Iterative Decoding of *Dual-K* Convolutional Codes

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Outline

- Motives
- *Dual-K* Recursive Systematic Convolutional Encoder
  - Parallel concatenation
- Symbol-by-Symbol *MAP* Decoder
  - Combined with M-ary *FSK* signaling
  - Iterative decoding algorithm
- Simulation Results
- Some Remarks

Motives

- Generally, M-ary orthogonal signaling is used for noncoherent modems in mobile communication systems.
  - Over the channels subject to fading or partial-band interference.
  - *Example 1*: a binary convolutional code + 64-ary orthogonal modulation in the reverse link of *IS-95(A)*.
  - *Example 2*: a nonbinary convolutional code called a *dual-K* code + M-ary orthogonal signaling in frequency hopping systems.
- A joint decoding algorithm of binary *Turbo* codes and M-ary orthogonal modulation in *IS-95(A)* systems was introduced by *Liang* and *Stark* in 2000.

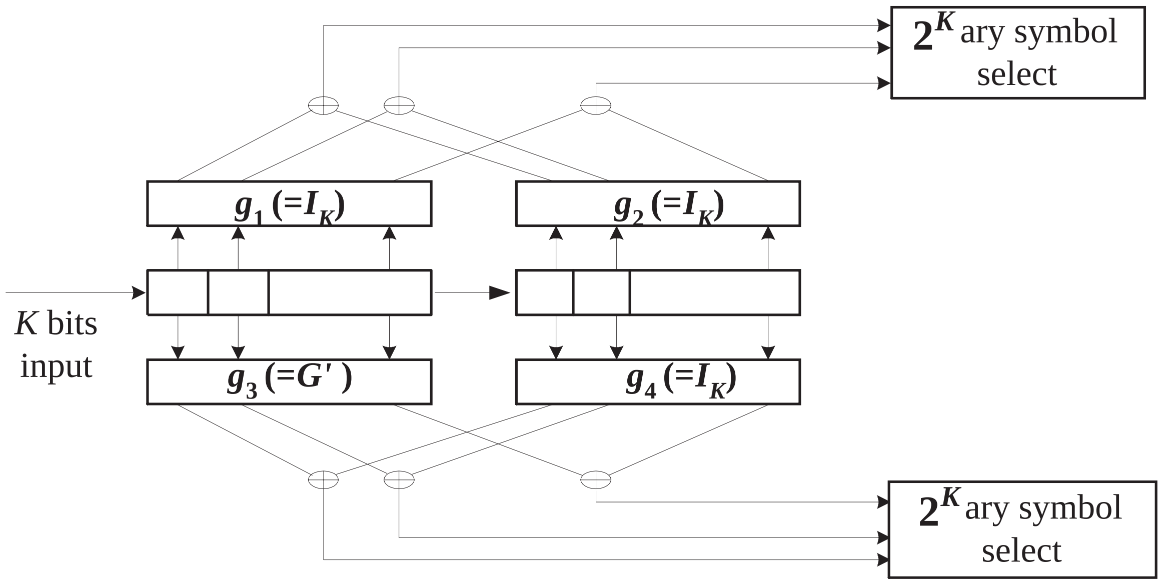
- How to apply iterative decoding to *FH* systems such as the last example.
  - The recursive systematic form of *dual-K* convolutional codes.
  - The parallel concatenated *Turbo-like* encoder.
  - *Bitwise* vs. *symbolwise* interleaver.
  - A nonbinary *MAP* decoder.
  - A joint decoding algorithm of nonbinary *Turbo* codes and *M*-ary orthogonal modulation.

