Reduced Memory Turbo MAP Decoding Algorithm for Non-binary Orthogonal Signaling

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Introduction

- *M*-ary orthogonal signaling (MFSK)
  - Binary turbo code
    - Decoding based on the LLR (log likelihood ratio) value of binary information
    - Channel measurement information of non-binary symbol -> that of bits
    - Some loss of information
  - Non-binary turbo code
    - Decoding based on the LL value of non-binary symbol information

The properties of non-binary Turbo Code

- The merit of non-binary turbo code
  - Reduction in the effective block length
  - Shorter Interleaver size
  - Fewer stage for the forward and backward recursions
    - ➔ less α values need to be stored
  - Better performance than binary turbo code in low SNR for Non-binary modulation

- The demerit of non-binary turbo code
  - More branches leaving each state
  - Non-binary symbol
  - Non-binary LLR values
    - ➔ more computations and memory storage

Non-binary MAP Decoding

- The MAP decision rule of non-binary turbo code

$$\hat{d}_k = i \quad \text{if} \quad \log\left(\frac{\Pr(d_k = i \mid \mathbf{y})}{\Pr(d_k = j \mid \mathbf{y})}\right) > 0 \text{ for all } j \text{ with } j \neq i.$$

or

$$\hat{d}_k = \arg_i \max \mathbf{M}(d_k = i) \quad \text{where} \quad \mathbf{M}(d_k = i) \equiv \log(\Pr(d_k = i \mid \mathbf{y}))$$

- A posteriori probability (APP)

$$\mathbf{M}(d_k = i) \equiv \log(p(d_k = i \mid \mathbf{y}))$$

$$= \log\left(\sum_{(s_{k-1}, s_k) \in S^i} p(s_{k-1} = s', s_k = s, \mathbf{y}) / p(\mathbf{y})\right)$$

- Turbo MAP decoding

$$\mathbf{M}(d_k = i) = \log\left(p\left(y_k^s \mid d_k = i\right)\right) + \log(P(d_k = i)) + \mathbf{M}^e(d_k = i)$$

Channel value

a priori value

Extrinsic value or LL value

Modified Channel Value(MCV) for Memory reduction

- Channel value in Non-binary turbo code (MFSK Case)
  - The conditional channel probability density function

$$p(r_m \mid d_k = i) = \begin{cases} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|r_m|^2 + E_s}{2\sigma^2}\right) I_o\left(\frac{|r_m|\sqrt{E_s}}{\sigma^2}\right) & \text{if } m = i \\ \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|r_m|^2}{2\sigma^2}\right) & \text{otherwise} \end{cases}$$

  - Channel value for turbo decoding : M channel values for each symbol

$$p(y_k^s \mid d_k = i) = A \cdot I_o\left(\frac{|r_m|\sqrt{E_s}}{\sigma^2}\right)$$

$$\text{where } A \equiv \left(\frac{1}{2\pi\sigma^2}\right)^M \exp\left(-\frac{|r_1|^2 + \cdots + |r_M|^2 + E_s}{2\sigma^2}\right)$$

Modified Channel Value (MCV) for Memory Reduction

- Proposed channel value: $i_{\max}$ and $\bar{p}_{\max}$ for each symbol
  - Choose the max value in M channel values

$$p(y_k \mid d_k = i_{\max}) \geq p(y_k \mid d_k = j), \quad \forall j = 1,2,\ldots,M$$

$$p_{\max}(y_k \mid d_k = i_{\max}) \equiv \max\{p(y_k \mid d_k = 1),\ldots, p(y_k \mid d_k = M)\}$$

  - Normalized

$$\bar{p}_{\max}(y_k \mid d_k = i_{\max}) \equiv \frac{p_{\max}(y_k \mid d_k = i_{\max})}{\dfrac{\left(\sum_{\substack{i=1 \\ i \neq i_{\max}}}^{M} p(y_k \mid d_k = i)\right)}{M-1} + p_{\max}(y_k \mid d_k = i_{\max})}$$

  - Other M-1 channel values

$$\bar{P}(y_k \mid d_k = j) \equiv 1 - \bar{p}_{\max}(y_k \mid d_k = i_{\max}), \quad j \neq i_{\max}$$

