Rate Allocation for Component Codes of Plotkin-Type UEP Codes

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Error Rate

Non-UEP

MSB

LSB

$E_b/N_0$
Plotkin-Type Codes

- Plotkin-type Code

$$C_p = \{|u|u + v|u \in C_1, v \in C_2\}$$

Channel

\[ u \rightarrow u + v \rightarrow L'' \rightarrow L''' \]

\(u: \text{Repeated}\)
Plotkin-type UEP Codes


\[ u \quad \quad w = u + v \]

- Code Rate of \( C_1 \): \( R_1 \) > Code Rate of \( C_2 \): \( R_2 \)

Overall Code Rate: \( R_p = (R_1 + R_2)/2 \)
Plotkin-type UEP Codes


\[
L_i^u = L_i^{y'} + (-1)^\hat{v}_i L_i^{y''}
\]

\[
L_i^v = 2 \tanh^{-1} \left( \tanh \left( \frac{L_i^{y'}}{2} \right) \tanh \left( \frac{L_i^{y''}}{2} \right) \right)
\]
According to their paper (with arbitrary rate allocation I)

\[ R_1 = 0.65 \quad R_2 = 0.47 \quad (R_p = 0.56) \]

Plotkin-type UEP Code works MUCH WORSE than Non-UEP Code
According to their paper (with arbitrary rate allocation II)

\[ R_1 = 0.47 \quad R_2 = 0.65 \quad (R_p = 0.56) \]

Plotkin-type UEP Code works still WORSE than Non-UEP Code
According to their paper (with arbitrary rate allocation III)

\[ R_1 = 0.87 \quad R_2 = 0.25 \quad (R_p = 0.56) \]

Plotkin-type UEP code works as "UEP", But MSB and LSB are switched!!
PROBLEM

We Give the
Reason & Method
Average Code

“(Ideal) Average EEP Code $C_a$”

- **Ideal** EEP code which **Achieves the Channel Capacity**
- **Code Rate** $R_a = R_p$
- **Threshold** $\sigma_{a,th}^2$
Notations

"Plotkin-type UEP Code $C_p$"

"$C_1$ in $C_p$"
Threshold: $\sigma_{1,th}^2$
Channel Noise: $\sigma_1^2$

"$C_2$ in $C_p$"
Threshold: $\sigma_{2,th}^2$
Channel Noise: $\sigma_2^2$

"(Ideal) Average EEP Code $C_a$"

"$C_i$ (for $i = 1, 2$) Only" (EEP)
For a Given $R_a = R_p$ (i.e., $\sigma_{a,th}^2$)

$R_p = (R_1 + R_2)/2$

$\sigma_{a,th}^2$

$\sigma_{1,th}^2$

$\sigma_{2,th}^2$

< Threshold >

Compare

< Equivalent Channel Noise >

By Monte Carlo Simulation

$\sigma_1^2$

$\sigma_2^2$

$\sigma_{a,th}^2$

put

$\sigma_{ch}^2$
For a Given $R_a = R_p$ (i.e., $\sigma_{a,th}^2$)

- $\sigma_{a,th}^2$ > $\sigma_{1,th}^2$
- $\sigma_{1,th}^2$ > $\sigma_1^2$
- $\sigma_1^2$ > $\sigma_{ch}^2$

Known

< Threshold >

Error Free

< Equivalent Channel Noise >
Design Example: $\sigma_{a,th}^2 = 0.8$ (i.e., $R_a = 0.5604$)
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Choice - Right

$\sigma_{1,th}^2 < \sigma_1^2$
Choice – Left \( (\sigma_{1,th}^2 = 0.3) \): \( R_1 = 0.87, R_2 = 0.25 \)
Choice – Middle ($\sigma_{1,\text{th}}^2 = 0.4$): $R_1 = 0.79, R_2 = 0.33$
Choice – Right ($\sigma_{1,th}^2 = 0.619$): $R_1 = 0.65, R_2 = 0.47$
Our Design Works as Expected

- ‘Choice-Left’ Combination
  - UEP capability ↑ - Slightly Worse than $C_a$

- ‘Choice-Middle’ Combination
  - UEP capability ↓ - Comparable to $C_a$

- ‘Choice-Right’ Combination
  - UEP capability ? - Much Worse than $C_a$
Design Example: $\sigma_{a,th}^2 = 2.0$ (i.e., $R_a = 0.2905$)
Conclusions

• Guideline for the rate allocation for the component codes of Plotkin-type UEP codes.

⇒ We can construct the Plotkin-type codes without brute force simulation of performance.

  – For a good overall performance, we should select the code rates near the “middle” region.

  – For a good UEP capability, we suggest that the code rates should be selected in the “left” region and use $C_2$ as MSB (instead of $C_1$).
Thank You for Listening!