Concatenated LDGM Codes with Single Decoder

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***Abstract*— We propose a design criterion for serially concatenated LDGM codes which require a single decoder and a bit-interleaver. The inner LDGM code can be obtained by expanding the rows of the parity check matrix of the outer LDGM code. The resulting codes can be decoded using only the inner LDGM decoder with slight modification. Simulation results show that the performance of the proposed codes is almost the same as that of serially concatenated LDGM codes's using both the inner and the outer decoders.**

***Index Terms*— Low-density generator matrix codes, concatenated schemes, belief-propagation algorithm.**

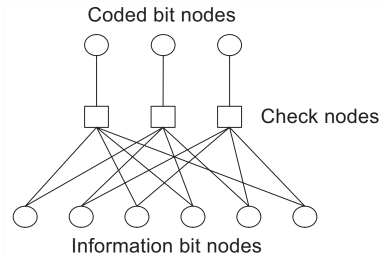

Fig. 1. Bipartite graph representation of a LDGM code.

I. Introduction

An LDPC code is a linear code with a parity-check matrix that contains a small number of ones. By using probabilistic iterative decoding algorithms, the performance of LDPC codes is known to approach the Shannon limit[1][2]. The main advantage of LDPC codes over turbo codes is a fully parallelizable decoder which allows fast decoding. On the other hand, the encoding complexity is much larger than that of Turbo codes since the encoding an LDPC code is usually based on the matrix multiplication of large size. Therefore, it is essential to search for (a family of) LDPC codes with efficient encoding algorithms as well as efficient decoding algorithms.

In this paper, we consider LDGM codes which can be regarded as a special type of LDPC codes[3]. Due to the sparseness of its generator matrix and the fact that the parity check matrix to/from generator matrix conversion is straightforward, the encoder complexity of LDGM codes is much less than that of LDPC codes. Furthermore, since the parity-check matrix is also sparse, LDGM codes can be decoded using the same techniques as those for LDPC codes. In spite of these advantages, the performance of LDGM codes is known to be asymptotically bad since they have too many degree-1 columns. Recently, Garcia-Frias and Zhong proposed a concatenated scheme of LDGM codes[4]. In this scheme, the output of the inner LDGM decoder can be regarded as a priori probability to initialize the bit nodes of the outer LDGM decoder. Concatenated LDGM codes proposed by them were shown to achieve near Shannon limit performance. We, however, would like to point out that the decoder hardware complexity becomes significantly higher than that of a single (non-concatenated, with comparable code-length) LDPC code since their code requires both inner and outer decoders.

Manuscript received November 21, 2005. The associate editor coordinating the review of this letter and approving it for publication was Prof. Marc Fossorier. Support for this work was provided in part by Samsung Electronics under the project on 4G wireless communication systems.

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Digital Object Identifier 10.1109/LCOMM.2006.04028.

In this paper, we first review Garcia-Frias's concatenated schemes. We then propose a design criterion of the inner and the outer LDGM codes for concatenation. The resulting code can be decoded using *only* the inner LDGM decoder with slight modification. This gives a much reduced decoder hardware complexity but still maintaining the performance, as confirmed by computer simulation at the end.

II. A Concatenated Scheme of LDGM Codes

LDGM codes are linear codes with generator matrix, $G = [I \ \ P]$, where I is a $k \times k$ identity matrix and P is a $k \times (n-k)$ sparse matrix[4]. Here, k denotes the number of information bits and n the number of bits of a codeword. The parity check matrix of the codes is $H = [P^T \ \ I]$. Figure 1 shows the bipartite graph representation of LDGM codes. There exist $(n-k)$ coded bit nodes of degree-1 and k bit nodes corresponding to the systematic bits. Since the messages propagated from the degree-1 coded bit nodes to their corresponding check nodes are always the same, LDGM codes have high error floors. But as mentioned in [4], the number of errors for the codewords in error decays very fast, and the outputs obtained from the decoding of LDGM codes can be seen as a priori probability produced by an equivalent channel introducing a small amount of erasures at specific locations. So if we use these outputs to initialize the bit nodes of the outer LDGM decoder in the decoding process, the number of residual errors can be reduced. Simulation results presented in [4, Fig. 3] show the good performance of these concatenated LDGM codes.

III. Proposed Design Criterion

Concatenated LDGM codes achieve a performance comparable to that of irregular LDPC and Turbo codes. But its decoder hardware complexity is higher than that of LDPC codes since it requires both inner and outer decoders. To reduce the decoder hardware complexity, we will design an inner LDGM code by expanding the outer LDGM code so that the inner LDGM decoder (with possibly a little modification) can also be used to decode the outer LDGM code.

