Performance Analysis of QC-LDPC codes constructed by using Golomb rulers

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Introduction

Introduction

- QC-LDPC codes are widely studied and used because of its simple encoding and parallel decoding (5GNR).

- Golomb ruler is a sequence of n marks in integer position such that every distances between two marks are distinct

- Recently, a paper about construction method for girth-8 QC-LDPC codes using Golomb rulers is published [1].

- We check and analyze the performance of half rate QC-LDPC codes of length 900, 1200 from the construction in [1]

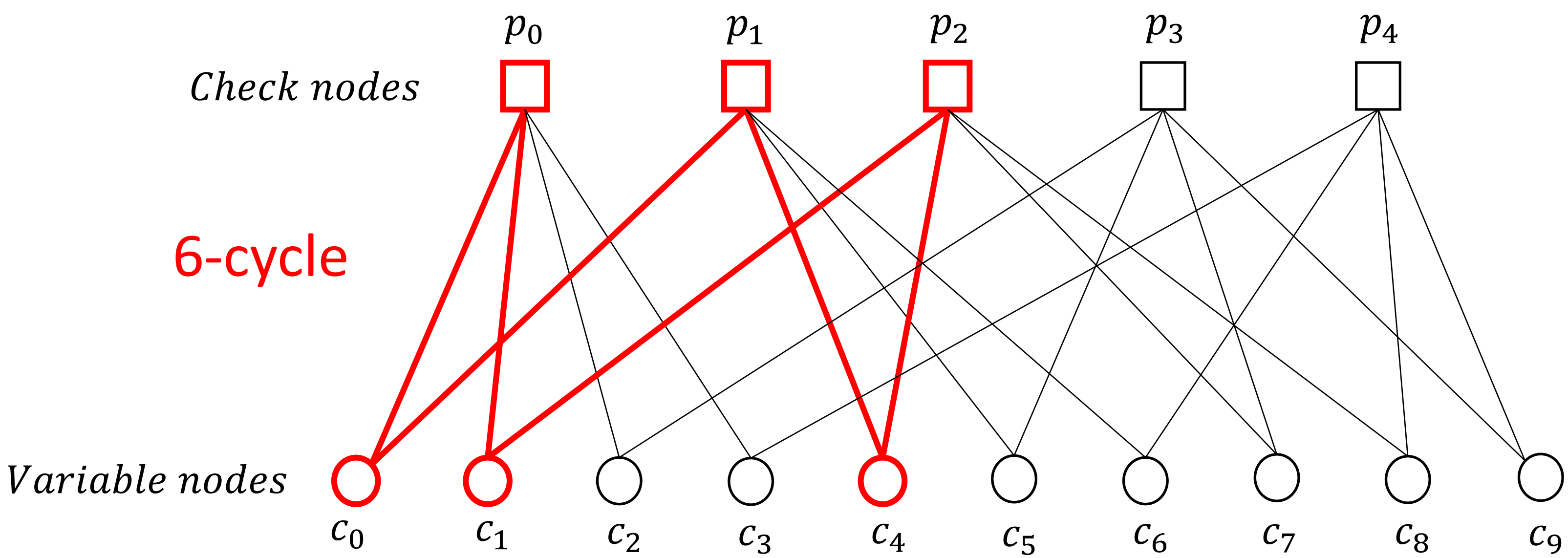
[1] I. Kim and H.-Y. Song, "A construction for girth-8 QC-LDPC codes using Golomb rulers," Electronic Letters, vol. 58, no. 15, pp. 582-584, July 2022.