Dual-K Convolutional Code

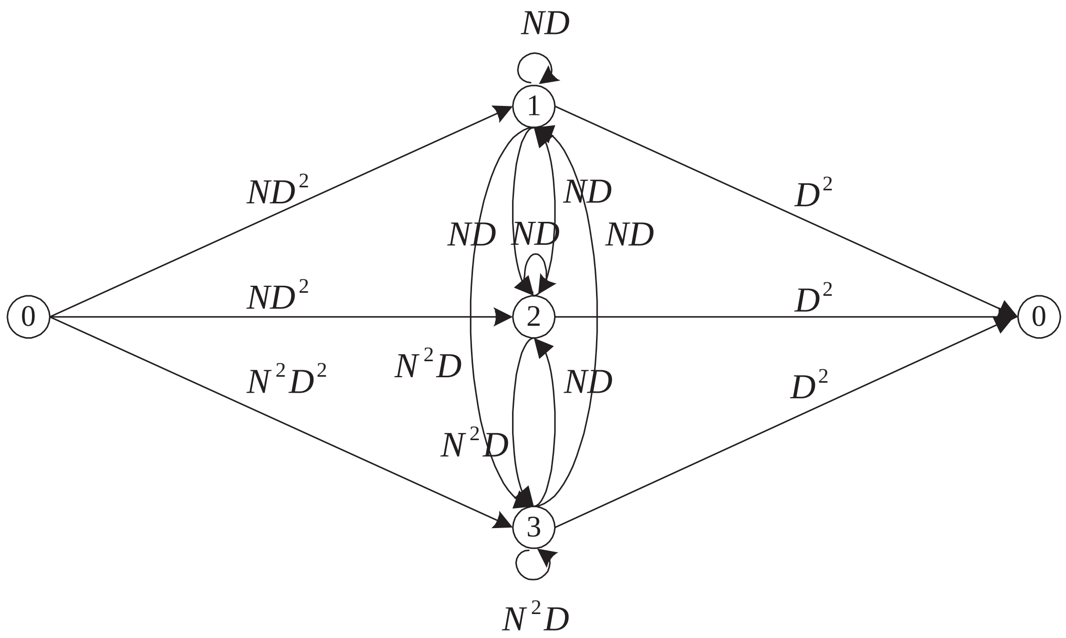
Definition : A $1/2$-rate *dual-K* convolutional code with the constraint length $2K$ has the generator matrix

$$G = \begin{bmatrix} I_K & G' \\ I_K & I_K \end{bmatrix}, \quad where \;\; G' = \begin{bmatrix} 1 \;\; 0 & \cdots & 0 \;\; 1 \\ & & 0 \\ & \mathbf{I}_{K-1} & \vdots \\ & & 0 \end{bmatrix}$$

where I_K denotes the $K \times K$ identity matrix.



State Diagram for $K = 2$



- The exponent on $N \triangleq$ the number of information bit errors.
- The exponent on $D \triangleq$ the *Hamming* distance for the 4-ary(2^K-ary) symbols.
- The minimum free distance D_{free} = 4 (4 K-bit sysmbols).

D-Transform Representation (1)

We consider each K-bit input as a K-bit vector, and input/output sequences as series of K-bit vectors. That is, the input sequence is expressed as a K-tuple vector of polynomials in D, *i.e.*,

$$\mathbf{x}(D) = (\mathbf{x}^{(1)}(D), \mathbf{x}^{(2)}(D), \ldots, \mathbf{x}^{(K)}(D)),$$

where the i-th component polynomial $\mathbf{x}^{(i)}(D)$ for $1 \le i \le K$ is defined as

$$\mathbf{x}^{(i)}(D) = \sum_{l \ge 1} d_l^{(i)} D^{l-1} = d_1^{(i)} + d_2^{(i)} D + d_3^{(i)} D^2 + \cdots ,$$

so that we may express

$$\mathbf{x}(D) = \sum_{l \ge 1} (d_l^{(1)}, d_l^{(2)}, \ldots, d_l^{(K)}) D^{l-1}.$$

D-Transform Representation (2)

