Trellis Structure and Performance Analysis of Space-Time Trellis Codes from Optimal Product Distance Codes

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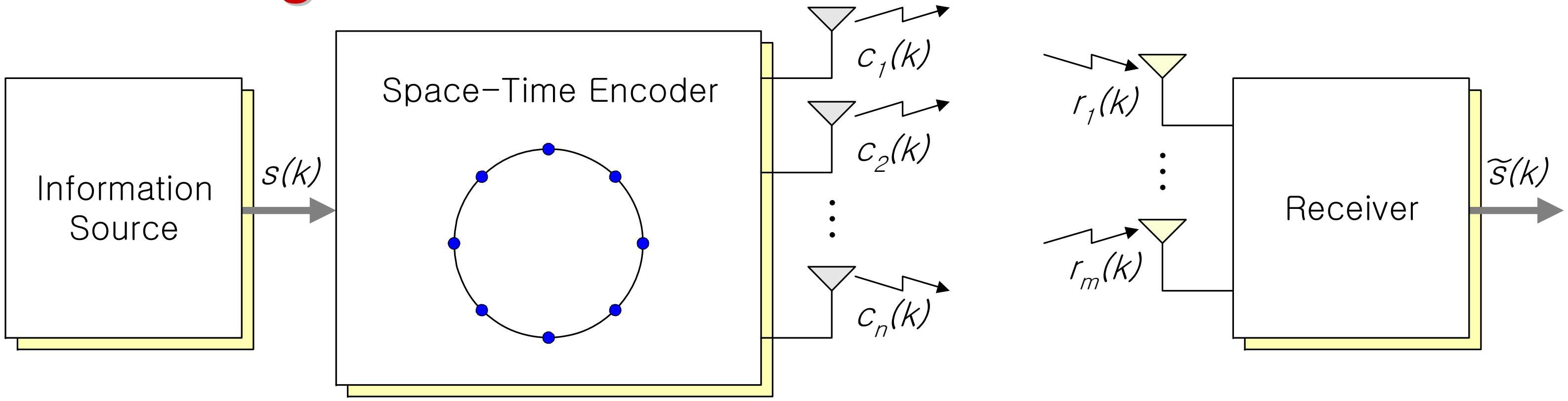
MIMO System with STTC

- Block Diagram



- Received Signal

$$r_t^j = \sqrt{E_s / n} \sum_{i=1}^{n} \alpha_{j,i} c_t^i + \eta_t^j \qquad \mathbf{r}_t = \sqrt{E_s / n}\, \mathbf{H} \cdot \mathbf{c}_t + \mathbf{n}_t$$

- ML Decoding (Space-Time Viterbi Decoding)

$$\sum_{t=1}^{l} \sum_{j=1}^{m} \left| r_t^j - \sqrt{E_s / n} \sum_{i=1}^{n} \alpha_{j,i}\, c_t^i \right|^2 \qquad \tilde{\mathbf{C}} = \arg\min_{\mathbf{C}} \sum_{t=1}^{l} \left\| \mathbf{r}_t - \sqrt{E_s / n}\, \mathbf{H} \cdot \mathbf{c}_t \right\|$$

Design Criteria for STTC

- Fundamental Bound (Tarokh et. al)

$$\Pr(\mathbf{c} \to \mathbf{e}) \le \left(\frac{\eta E_s}{4N_0} \right)^{-rL_r}$$

- Rank Criterion
  - Maximize the diversity advantage

$$r = \operatorname{rank}\left(f(\mathbf{c}) - f(\mathbf{e}) \right)$$

over all pairs of distinct codewords **c**, **e** ∈ C

- Determinant Criterion
  - Maximize the coding advantage

$$\eta = \left(\lambda_1 \lambda_2 \cdots \lambda_r \right)^{1/r}$$

over all pairs of distinct codewords **c**, **e** ∈ C

Where η is the geometric mean of nonzero eigenvalues of

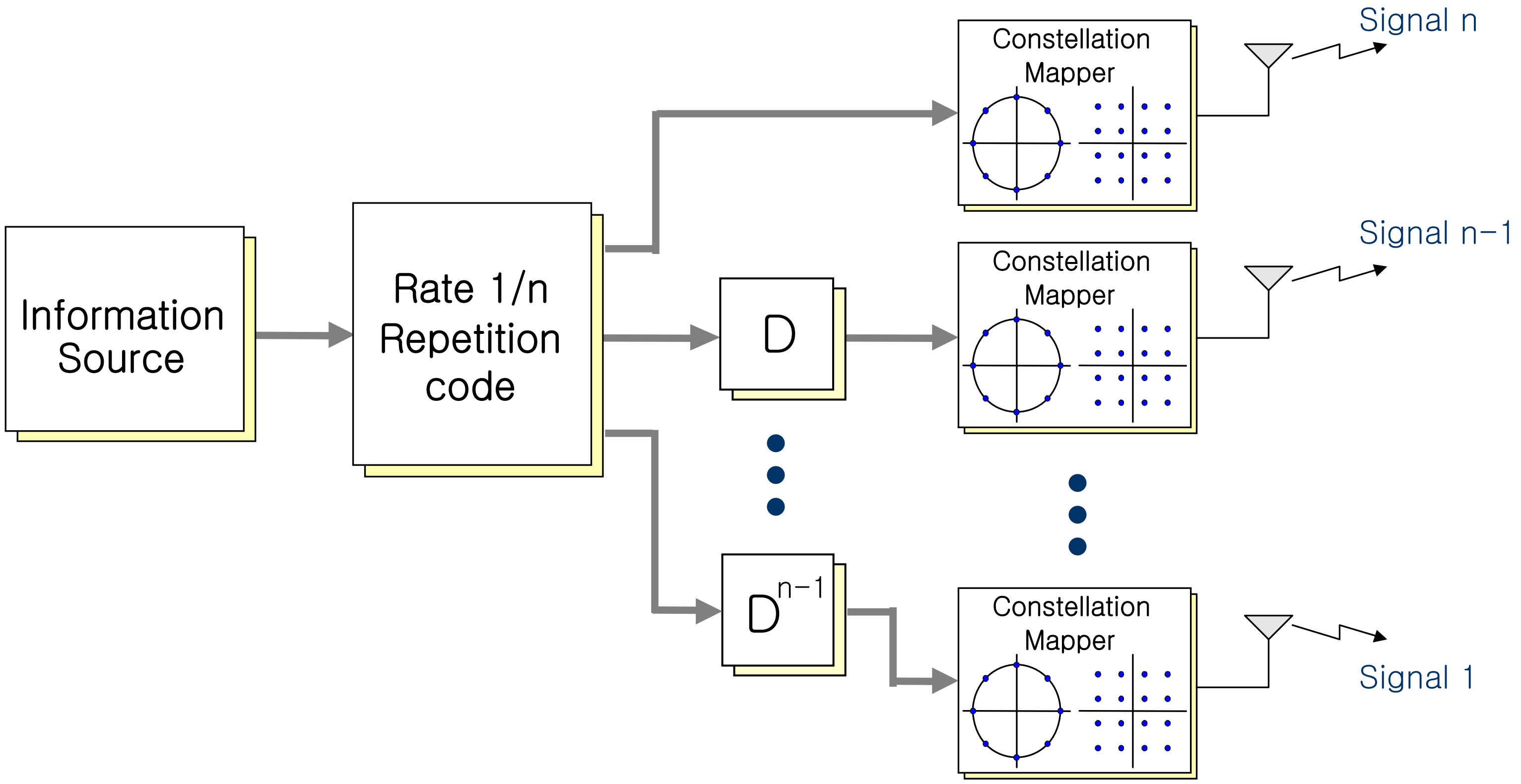
$$\mathbf{A} = \left(f(\mathbf{c}) - f(\mathbf{e}) \right)\left(f(\mathbf{c}) - f(\mathbf{e}) \right)^H$$