Reduced Memory (RM) MAP Algorithm

- The forward recursions of the MAP
  - BCJR algorithm

$$\alpha_k(s_k) = \sum_{s_{k-1}} \alpha_{k-1}(s_{k-1})\gamma_k(s_{k-1}, s_k)$$

  - Proposed forward recursions of the MAP
    - Choose the max value in M channel values

$$\alpha_k(s_k = i_{\max}) \geq \alpha_k(s_k = j), \quad \forall j = 1,2,\ldots,2^m$$

$$\alpha_k^{\max}(s_k = i_{\max}) \equiv \max\{\alpha_k(s_k = 1),\ldots,\alpha_k(s_k = M)\}$$

    - Normalized

$$\overline{\alpha}_k^{\max}(s_k = i_{\max}) \equiv \frac{\alpha_k^{\max}(s_k = i_{\max})}{\sum_{s_k} \alpha_k(s_k)}$$

    - Other M-1 channel values

$$\overline{\alpha}_k(s_k = j) \equiv \left(1 - \overline{\alpha}_k^{\max}(s_k = i_{\max})\right)/(M-1), \quad j \neq i_{\max}$$

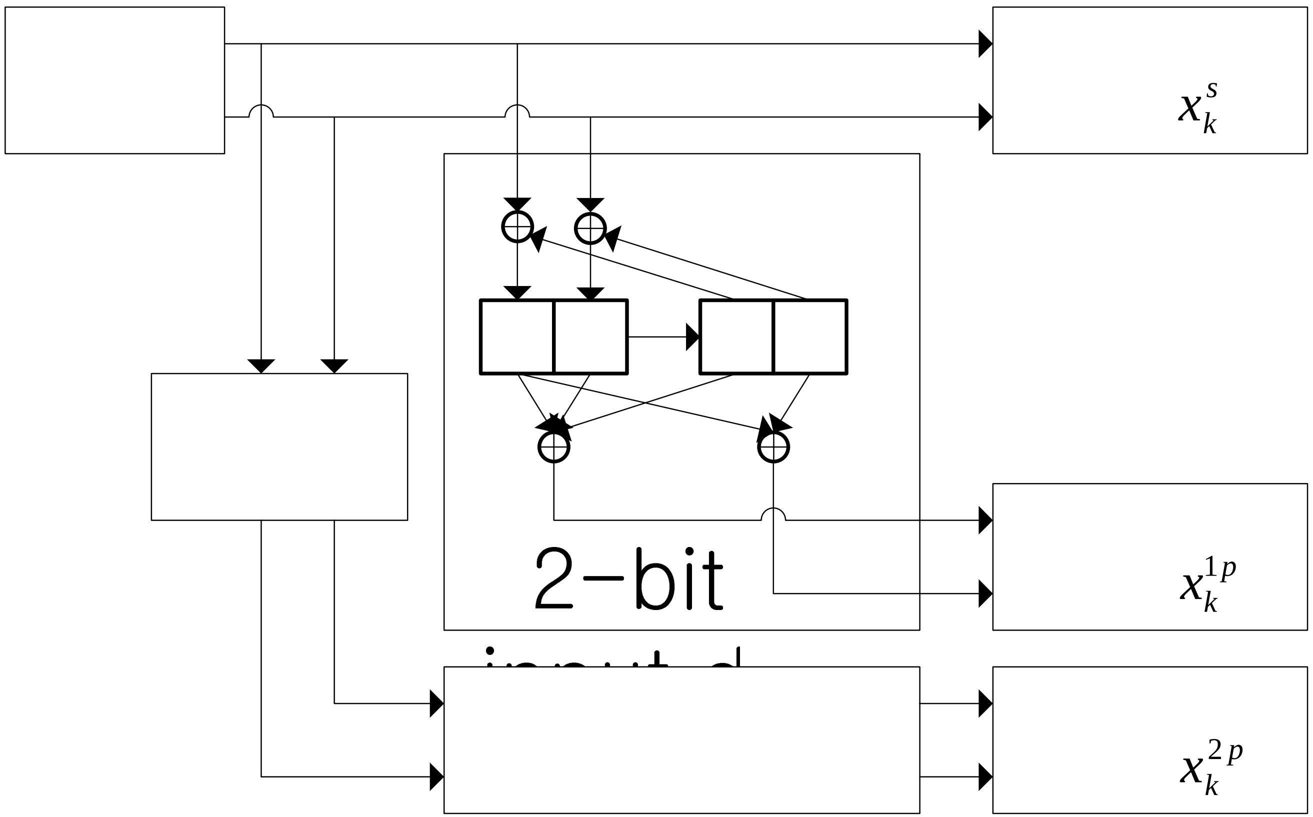
Memory Comparision between decoding method