- The nonsystematic transfer-function matrix $\mathbf{G}(D)$ corresponding to the binary generator matrix G can be represented as

$$\mathbf{G}(D) \triangleq \left[\begin{array}{c|cccccc} & 1{+}D & 0 & 0 & \cdots & 0 & 1 \\ & 1 & D & 0 & \cdots & 0 & 0 \\ (1{+}D)\,\mathbf{I}_K & 0 & 1 & D & 0 & \cdots & 0 \\ & \vdots & & & \ddots & & \vdots \\ & 0 & 0 & \cdots & 0 & 1 & D \end{array} \right] = \left[\; (1+D)\,\mathbf{I}_K \; \middle| \; \mathbf{G}'(D) \; \right].$$

- The D-transformed encoding process will be

$$\mathbf{x}(D)\mathbf{G}(D) = [\; \mathbf{x}^s(D) \mid \mathbf{x}^p(D) \;],$$

where $\mathbf{x}^s(D) \triangleq (\mathbf{x}^{s(1)}(D), \mathbf{x}^{s(2)}(D), \ldots, \mathbf{x}^{s(K)}(D))$ and $\mathbf{x}^p(D) \triangleq (\mathbf{x}^{p(1)}(D), \mathbf{x}^{p(2)}(D), \ldots, \mathbf{x}^{p(K)}(D))$ represent the two 2^K-ary output sequences similarly defined as $\mathbf{x}(D)$.

- The recursive systematic transfer-function matrix $\mathbf{G}_S(D)$ can be obtained as

$$\mathbf{G}_S(D) = \mathbf{G}(D)/(1+D) = [\; \mathbf{I}_K \mid \mathbf{G}'(D)/(1+D) \;].$$

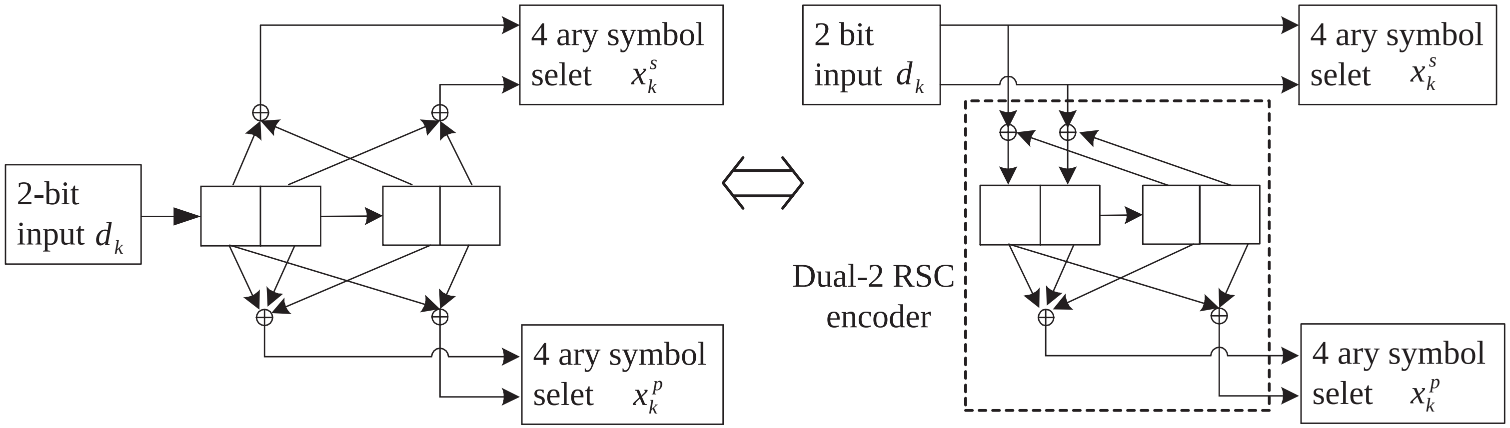
Dual-K Recursive Systematic Convolutional Encoder

- The *dual-K RSC* encoder output will be

$$\mathbf{x}(D)\mathbf{G}_S(D) = [\ \mathbf{x}(D)\ |\ \frac{\mathbf{x}(D)}{(1+D)}\mathbf{G}'(D)\ ] \triangleq [\ \mathbf{x}^s(D)\ |\ \mathbf{x}^p(D)\ ].$$