STTC Design (So far)

- Tarokh et.al
  - Geometrically Uniform Code
    - Reduced complexity in computing the coding gain
  - 4PSK : 4,8,16,32 state, 8PSK : 8,16,32 state
- Search Optimal STTC
  - Based on Exhaustive Search
    - Baro: With Generator Matrix (2Tx, QPSK, 4, 8,16states)
    - Grimm: Within Zero-Symmetry Domain
    - Blum: With Coding gain Algorithm
  - Mostly on the case of the number of Tx Antenna is 2
  - BPSK, QPSK
- Systematic Construction
  - Hammons: From optimum d_{free} convolutional code
  - BPSK only

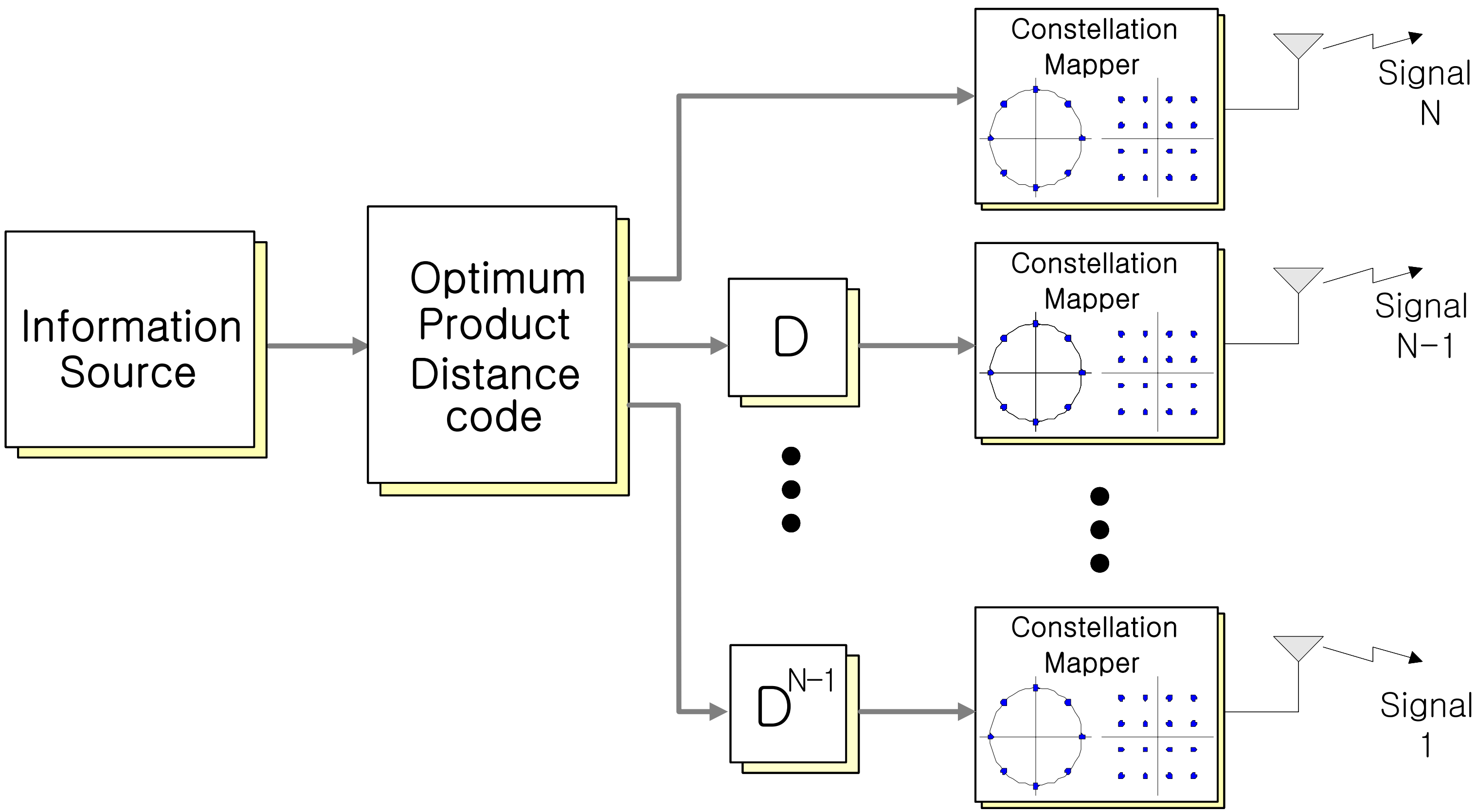
Delay Diversity

- Delay Diversity = Repetition code + Delay element between multiple Tx Antennas



STTC using Opt. PDC

Transmitted Signal with Opt. PDC



Product Distance Code

- Consider a block Code *C*

$$C = \{\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_M\}$$

  - M codewords (with length N)
  - i-th codeword

$$\mathbf{c}_i = c_i^1 c_i^2 \cdots c_i^N, \quad c_i^m \in Z_M$$

- Product Distance

$$D_{(\mathbf{c}_i, \mathbf{c}_j)} = \prod_{m=1}^{N} \left| f(c_i^m) - f(c_j^m) \right|$$