LDPC codes and Golomb ruler

Low-Density Parity-Check (LDPC) code

- LDPC code is a linear block code whose parity check matrix have sparse non-zero elements

- Tanner graph can represent parity relations of codewords

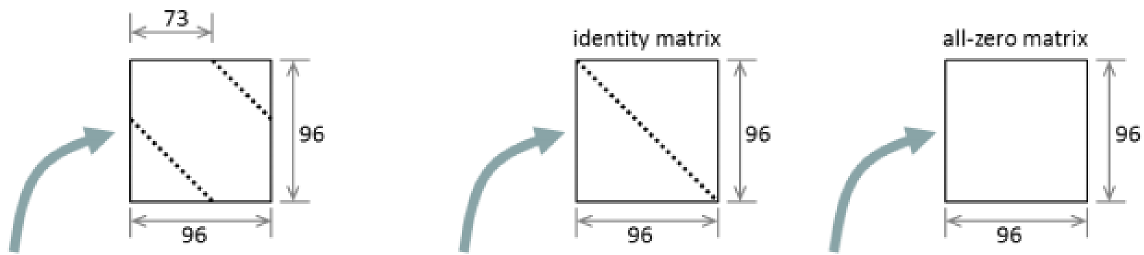


- Avoid short length cycles in Tanner graph for designing LDPC codes

Quasi-Cyclic(QC) LDPC codes

- Widely studied since its simple encoding scheme, parallel decoding

- Adopted in many mobile communications like WIMAX, 5G NR, ...
  - WIMAX LDPC code (N=2304)



$B =$

-1	94	73	-1	-1	-1	-1	-1	55	83	-1	-1	7	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	27	-1	-1	-1	22	79	9	-1	-1	-1	12	-1	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	24	22	81	-1	33	-1	-1	-1	0	-1	-1	0	0	-1	-1	-1	-1	-1	-1	-1	-1
61	-1	47	-1	-1	-1	-1	-1	65	25	-1	-1	-1	-1	-1	0	0	-1	-1	-1	-1	-1	-1	-1
-1	-1	39	-1	-1	-1	84	-1	-1	41	72	-1	-1	-1	-1	-1	0	0	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	46	40	-1	82	-1	-1	-1	79	0	-1	-1	-1	-1	0	0	-1	-1	-1	-1	-1
-1	-1	95	53	-1	-1	-1	-1	-1	14	18	-1	-1	-1	-1	-1	-1	-1	0	0	-1	-1	-1	-1
-1	11	73	-1	-1	-1	2	-1	-1	47	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	-1	-1	-1
12	-1	-1	-1	83	24	-1	43	-1	-1	-1	51	-1	-1	-1	-1	-1	-1	-1	-1	0	0	-1	-1
-1	-1	-1	-1	-1	94	-1	59	-1	-1	70	72	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	-1
-1	-1	7	65	-1	-1	-1	-1	39	49	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
43	-1	-1	-1	-1	66	-1	41	-1	-1	-1	26	7	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0

← Layer 1 (96 rows)
← Layer 2 (96 rows)
(when layered scheduling is used)
← Layer 12 (96 rows)

- Constructed from exponent matrix (may have some algebraic structure)