- For $K = 2$, the encoding process is described as

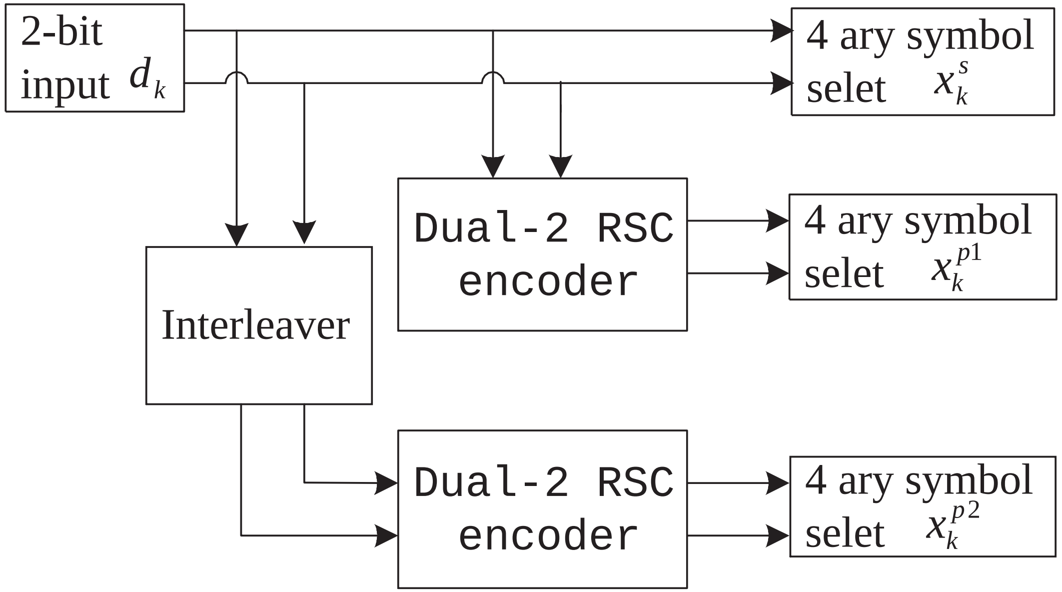
$$\left[\mathbf{x}^{s(1)}(D)\ \mathbf{x}^{s(2)}(D)\middle|\mathbf{x}^{p(1)}(D)\ \mathbf{x}^{p(2)}(D)\right] = \left[\mathbf{x}^{(1)}(D)\ \mathbf{x}^{(2)}(D)\right]\begin{bmatrix} 1 & 0 & | & 1 & \frac{1}{(1+D)} \\ 0 & 1 & | & \frac{1}{(1+D)} & \frac{D}{(1+D)} \end{bmatrix}.$$



(a) (b)

Parallel Concatenation

- For $K = 2$



- An 1/3-rate *Turbo-like* encoder
- *Bitwise* vs. *Symbolwise* interleaver

Symbol-by-Symbol *MAP* decoder

Let's define

- $d_k \triangleq k$-th K-bit input symbol into an *RSC* encoder for $k = 1, 2, ..., N$.
- $\mathbf{x}^s \triangleq (x_1^s, x_2^s, \ldots, x_N^s) = (d_1, d_2, \ldots, d_N)$.
- $\mathbf{x}^p \triangleq (x_1^p, x_2^p, \ldots, x_N^p)$.
- $y_k \triangleq (y_k^s, y_k^p)$ (the k-th demodulated output of the orthogonal 2^K-ary *FSK* signals corresponding to x_k^s and x_k^p through the *AWGN* channel).
- $\mathbf{y} \triangleq (y_1, y_2, \ldots, y_N)$ (the sequence of the demodulated outputs)