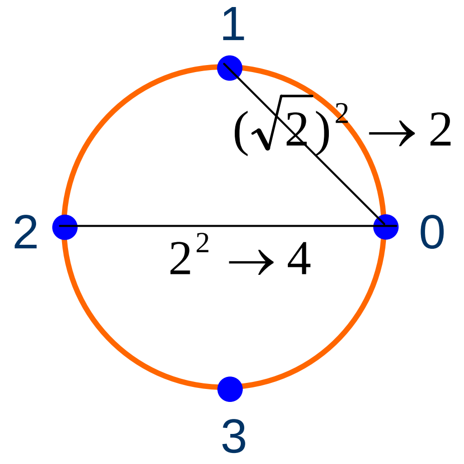
  - With M-ary Modulation
  - f maps the symbol element to signal constellation

Product Distance Code

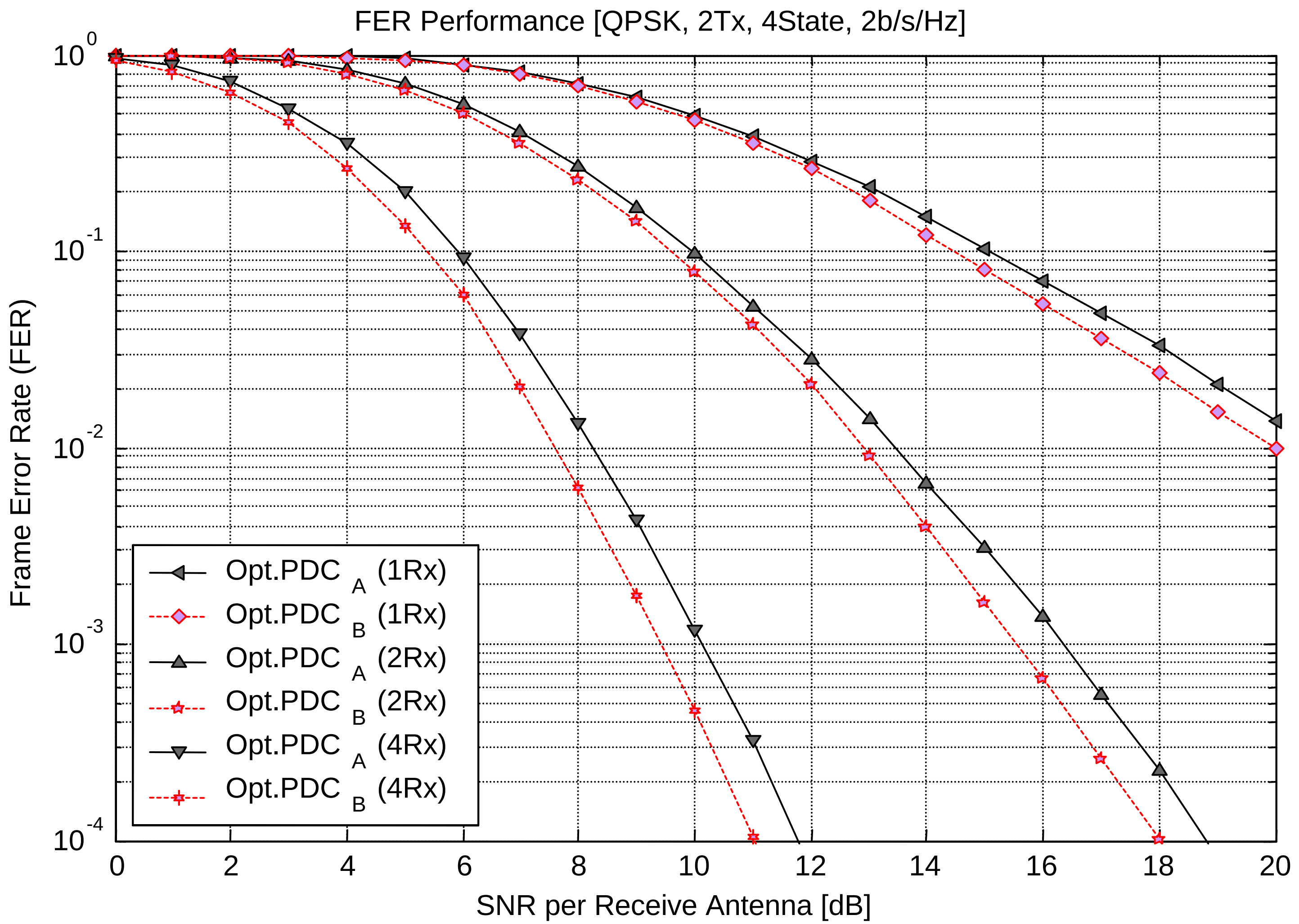
Example(QPSK, Tx Ant.: 3)

- Opt. Product Distance Code {000, 112, 231, 323}

Delay Diversity	Opt. PDC
{000, 111, 222, 333}	{000, 112, 231, 323}
8 8 8	16 16 16
64 64	16 16
8	16
Minimum Dist. = 8	Minimum Dist. = 16
# of Minimum = 4	# of Minimum = 6
Avg. Dist. = 26.67	Avg. Dist. = 16.00



STTC using Opt. PDC



STTC Using Opt. PDC

Example(QPSK, Tx Ant.: 2)