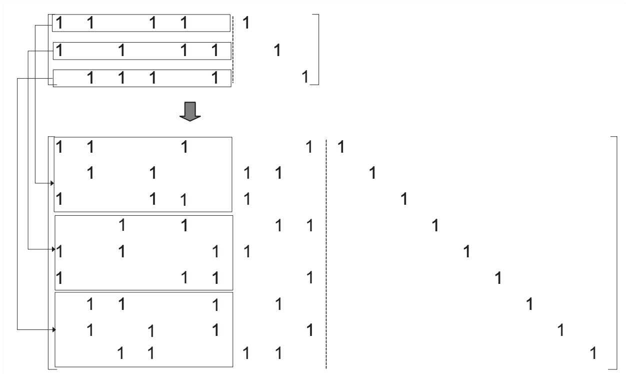
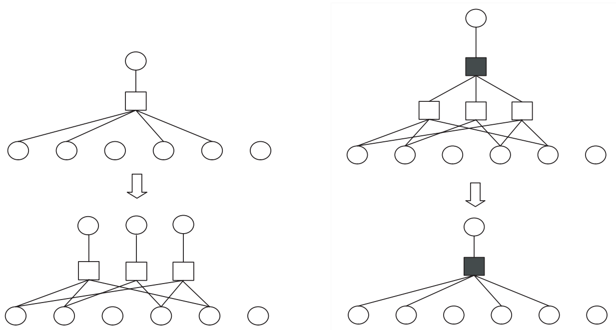


Fig. 2. An $(18, 9, 4)$ inner LDGM code obtained by expanding the rows of the parity check matrix of a $(9, 3, 2)$ outer LDGM code.



(a) Expansion of a check node (b) Merge of the expanded check nodes

Fig. 3. Bipartite graph representation of the first row shown in Fig. 2.

Let an $(n, n-k, p)$ LDGM code be an LDGM code of code length n, the number of check nodes $n-k$ and all the information bit nodes of degree p. In this paper, we will use an (n_1, n_1-k, p) outer LDGM code and an (n_2, n_2-n_1, rp) inner LDGM code, where k is the number of information bits, and r and $s = (n_2-n_1)/(n_1-k)$ must be natural numbers. Note that the fact that s must be a natural number imposes a constraint in the choice of inner and outer codes. First, we generate the parity check matrix of the outer LDGM code in a pseudorandom way. Then the parity check matrix of the inner LDGM code can be derived from the outer LDGM code by expanding each row of the parity check matrix of the outer LDGM code into s rows such that the OR operation of the expanded rows must be the same as the row of the outer LDGM code. This enables us to make the parity check matrices of both codes to have essentially the same decoder structure. These expanded rows are to be used to construct the parity check matrix of the inner LDGM code, where the number of the edges of each bit node is increased r times. For example, in Fig. 2, we consider a $(9, 3, 2)$ outer LDGM code and an $(18, 9, 4)$ inner LDGM code where $k = 6$, $n_1 = 9$ and $n_2 = 18$, so that $r = 2$ and $s = 3$ are natural numbers. Here, the inner code can be obtained by expanding each row of the parity check matrix of the outer code into 3 rows, where the number of the edges of each bit node is increased twice. In this expansion process, we can use a progressive edge growth algorithm to maximize the performance[5]. The residual part of the information bit nodes of the inner LDGM code can be generated in a pseudorandom way. Figure 3(a) shows the bipartite graph representation of the expansion of the first row of the outer LDGM code shown in Fig. 2. In decoding process, if all the messages from the expanded check nodes of the inner decoder are merged into another new check node, as shown in Fig. 3(b), the result will be the same as the check node of the outer decoder. By designing the inner code that is derived with afore-mentioned properties from the given outer code, we can use the inner decoder in the decoding of the outer code with a slightly modified belief-propagation algorithm.

The concatenated LDGM codes as presented has no computational savings but has achieved the goal of reducing the decoder hardware complexity. The decoded performance, however, turns out to be worse. In fact, the performance of the proposed concatenated codes turned out to be worse than that of serially concatenated LDGM codes reviewed in Section II. The reason seems to be that for some information bit i, all of the bits that participated in bit i for the inner code are also participated in bit i for the outer code. Therefore, the probability that the erroneous bits in the inner decoder output are still in error at the outer decoder output is high. Fortunately, this degradation can be overcome by the introduction of a bit-interleaver between the inner and the outer code.

Therefore the decoding procedure of the proposed code is as follows: The received channel outputs are fed into the inner decoder. Then the inner decoded outputs are de-interleaved, and fed into the slightly modified inner decoder in order to decode the outer code. Finally the decoded outputs are hard-decisioned.

IV. Modified Belief-Propagation Algorithm for Proposed Codes

Let z_{mn} be the log-likelihood ratio(LLR) of bit n which is sent from the bit node n to check node m, and z_m be the a priori probability of the degree-1 bit node in check m. Then, the check node message updating rule of a belief-propagation algorithm can be given as[6]

$$T_m = \prod_{n' \in N(m)} \frac{1-\exp(z_{mn'})}{1+\exp(z_{mn'})} \tag{1}$$

$$T_{mn} = T_m \times \frac{1-\exp(z_m)}{1+\exp(z_m)} \bigg/ \frac{1-\exp(z_{mn})}{1+\exp(z_{mn})} \tag{2}$$

$$L_{mn} = \ln \frac{1-T_{mn}}{1+T_{mn}}, \tag{3}$$

where L_{mn} denotes the LLR of bit n and $N(m)$ denotes the set of bits that participate in check m except for the degree-1 bit node. The bit node message updating rule can be similarly given as

$$z_{mn} = F_n + \sum_{m' \in M(n) \setminus m} L_{m'n}, \tag{4}$$

where F_n denotes the LLR of bit n which is derived from the received value y_n and $M(n) \setminus m$ denotes the set of checks that participate in bit n except for the check m.