MAP Decision Rule :

$$\hat{d}_k = i \text{ if } \log\left(\frac{Pr(d_k = i|\mathbf{y})}{Pr(d_k = j|\mathbf{y})}\right) > 0 \text{ for all } j \text{ with } j \neq i.$$

or

$$\hat{d}_k = \arg_i \max \ \mathbb{M}(d_k = i) \text{ where } \mathbb{M}(d_k = i) \triangleq \log(Pr(d_k = i|\mathbf{y}))$$

- The metric can be obtained by extending the modified *BCJR* algorithm to the nonbinary case as

$$\mathbb{M}(d_k = i) = \log\left(\sum_{S^i} \alpha_{k-1}(s')\,\gamma_k(s',s)\,\beta_k(s)\right)$$

where S^i is the set of ordered pairs (s', s) corresponding to all the state transitions from $s_{k-1} = s'$ to $s_k = s$ caused by the nobinary input data $d_k = i$.

- Since it is assumed that y_k^s and y_k^p are independent, and y_k^s is irrespective of the trellis, we can express

$$\gamma_k(s',s) = P(d_k)\,p(y_k^s|d_k)\,p(y_k^p|d_k)$$

.

- The metric can be split into three terms (*channel*, *a priori*, and *extrinsic* values) as

$$\mathbb{M}(d_k = i) = \log(p(y_k^s|d_k = i)) + \log(P(d_k = i)) + \log\left(\sum_{S^i} \alpha_{k-1}(s')\,\gamma_k^e(s',s)\,\beta_k(s)\right) \;.$$

Channel Transition Probability

- To emnumerate $\gamma_k(s', s)$, the channel transition probabilities $p(y_k^s|d_k)$ and $p(y_k^p|d_k)$ have to be determined.

- The conditional probability from the m-th envelope detector

$$p(r_m|d_k = i) = \begin{cases} \frac{1}{2\pi\sigma^2}\, exp\Big(-\frac{|r_i|^2+E_s}{2\sigma^2}\Big)\, I_o\Big(\frac{|r_i|\sqrt{E_s}}{\sigma^2}\Big) & \text{if } m = i, \\ \frac{1}{2\pi\sigma^2}\, exp\Big(-\frac{|r_m|^2}{2\sigma^2}\Big) & otherwise. \end{cases}$$

where $I_o(x) \triangleq$ the modified Bessel function of the first kind.

- The product of all of the probabilities for $m = 1, 2, \ldots, M$ leads to

$$p(y_k^s|d_k = i) = A \cdot I_o\Big(\frac{|r_v|\sqrt{E_s}}{\sigma^2}\Big)_{v=x_k^s},$$

where

$$A \triangleq \Big(\frac{1}{2\pi\sigma^2}\Big)^M exp\Big(-\frac{|r_1|^2 + \cdots + |r_M|^2 + E_s}{2\sigma^2}\Big),$$