Opt. PDC A

{00, 11, 22, 33}

4 4 4

16 16

4

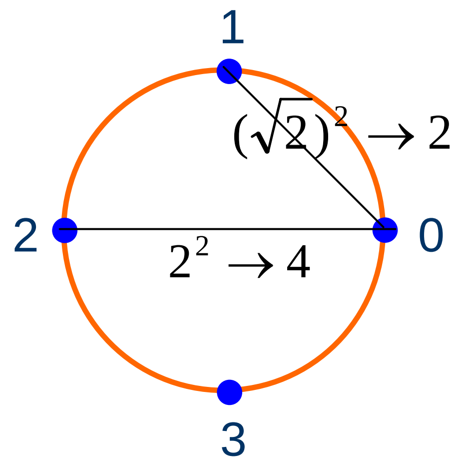
Opt. PDC B

{00, 11, 23, 32}

4 8 4

8 8

8



Minimum Dist. = 4

# of Minimum = 4

Avg. Dist. = 8

Minimum Dist. = 4

# of Minimum = 2

Avg. Dist. = 6.67

Design Criterion

Definitions

- To Maximize the coding gain

$$D_{\min} = \min_{i \neq j} D_{(\mathbf{c}_i, \mathbf{c}_j)} = \min_{i \neq j} \prod_{m=1}^{N} \left| f(c_i^m) - f(c_j^m) \right|$$

- $N_{\min}$: the number of distinct codeword pairs (ci, cj) with

$$D_{(\mathbf{c}_i, \mathbf{c}_j)} = D_{\min}$$

- D_{avg}: the average product distance between pairs of distinct codewords

Criterion

- Opt. PDC with $N_{\min}$ minimized
- Opt. PDC with D_{avg} maximized

Search Results (QPSK Opt. PDC)

QPSK

Tx	PDC	D_{opt}	N_{min}	D_{avg}
2	0 1 2 3 0 1 3 2	4	2	6.667
	0 1 2 3 0 1 2 3		4	8
3	0 1 2 3 0 1 3 2 0 2 1 3	16	6	16
4	0 1 2 3 0 1 2 3 0 1 3 2 0 2 1 3	32	4	42.667
5	0 1 2 3 0 1 2 3 0 1 3 2 0 1 3 2 0 2 1 3	64	2	106.667
	0 1 2 3 0 1 2 3 0 1 2 3 0 1 3 2 0 2 1 3		4	128

Search Results (8PSK, 16QAM)

8PSK

Tx	PDC	D_{opt}	N_{min}	D_{avg}
2	0 1 2 3 4 5 6 7 0 3 6 1 4 7 2 5	2	16	4.57143
3	0 1 2 3 4 5 6 7 0 2 5 7 3 1 6 4 0 3 7 4 1 6 2 5	4	12	6.28571
	0 1 2 3 4 5 6 7 0 2 4 7 1 6 3 5 0 3 6 2 5 7 1 4		18	8.28571
4	0 1 2 3 4 5 6 7 0 1 5 3 7 2 6 4 0 3 2 5 1 7 6 4 0 3 7 5 4 2 6 1	4	2	25.5714
	0 1 2 3 4 5 6 7 0 1 2 3 4 5 6 7 0 3 6 1 5 2 7 4 0 3 7 4 1 6 2 5		16	17.2857

16QAM

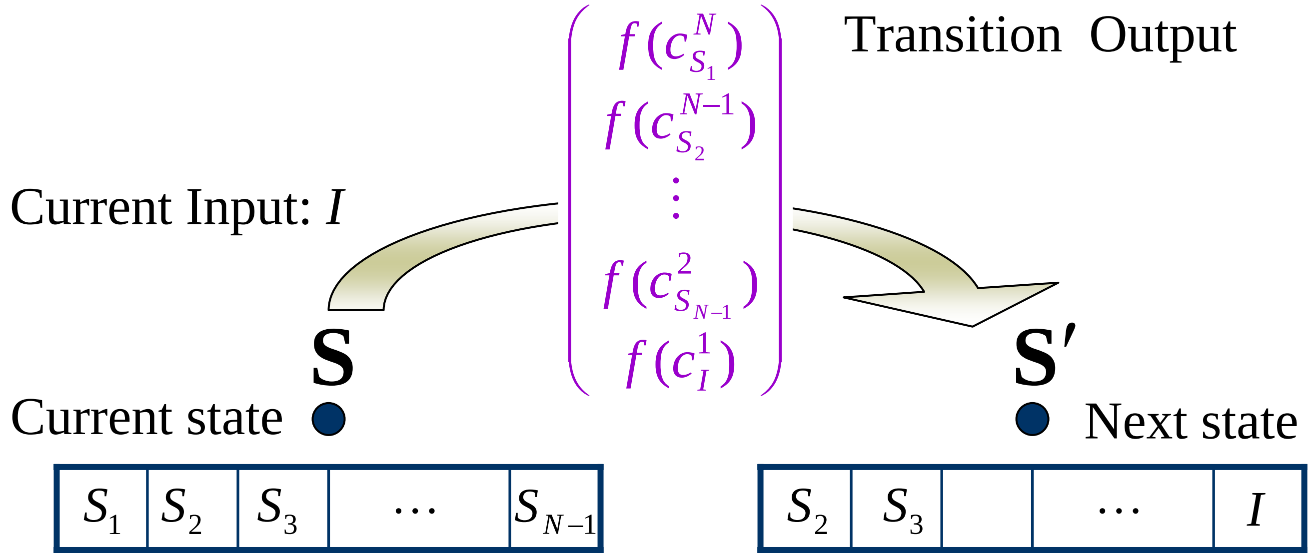
2	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 0 6 13 11 9 15 4 2 7 1 10 12 14 8 3 5	64	4	426.667
	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 0 2 4 6 8 10 12 14 1 3 5 7 9 11 13 15		33	494.933

Trellis Structure of Opt. PDC

- Given Opt. PDC (M-ary, n Tx Ant)

$$\begin{pmatrix} 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{M}-1 & \mathrm{M}-1 & \cdots & \mathrm{M}-1 \end{pmatrix} \Rightarrow \begin{pmatrix} c_0^1 & c_0^2 & \cdots & c_0^N \\ c_1^1 & c_1^2 & \cdots & c_1^N \\ \vdots & \vdots & \ddots & \vdots \\ c_{M-1}^1 & c_{M-1}^2 & \cdots & c_{M-1}^N \end{pmatrix}$$