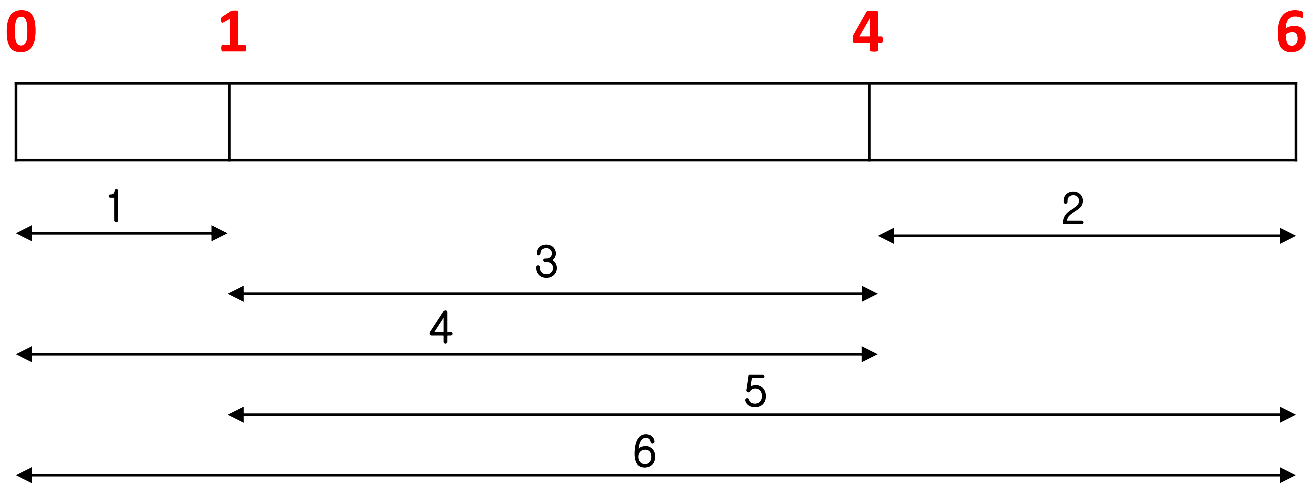
Golomb ruler

- Golomb ruler is a sequence of n-marks in integer position such that every distance between two marks is distinct

- n-mark Golomb ruler can be represented in ascending order like

$$\{ g_1, g_2, \ldots, g_n \}$$

- Example of 4-mark Golomb ruler $\{0, 1, 4, 6\}$



Distances are all distinct

Constructing QC-LDPC codes using Golomb ruler[1]

- Generate exponent matrix from multiplication table

×	0	1	8	12	14	17
1	0	1	8	12	14	17
2	0	2	16	24	28	34
3	0	3	24	36	42	51

$$E = \begin{bmatrix} e(0,0) & e(0,1) & e(0,2) & e(0,3) & e(0,4) & e(0,5) \\ e(1,0) & e(1,1) & e(1,2) & e(1,3) & e(1,4) & e(1,5) \\ e(2,0) & e(2,1) & e(2,2) & e(2,3) & e(2,4) & e(2,5) \end{bmatrix}$$

3×6 size

$I : P \times P$ size Identity matrix
$I(t)$: t time cyclic shift of I

$3P \times 6P$ size

$$H = \begin{bmatrix} I(e(0,0)) & I(e(0,1)) & I(e(0,2)) & I(e(0,3)) & I(e(0,4)) & I(e(0,5)) \\ I(e(1,0)) & I(e(1,1)) & I(e(1,2)) & I(e(1,3)) & I(e(1,4)) & I(e(1,5)) \\ I(e(2,0)) & I(e(2,1)) & I(e(2,2)) & I(e(2,3)) & I(e(2,4)) & I(e(2,5)) \end{bmatrix}$$

* I. Kim and H.-Y. Song, "A construction for girth-8 QC-LDPC codes using Golomb rulers," Electronic Letters, vol. 58, no. 15, pp. 582-584, July 2022.

Girth property

- Girth is the shortest length of cycle(s) in a graph

Theorem [1]
The QC-LDPC codes from exponent matrix constructed from Golomb ruler have **girth 8** if **P > 2L,** where P is the modulus in the construction of the exponent matrix and L is the length of the Golomb ruler.

- The length L of n-mark Golomb ruler $\{ g_1, g_2, \dots, g_n \}$ is

$$L = g_n - g_1$$

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Constructing another Golomb ruler

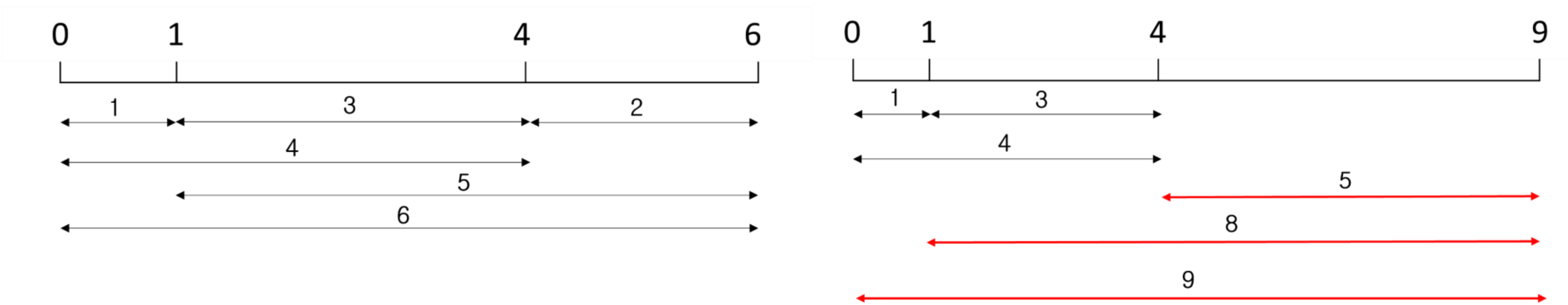
- If $g_1 = 0$, change g_n (of n-mark Golomb ruler) to g_n^* as