Iterative Decoding

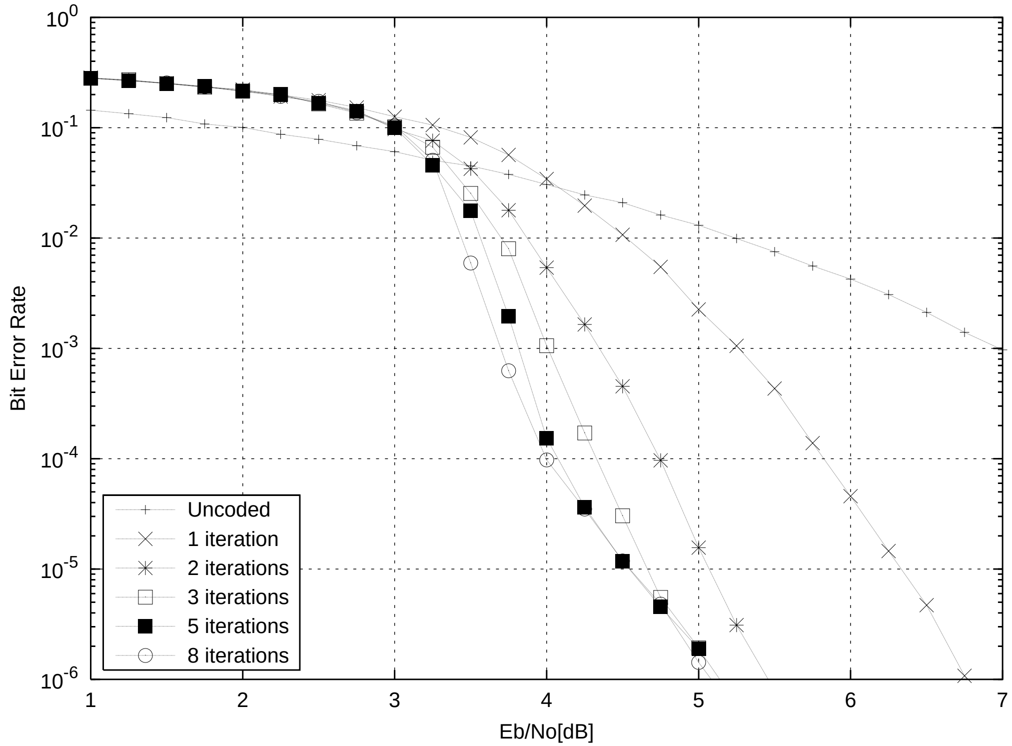
- The iteration occurs between two extrinsic values.
- When a bitwise (de)interleaver is used, an extrinsic information should be decomposed first into bitwise probabilities by utilizing the well-known joint combining technique.

as a priori prob.

$$\mathbf{M}(d_k) = \log(P(y_k^s \mid d_k)) + \mathbf{M}_1^e + \mathbf{M}_2^e$$

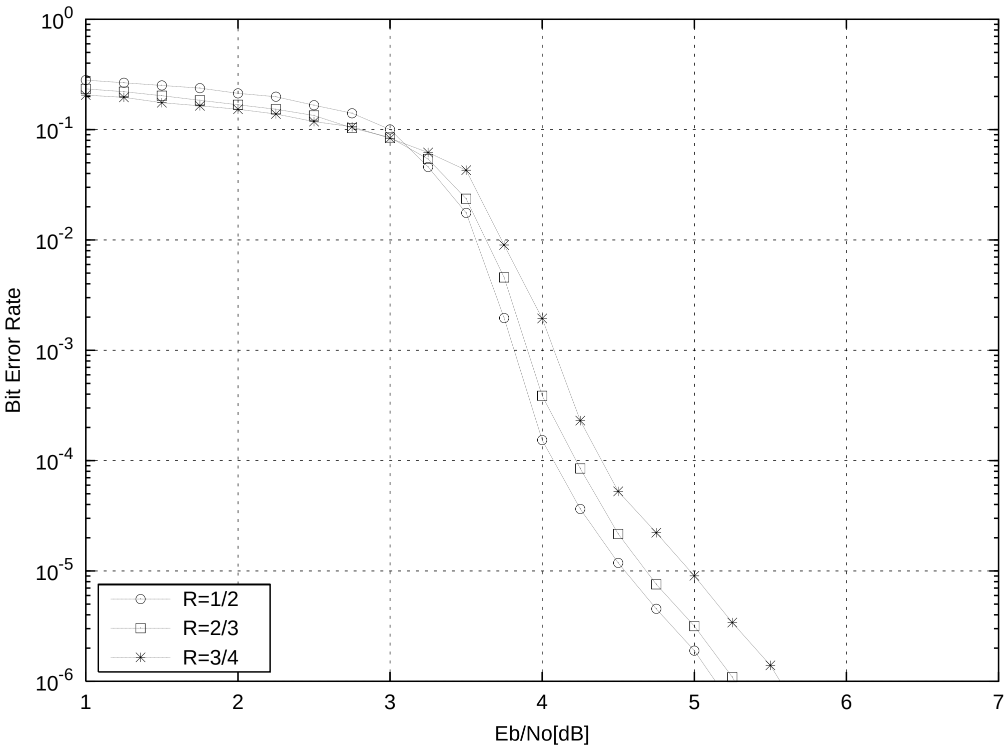
Simulation Results (1)

- The *dual-3* parallel concatenated *RSC* (*PC-RSC*) code of the rate $1/2$ obtained by simple even-odd puncturing. (8-state *MAP* decoder)
- Orthogonal 8-ary *FSK* signaling over *AWGN*.
- The 8192 *symbolwise* interleaver derived from an *M*-sequence.