- Trellis Structure



System Description

Channel Model

- Quasi-static flat fading channel + AWGN Channel
- 1 frame = 130 symbols (l=130, IS-54)

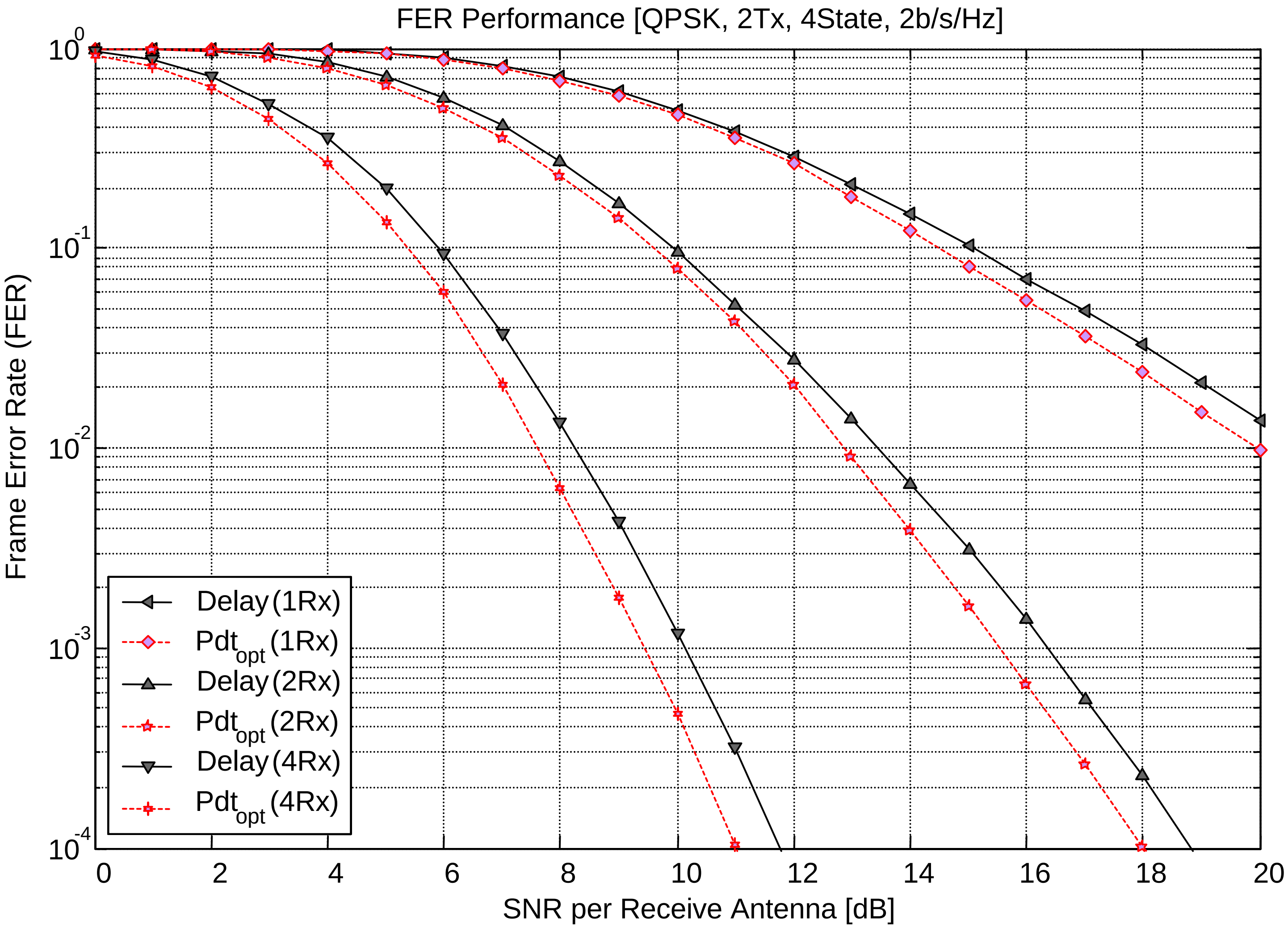
$$\mathbf{r} = \mathbf{H}\mathbf{c} + \mathbf{n} \qquad \mathbf{H} = [\alpha_{ij}] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix}$$