Original Golomb ruler

$$2g_{n-1} < g_n^*$$

New Golomb ruler

$$\{ g_1, g_2, \dots, g_{n-1}, \boldsymbol{g_n} \} \longrightarrow \{ g_1, g_2, \dots, g_{n-1}, \boldsymbol{g_n^*} \}$$



- To satisfy girth 8 in QC-LDPC construction ($g_1 = 0$)

$$2g_{n-1} < g_n^* = L < P/2$$

Performance Comparison

Simulation

- QC-LDPC codes
  - construct using 6-mark Golomb ruler $\{0,1,8,12,14, g_6\}$ where

$$g_6 = 29, 30, \dots, 74 \quad (P = 150)$$
$$g_6 = 29, 30, \dots, 99 \quad (P = 200)$$

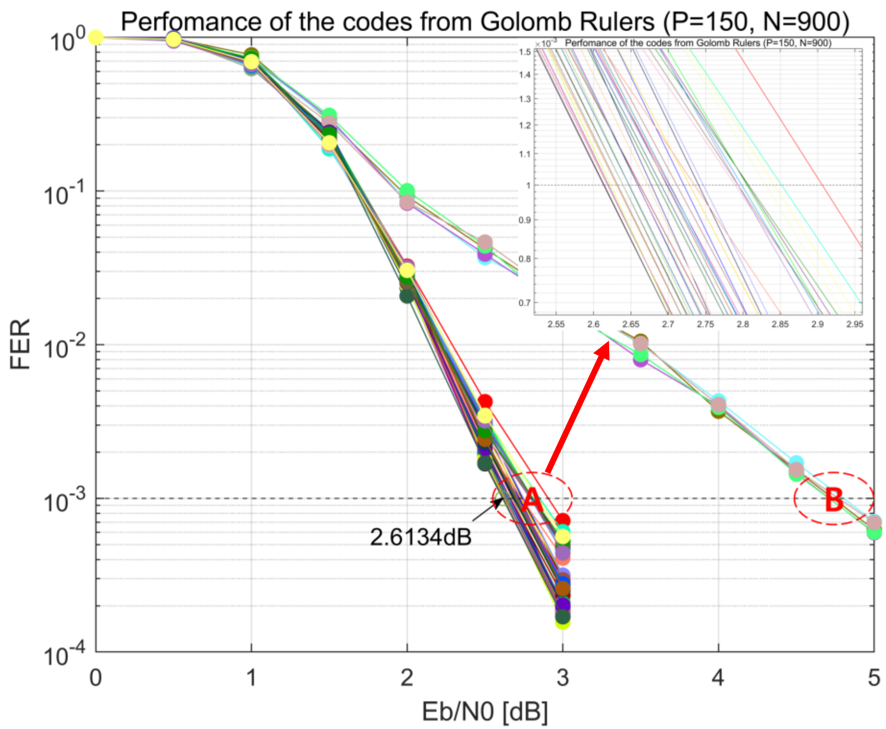
⇒ Satisfying girth 8

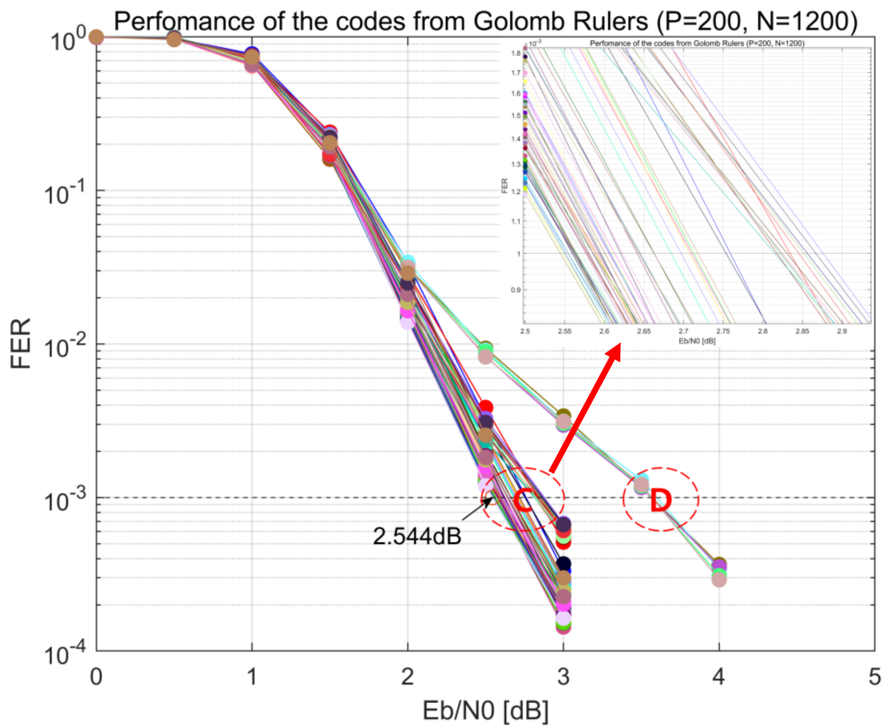
- Assume AWGN channel, BPSK modulation

- Decoded using sum-product algorithm (maximum 50 iterations)

- Checked 200 frame errors to estimate Frame Error Rate(FER) using Monte-Carlo method