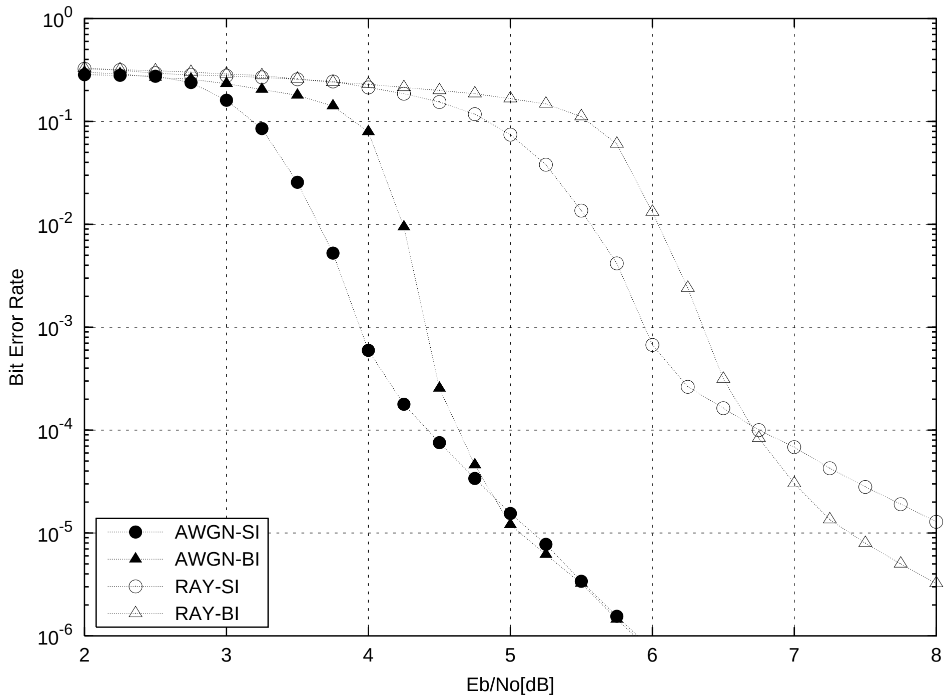
Simulation Results (2)

- The *dual-3 PC-RSC* codes of the rate $1/2$, $2/3$, and $3/4$ by some puncturings.
- 8 iterations with 8192 *symbolwise* interleaver over *AWGN*.
- Less sensitive to puncturing $\rightarrow$ high code rates with little degradation in *BER*.



Simulation Results (3)

- The *dual-3 PC-RSC* code of the rate $1/3$ with a *bitwise* interleaver (16383 bits) and a *symbolwise* interleaver (16383/3=5461 symbols).
- 10 iterations over both *AWGN* and *Rayleigh* fading channels.
- The *symbolwise* interleaving (SI) looks better than the *bitwise* interleaving (BI) in low *SNR*.
- The *bitwise* interleaving in high *SNR* looks more effective over the *Rayleigh* fading channel.



Some Remarks

- The recursive systematic form of *dual-K* convolutional codes.
- The parallel concatenation scheme of *dual-K RSC* codes.
- Symbol-by-Symbol *MAP* decoder $\rightarrow$ A joint decoding algorithm of *turbo-like dual-K* code and *M*-ary orthogoanl signaling for frequency hopping systems.
- Less sensitive to puncturing for high code rates.
- A *symbolwise* interleaver looks better in the most case except for the case of the *Rayleigh* fading channel in high *SNR*.