- Ideal channel estimation

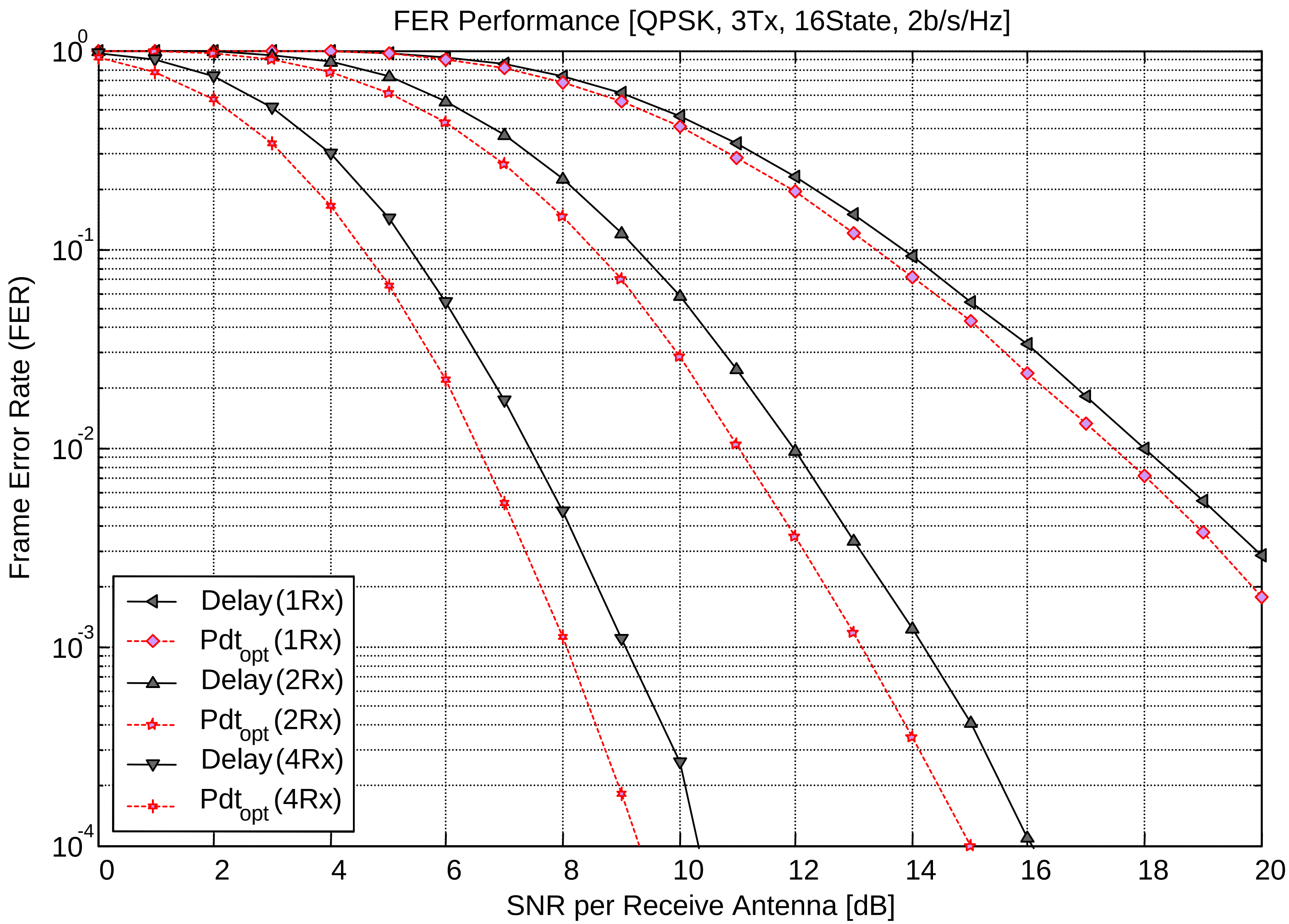
System model

- Modulation: QPSK, 8PSK, 16QAM
- Space-Time Viterbi Decoder (ML)
- Unquantized Soft Decision

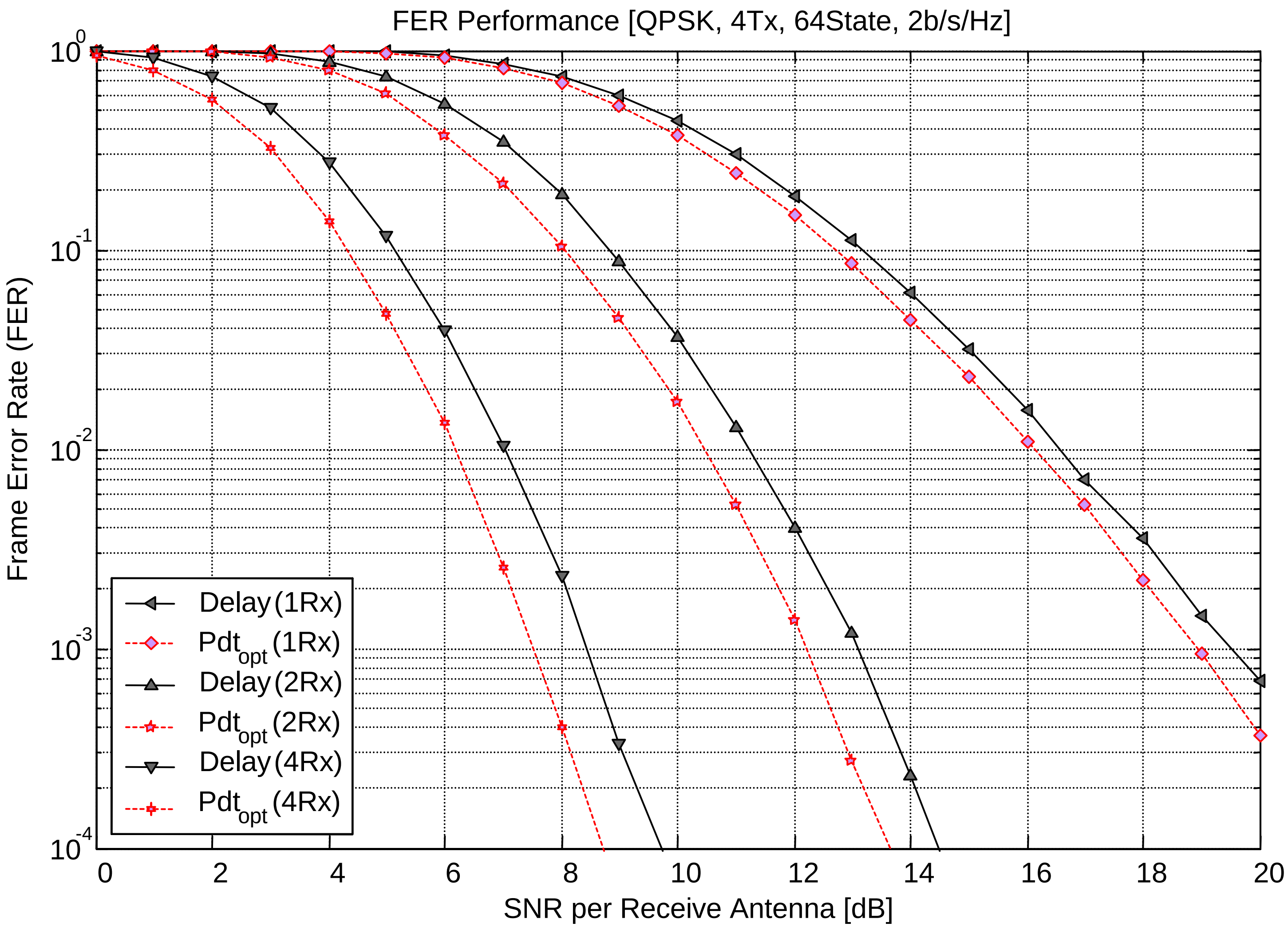
Delay VS Opt. PDC [QPSK, 2Tx]