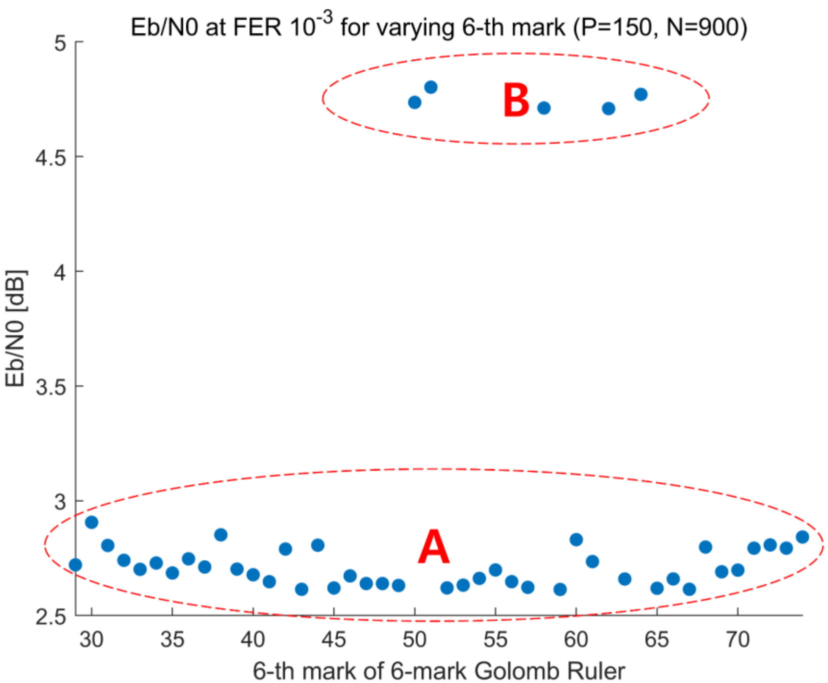
Performance comparison

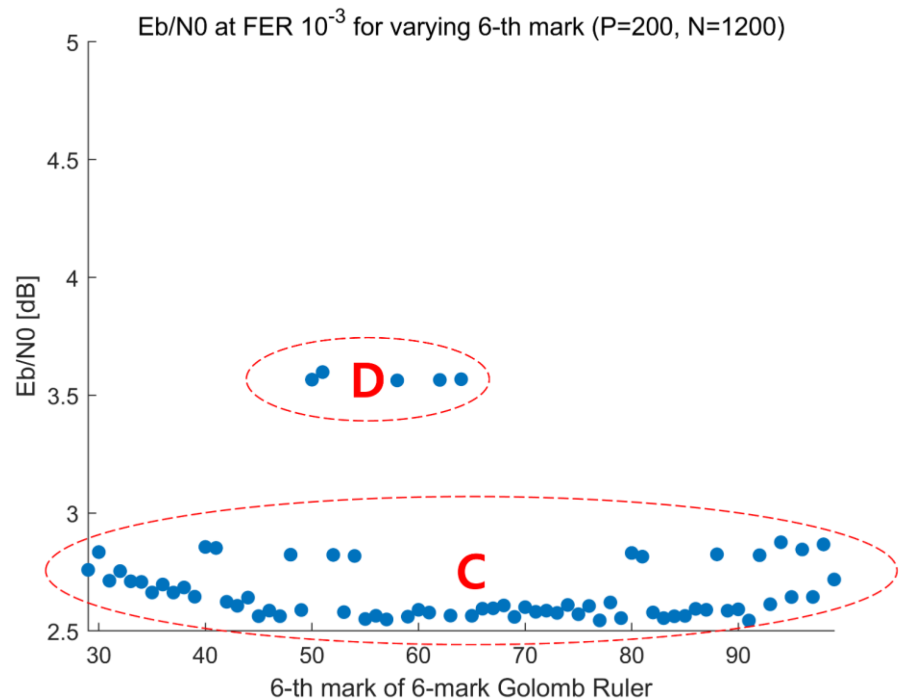




- Best($FER\ 10^{-3}$) : $g_6 = 59$ (P = 150, N = 900), $g_6 = 91$ (P = 200, N = 1200),
- Most cases(group A, C) shows 2.5~3dB at FER 10^{-3}
- Few cases(group B, D) show apparently worse performance

Performance comparison





Golomb ruler

Both case
(P=150, P=200)

$$g_6 = 50, 51, 58, 62, 64$$

{**0**, 1,8,12,14, **50**}

{0, **1**, 8,12,14, **51**}

{0,1, **8**, 12,14, **58**}

{0,1,8, **12**, 14, **62**}

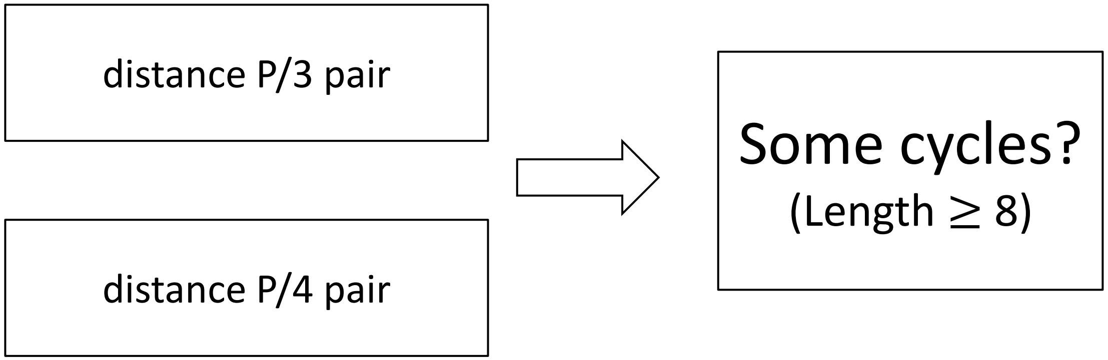
{0,1,8,12, **14**, **64**}

distance 50(=P/3) pair (P=150)

distance 50(=P/4) pair (P=200)

Conclusion

- Constructed half rate QC-LDPC codes with 900, 1200 length using previously proposed method [1]
- Checked that most constructed codes shows performance between 2.5dB~3dB at FER 10^{-3}
- Most of constructed QC-LDPC codes shows 2.5~3dB at FER 10^{-3}
- But there are few cases that show worse performance
- Additional analysis considering other factors is required



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Thank for Your Attention