Reduced Complexity-and-Latency Variable-to-Check Residual Belief Propagation for LDPC Codes

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The Fourth International Workshop on Signal Design and its Application in Communications
October 19 – 23, 2009
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- Sign-based NVCRBP (S-NVCRBP)

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- Summary and Conclusions
Future system requires higher data throughput
- Fast and accurate decoder (fast convergence, high performance)

Standard BP (simultaneous scheduling)
Layered / Shuffled BP (serial scheduling)

Residual BP (Dynamic Scheduling)

RBP, Node-wise RBP (NRBP)
Variable-to-Check RBP (VCRBP), Node-wise VCRBP (NVCRBP)
Forced-Convergence NVCRBP (FC-NVCRBP), Sign-based NVCRBP (S-NVCRBP)
Example Procedure of BP Decoding for LDPC Codes [1]

- Simultaneously update all the check nodes and variable nodes

![Example Diagram]

Black circles and squares present updated variable nodes and updated check nodes respectively.

Example Procedure of LBP Decoding for LDPC Codes [2]

- Serially update toward check nodes

Black circles and squares present updated variable nodes and updated check nodes respectively.

Example Procedure of SBP Decoding for LDPC Codes [3]

- Serially update toward variable nodes

Black circles and squares present updated variable nodes and updated check nodes respectively.

Convergence speed of the decoding by serial schedule algorithms (LBP [2], SBP [3]) are twice faster in terms of iterations than standard BP [1] algorithm.

FER performance comparison of BP, LBP, SBP decoding using IEEE 802.16e block length-576 rate-1/2 code up to 50 iterations at 2.5dB.


(CHECK-TO-VARIABLE) RESIDUAL BP  = RBP

AND

VARIABLE-TO-CHECK RESIDUAL BP  = VCRBP
RBP FOR LDPC CODES

Residual Belief Propagation [4]

\[ r(m_k) = |m_k^* - m_k|, \]

where \( m_k^* \) is a newly computed \( m_k \) by some update function.

RBP for LDPC codes [5]

\[ r(m_{c\rightarrow v}) = |m_{c\rightarrow v}^* - m_{c\rightarrow v}| \]

VCRBP for LDPC codes [6]

\[ r(m_{v\rightarrow c}) = |m_{v\rightarrow c}^* - m_{v\rightarrow c}| \]


RBP FOR LDPC CODES

Example Procedure of RBP Decoding for LDPC Codes [5]

Black circles and squares present updated variable nodes and updated check nodes respectively.

Example Procedure of VCRBP Decoding for LDPC Codes [6]

Black circles and squares present updated variable nodes and updated check nodes respectively.

NODE-WISE RBP = NRBP = LAYERED RBP

AND

NODE-WISE VCRBP = NVCRBP = SHUFFLED VCRBP
Node-wise RBP (NRBP) = Layered RBP

Example Procedure of NRBP Decoding for LDPC Codes [7]

Black circles and squares present updated variable nodes and updated check nodes respectively.

Node-wise VCRBP (NVCRBP) = Shuffled VCRBP

Example Procedure of NVCRBP Decoding for LDPC Codes [8]

Black circles and squares present updated variable nodes and updated check nodes respectively.

FER performance comparison of BP, LBP, NRBP, RBP, NVCRBP, and VCRBP decoding using IEEE 802.16e block length-576 rate-1/2 code with at most 8 iterations
Performance Comparison at **Constant** Complexity of Computation

### Approximate Decoding Complexity

<table>
<thead>
<tr>
<th></th>
<th>BP / SBP</th>
<th>NRBP</th>
<th>NVCRBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of computations for $m_{c \rightarrow v}$ in one iteration</td>
<td>1</td>
<td>$(d_v - 1)(d_c - 1) + 1$</td>
<td>$d_c - 1$</td>
</tr>
<tr>
<td>The number of maximum iterations × The number of computations for $m_{c \rightarrow v}$</td>
<td>$100 \times 1$</td>
<td>$8 \times ((3.17 - 1)(6.33 - 1) + 1)$</td>
<td>$19 \times (6.33 - 1)$</td>
</tr>
<tr>
<td>in one iteration = Approximate decoding complexity (in Fig. 1)</td>
<td>$= 100$</td>
<td>$\geq 100$</td>
<td>$\geq 100$</td>
</tr>
</tbody>
</table>

NRBP decoding does **WORSE** than standard BP decoding at the constant complexity of computation!!

FER performance comparison of BP, SBP, NRBP and NVCRBP decoding using IEEE 802.16e block length-576 rate-1/2 code with maximum 100, 100, 8, and 19 iterations, separately.
FORCED-CONVERGENCE NVCRBP = FC-NVCRBP

AND

SIGN-BASED NVCRBP = S-NVCRBP
Forced Convergence
FC-NVCRBP FOR LDPC CODES

Forced Convergence NVCRBP Decoding for LDPC Codes

Convergence criterion
\[ r(m_{v\rightarrow c}) = \left| m_{v\rightarrow c}^* - m_{v\rightarrow c} \right| \leq r_{th} \]

FC-NVCRBP skips updates of convergent nodes.

Approximate version for \( r_{th} \)
Take some appropriate portion of the residuals, say, \( 1/2, 1/3, \) or \( 1/4 \)

FER performance comparison of NRBP, FC-NVCRBP(1/4), FC-NVCRBP(1/3), FC-NVCRBP(1/2), NVCRBP decoding using IEEE 802.16e block length-576 rate-1/2 code with at most 8 iterations.
Sign-based NVCRBP Decoding for LDPC Codes

New ordering measure $s(\cdot)$:

$$s(m_{v \rightarrow c}) = \text{sign}(m_{v \rightarrow c}^*) \times \text{sign}(m_{v \rightarrow c})$$

where if $m_{v \rightarrow c}$ has a positive value then let $\text{sign}(m_{v \rightarrow c}) = 1$,

otherwise if $m_{v \rightarrow c}$ has a negative value then let $\text{sign}(m_{v \rightarrow c}) = -1$.

S-NVCRBP does not need to calculate all residuals
And neither to compare all residuals each other.
Performance Comparison

FER performance comparison of BP, LBP, NRBP, FC-NVCRBP(1/2), S-NVCRBP, and NVCRBP decoding using IEEE 802.16e block length-576 rate-1/2 code with at most 8 iterations.
Performance as the iteration continues

FER performance comparison of BP, LBP, NRBP, FC-NVCRBP(1/2), S-NVCRBP, and NVCRBP decoding using IEEE 802.16e block length-576 rate-1/2 code up to 50 iterations at 2.5dB.
FINALLY...
Summary and Conclusions

- **VCRBP**
  - makes LDPC decoding converge much faster in terms of the number of iterations than RBP (=CVRBP).
  - guarantees better performance with lower decoding complexity than RBP within only 8 iterations.
  - performs similarly better after sufficiently many iterations than RBP.

- **NVCRBP**
  - has very close performance to VCRBP with significantly lower decoding complexity.

- **FC-NVCRBP and S-NVCRBP**
  - very close performance with much lower complexity and latency compared to NVCRBP within only 8 iterations as well as after sufficiently many iterations.
THANK YOU!!