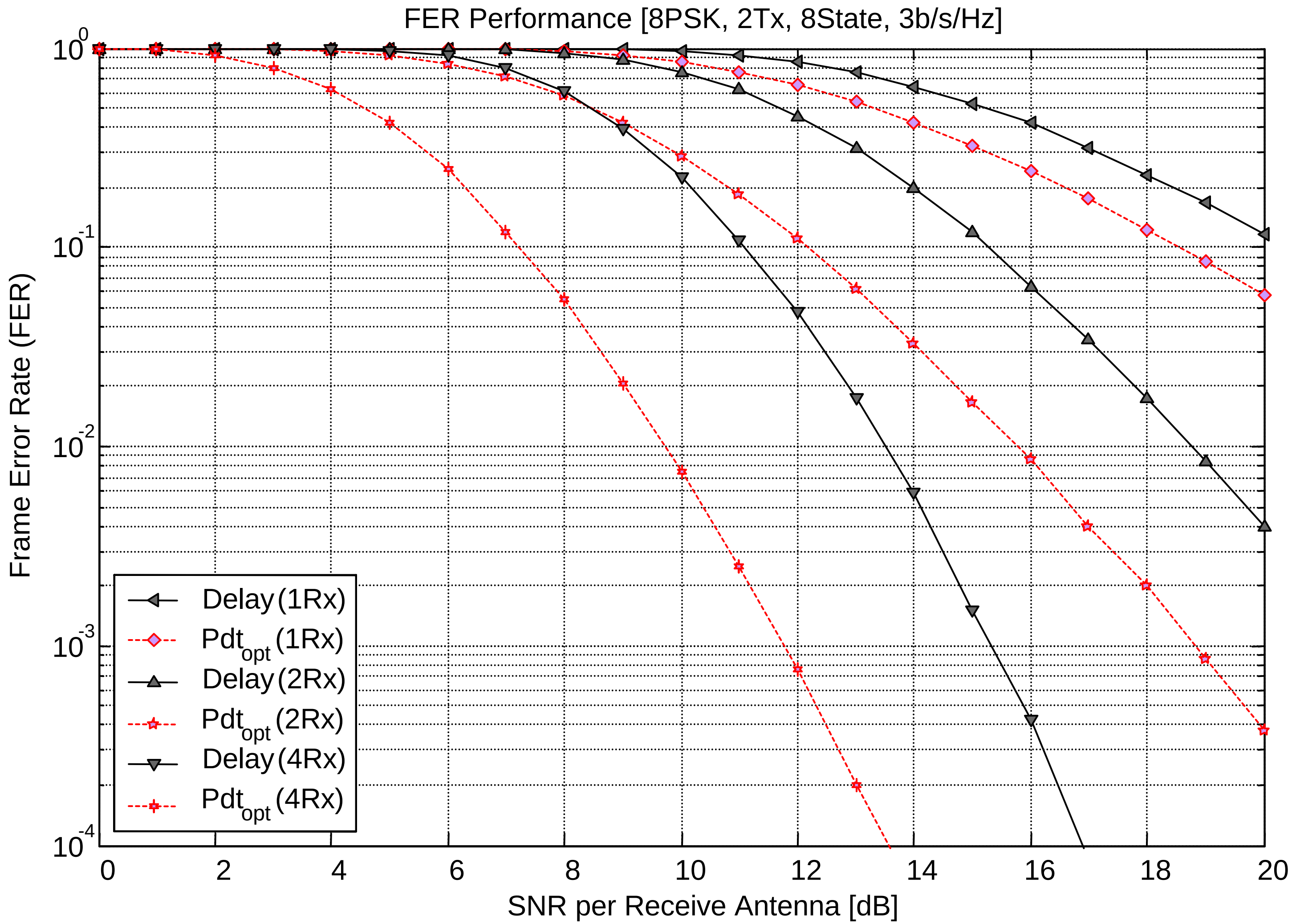
Delay VS Opt. PDC [QPSK, 3Tx]



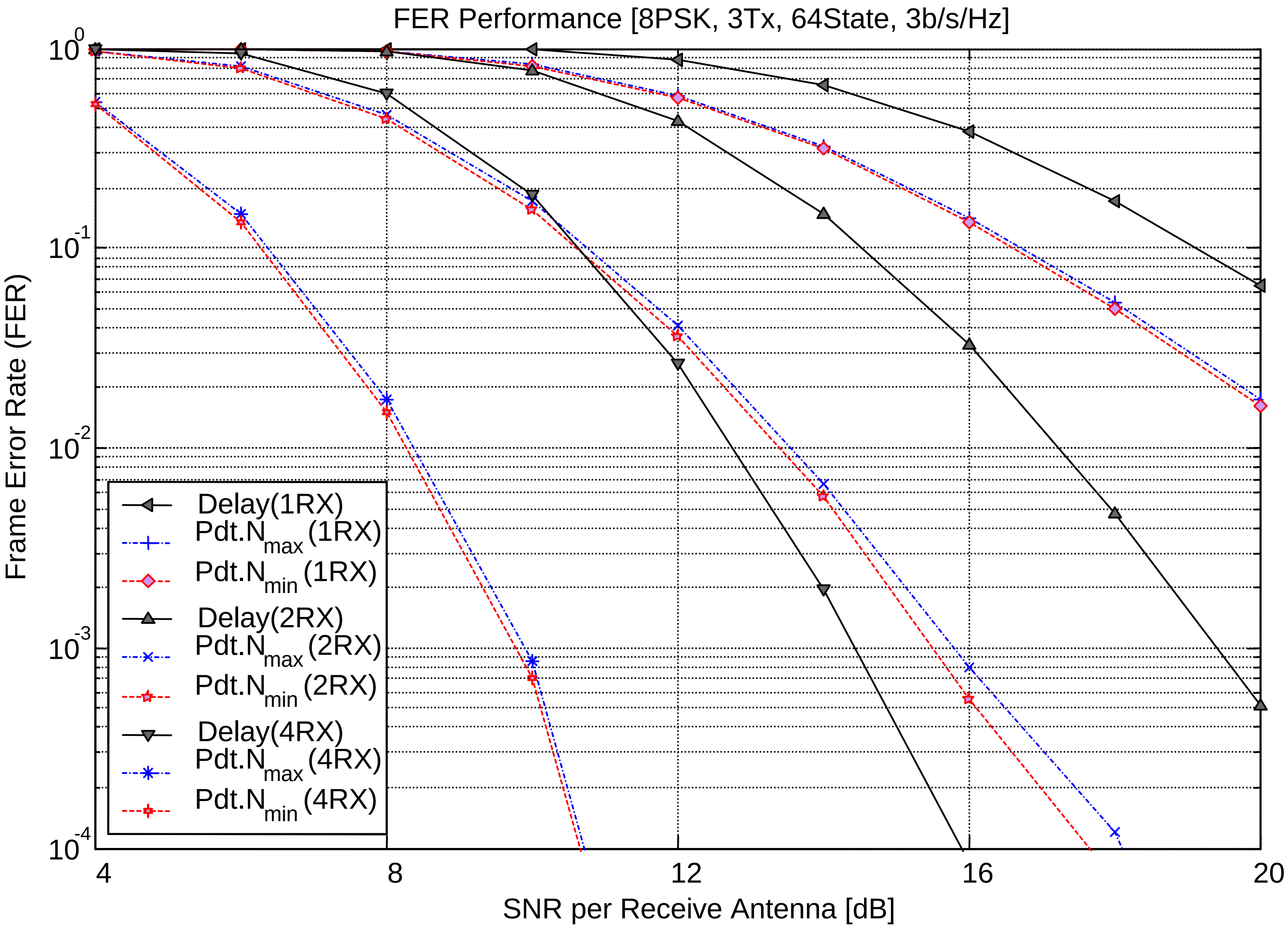
Delay VS Opt. PDC [QPSK, 4Tx]