- N : the length of the input symbols
- m : memory order, 2^m : the number of states
- M : the number of signals ($=2^K$)

	Channel Value	α
Non-binary MAP	(N/K)*M*3	$N*2^m$
Non-binary MAP with MCV	(N/K)*2*3	$N*2^m$
RM Non-binary MAP with MCV	(N/K)*2*3	N*2

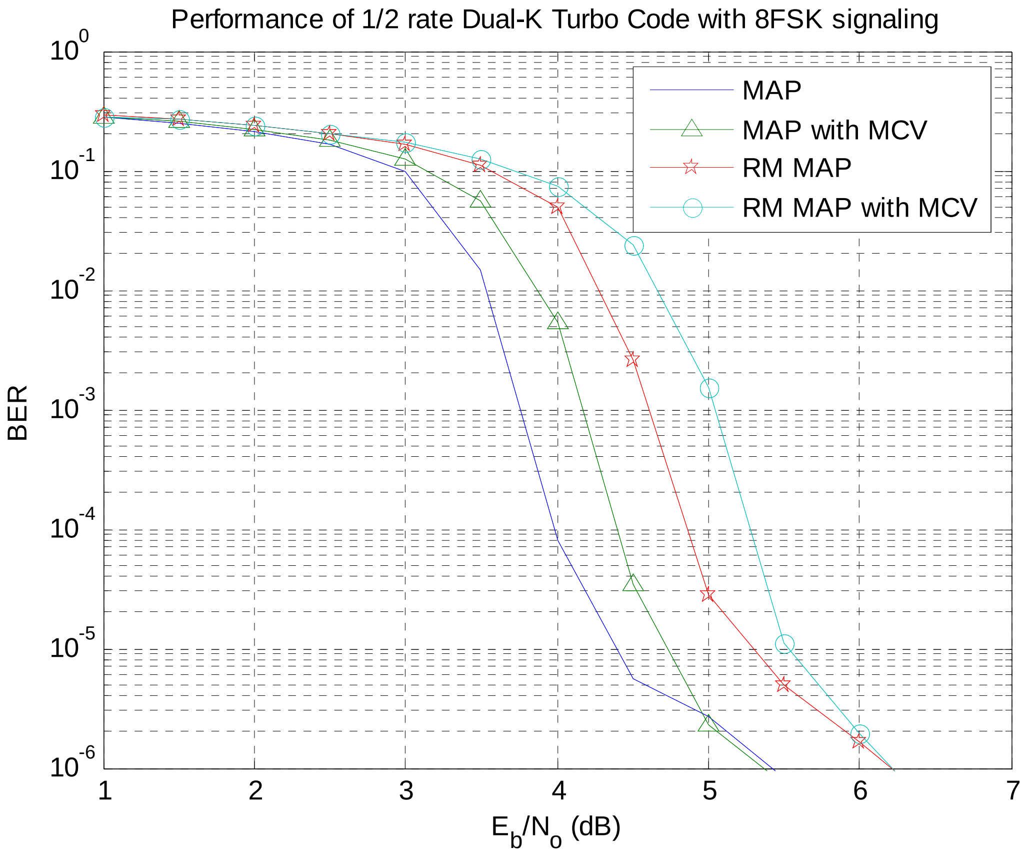
The Dual-*K* Turbo code

- Dual-2 Turbo encoder



Simulation Result

- 8FSK Modulation
- Non-coherent detection
- AGWN channel
- Dual-3 turbo encoder
- Non-binary MAP decoder
- 5 iterations
- 10000 symbol interleaver
- ½ code rate



Conclusion

- Non-binary turbo code has many merits.
- We proposed memory reduction method for non-binary signaling which can dramatically save the memory for channel information with a little loss that can be neglected
- We proposed reduced memory MAP decoding algorithm which can save the memory with performance loss.