For the decoding of the proposed concatenated LDGM codes, we first decode the inner code by using the inner decoder. Then the inner decoded outputs are de-interleaved,

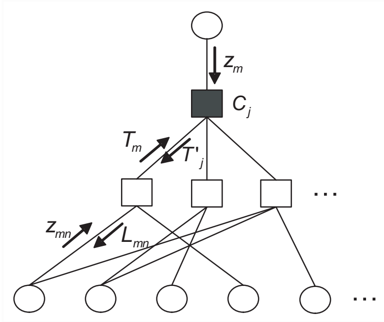


Fig. 4. Modified belief propagation algorithm.

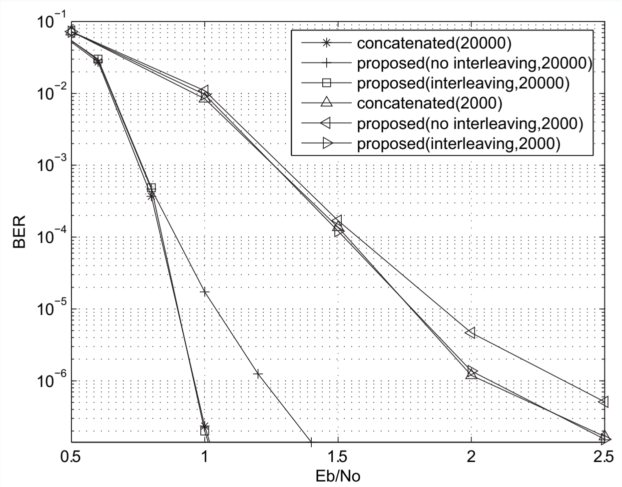


Fig. 5. BER performance of each type of concatenated LDGM codes.

and we decode the outer code by using the same inner decoder with the following modification. Let $S(j)$ be the set of the inner check nodes which were expanded from the same check node j of the outer code, and C_j be a new check node participated in $S(j)$. Then in C_j we derive T'_j between (1) and (2) as follows:

$$T_j^{'} = \left\{ \prod_{m \in S(j)} T_m \right\}^{1/r} \tag{5}$$

In the above equation, since the number of the edges among the inner bit nodes is r times larger than that among the outer bit nodes, the same messages propagated from bit nodes to the check node C_j is multiplied by itself r times. For this reason, in the check node C_j, we should extract the rth root given in (5). Then, the remaining message updating procedure (2) and (3) are the same except that T_m is replaced by T'_j.

For the bit node message updating rule, since the same messages from the check node C_j is added by itself r times in every bit node, (4) is modified as follows:

$$z_{mn} = F_n + \frac{1}{r} \times \sum_{m' \in M(n) \backslash m} L_{m'n}$$

The entire modified belief-propagation algorithm for the proposed concatenated LDGM codes is described in Fig. 4. In conclusion, the inner decoder can be used to decode the outer code with a few additional computations.

V. Simulation Results

The simulation results of comparisons between Garcia-Frias's concatenated scheme and the proposed scheme are presented. Figure 5 shows the performance of rate 0.475 concatenated LDGM codes of length 20000 and 2000 over AWGN channel. We use rate 0.5 $(20000, 10000, 6)$ inner LDGM code and a rate 0.95 $(10000, 500, 3)$ outer LDGM code for length 20000. The same outer code is used in both Garcia-Frias's and the proposed scheme. Garcia-Frias's inner code is randomly constructed by a PEG algorithm, but in the proposed scheme, the inner code is constructed by expanding the outer code as described earlier. For both codes, a maximum of 50 iterations for inner code and 20 iterations for outer code are performed. At a bit error rate of 10^{-5}, the performance of the proposed code without a bit-interleaver is about 0.2 dB away from Garcia-Frias's code. With a bit-interleaver, the proposed code has almost the same performance as Garcia-Frias's. For concatenated LDGM code of length 2000 in Fig. 5, we can confirm similar results. The results show that the proposed concatenated codes with a single decoder and a bit-interleaver achieves a performance close to Garcia-Frias's code.

VI. Conclusion

We propose a design criterion of concatenated LDGM codes which require a single decoder and a bit-interleaver. The inner LDGM code can be obtained from the outer LDGM code by expanding each row of the parity check matrix of the outer LDGM code into multiple rows and these rows are used to construct the parity check matrix of an inner LDGM code. For the proposed codes, we can use the inner decoder to decode the outer code with slight modification to the belief-propagation algorithm. Simulation results show that the performance of the proposed codes is almost the same as that of Garcia-Frias's with much reduced decoder hardware complexity.

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