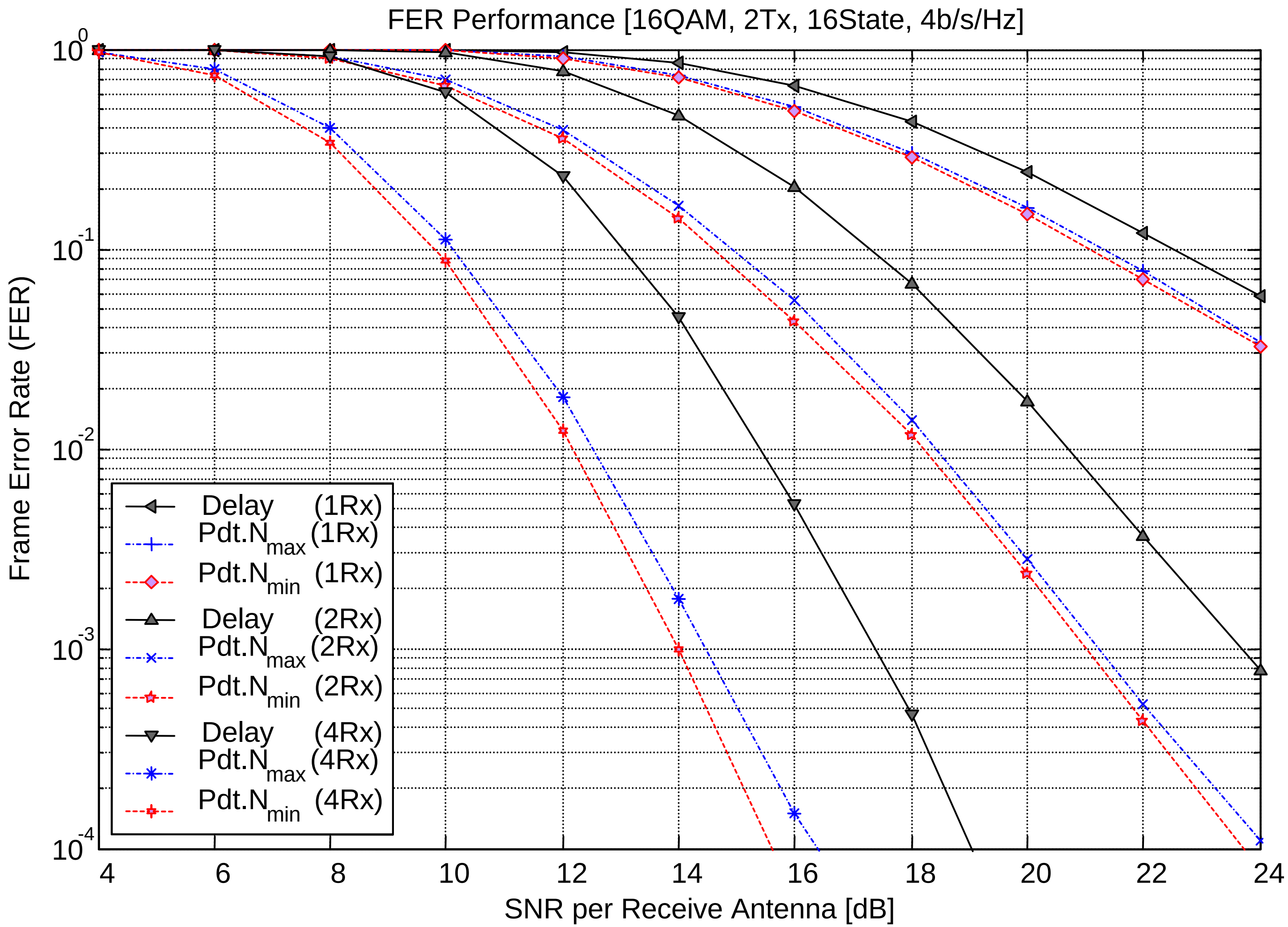
Delay VS Opt. PDC [8PSK, 2Tx]



Delay VS Opt. PDC [8PSK, 3Tx]



Delay VS Opt. PDC [16QAM, 2Tx]



Concluding Remarks

- STTC from Opt. PDC
  - From the Optimum Product Distance Codes
  - Modified Design Criterion by Product Distance Profile
  - Applicable to the case of the number of Tx Ant. is 2,3,4…
- Search Optimal Product Distance Code
  - 4PSK, 8PSK, 16QAM
  - Set up Trellis structure from Opt. PDC
- Performance Simulation
  - Comparison with Delay Diversity (←Baseline Performance)