make $B_{RS}(d, t)$ by choosing $t$ columns of $B_{RS}(d, n)$
  - $\Lambda_t = \{l_1, l_2, \ldots, l_t\}$: Index set of selected $t$ columns and satisfying following equations
    \[
    \begin{align*}
    &l_i \neq 2l_{i_2} - l_{i_1}, l_i \neq 3l_{i_2} - 2l_{i_1}, l_i \neq \frac{3l_{i_2} - l_{i_1}}{2}, \\
    &n \nmid l_{i_2} + l_{i_3} - 2l_{i_1}, n \nmid l_{i_2} + 2l_{i_3} - 3l_{i_1}, n \nmid 2l_{i_2} + l_{i_3} - 3l_{i_1}, \\
    &n \nmid 2l_{i_3} - l_{i_1} - l_{i_2}, n \nmid 3l_{i_3} - 2l_{i_1} - l_{i_2}, n \nmid 3l_{i_3} - l_{i_1} - 2l_{i_2},
    \end{align*}
    \]
  - Column selection is based on extending cycles of length 6 $\rightarrow$ length 8
  - RS based QC-LDPC codes with parity-check $B_{RS}(d, t)$ has girth at least 8

QC-LDPC codes based on protograph

- Protograph: allow more than 2 edges b/w check and variable nodes
- Copy small Tanner graph and lift the edges to copied graph

Fig. 3 Example of protograph of QC-LDPC codes

Fig. 4 Extended Tanner graph of protograph of QC-LDPC codes
QC-LDPC codes based on protograph

- Calculate upper bound of minimum Hamming distance of QC-LDPC codes based on protograph\[2\]

- Using a function of weight matrix of protograph, they explicit the upper bound of minimum Hamming distance of protograph.

- They found out that well constructed protograph with multi-edge have better upper bound of minimum Hamming distance than protograph with single edge

\[
H = \begin{bmatrix}
I_0 + I_1 & I_1 & 0 & I_2 \\
I_2 & I_0 & I_1 & I_2 \\
0 & I_1 & I_0 + I_1 & I_1 \\
\end{bmatrix}
\]

Parity matrix of Protograph of QC-LDPC codes(Fig.4)

\[
wt(H) = \begin{bmatrix}
2 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 \\
0 & 1 & 2 & 1 \\
\end{bmatrix}
\]

Weight matrix

Protograph based RS-QC-LDPC codes

- Procedures of proposed constructions

1. Consider $B_{RS}(d, t)$ satisfying the equations as mentioned before.

2. Construct the weight matrix of protograph by following conditions: make the degree of check nodes or variable nodes irregular.

3. With constructed the weight matrix of protograph, extend the graph as parity-check matrix of LDPC codes.
Propose the construction of QC-LDPC codes combining RS based QC-LDPC codes with girth 8 and protograph with increasing minimum Hamming distance.

- Protograph based RS-QC-LDPC codes still holds the property that the girth is at least 8.
- Protograph based RS-QC-LDPC codes would have better error-correcting performance than RS based QC-LDPC codes because it has increased upper bound of minimum Hamming distance.
Protograph based RS-QC-LDPC codes

Simulation

- Assume AWGN channel and BPSK modulation
- Attain BER performance under sum-product decoding with maximum 50 iterations
- Use the parity-check matrix of RS codes over $GF(2^8)$
- Construct 2 weight matrix of Protograph based RS-QC-LDPC codes with rate 1/2 and 2/3.

$\Lambda_8 = \{2, 5, 7, 13, 20, 32, 54, 60\}$

$wt(H_{PRS}(4,8)) = \begin{bmatrix} 2 & 2 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 2 & 2 \end{bmatrix}$

Rate 1/2

$\Lambda_{12} = \{0, 1, 4, 9, 11, 20, 24, 35, 41, 49, 90, 225\}$

$wt(H_{PRS}(4,12)) = \begin{bmatrix} 2 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 0 & 0 \end{bmatrix}$

Rate 2/3
Protograph based RS-QC-LDPC codes

- Simulation

- observe that BER gain is about 0.15 dB at BER $10^{-5}$ and BLER gain is about 0.12 dB at BLER $10^{-4}$.

- Error correcting performance of Proto-RS-QC-LDPC have more gain when the codes have high rate.
Conclusion

- Propose the construction of Protograph based RS-QC-LDPC codes combining two existing QC-LDPC codes construction
  - RS based QC-LDPC codes with girth 8
  - Protograph which increase the upper bounds of minimum Hamming distance

- We observe that Protograph based RS-QC-LDPC codes have better error-correcting performance than existing RS based QC-LDPC codes
  - We found out that the high rate codes have more coding gain.

- Future work
  - Can we propose protograph construction which will increase the minimum Hamming distance itself?
  - Can we propose the performance analysis of Protograph based RS-QC-LDPC codes with enumerator function or EXIT chart?
Thank you for listening

Any Questions? Or comments?