Properties and Crosscorrelation of Decimated Sidelnikov Sequences**

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SUMMARY In this paper, we show that if the \(d\)-decimation of a \((q-1)\)-ary Sidelnikov sequence of period \(q-1 = p^m-1\) is the \(d\)-multiple of the same Sidelnikov sequence, then \(d\) must be a power of a prime \(p\). Also, we calculate the crosscorrelation magnitude between some constant multiples of \(d\)- and \(d'\)-decimations of a Sidelnikov sequence of period \(q-1\) to be upper bounded by \((d + d' - 1) \sqrt{q} + 3\).

Section 2 introduces some preliminary concepts for the main results: notation, definition of Sidelnikov sequences, correlation, Weil bound on character sums, and decimation. Main results are given in Sect. 3. Section 4 gives a brief concluding remark.

2. Preliminaries

2.1 Notation and Convention

We will use the following notation:

- \(p\) : a prime number
- \(q\) : a prime power \(p^m\) with positive integer \(m\)
- \(GF(q)\) : the finite field with \(q\) elements
- \(d\) : a decimation factor with gcd\((q-1, d) = 1\)
- \(M\) : a divisor of \(q-1\) with \(M \geq 2\)
- \(\omega_M = \exp\left(\frac{2\pi j}{M}\right)\) where \(j = \sqrt{-1}\)
- \(\beta\) : a fixed primitive element of \(GF(q)\)
- \(\psi\) : the multiplicative character of \(GF(q)\) of order \(M\) defined by

\[
\psi(x) = \exp\left(\frac{2\pi j}{M} \log_M x\right) = \omega_M^{\log_M x}
\]

We keep \(\log(0) = 0\) and \(\psi(0) = 1\) in this paper for convenience.

2.2 Sidelnikov Sequences

**Definition 1** [1] For any fixed primitive element \(\beta\) of \(GF(q)\), let \(D_k = \{\beta^{M+1} - 1 \mid 0 \leq i < \frac{q-1}{M}\}\). Then \(M\)-ary Sidelnikov sequence \(\{s(t)\}\) of period \(q-1\) is defined as

\[
s(t) = \begin{cases} 
0, & \text{if } \beta^t = 1 \\
 k, & \text{if } \beta^t \in D_k.
\end{cases}
\]

We will say that \(s(t)\) above is defined by the primitive element \(\beta\).

Equivalently, it can also be expressed, with the new convention that \(\log 0 = 0\), as

\[
s(t) \equiv \log_M(\beta^t + 1) \mod M
\]

for \(0 \leq t \leq q-2\).
2.3 Correlation

A correlation is a measure of distance between two sequences or sequence families. If two sequences are the same, the correlation is called auto-correlation. Otherwise, we call it cross-correlation. The following definition has been well-known [4].

**Definition 2** Let \( (a(t)) \) and \( (b(t)) \) be \( M \)-ary sequences of period \( L \), where \( 0 \leq t \leq L-1 \). A periodic correlation between \( (a(t)) \) and \( (b(t)) \) is defined by

\[
C_{a,b}(\tau) = \sum_{t=0}^{L-1} \omega_M^{a(t)-b(t+\tau)}
\]

for \( 0 \leq \tau \leq L-1 \), where \( \omega_M = \exp(i \frac{2\pi}{M}) \) and \( t+\tau \) is computed modulo \( L \).

For \( \tau \equiv 0 \mod L \) when \( (a(t)) \) and \( (b(t)) \) are the same sequence, the value above becomes \( L \), which is regarded as trivial. The values at all other cases (for all \( \tau \) when \( (a(t)) \) and \( (b(t)) \) are two different sequences or for \( \tau \equiv 0 \mod L \) when \( (a(t)) \) and \( (b(t)) \) are the same sequence) are called non-trivial.

For a sequence set \( S \), \( C_{\max}(S) \) is the maximum magnitude of all the non-trivial correlations of the pairs of sequences in \( S \).

2.4 Weil Bound

The Weil bound gives an upper bound on the multiplicative character sums, and has been used to calculate upper bound on the crosscorrelation of various sequences [4]. Yu and Gong [9], [13] introduced a refined version from some of the previously well-known ones, with an additional assumption that \( \psi(0) = 1 \). We will use a further refined version by Kim [10].

**Theorem 1** [10] Let \( f_1(x), \ldots, f_m(x) \) be distinct monic irreducible polynomials over \( GF(q) \) with degrees \( d_1, \ldots, d_m \), with \( e_j \) the number of distinct roots of \( f_j(x) \) (\( j = 1, \ldots, m \)). Let \( \psi_1, \ldots, \psi_m \) be nontrivial multiplicative characters of \( GF(q) \), with \( \psi_j(0) = 1 \) (\( j = 1, \ldots, m \)). Then, for \( a_1, \ldots, a_m \in GF(q)^* \), we have the estimate

\[
\left| \sum_{x \in GF(q)} \psi_1(a_1 f_1(x)) \cdots \psi_m(a_m f_m(x)) \right| \leq \prod_{j=1}^{m} (d_j - 1) \sqrt{q} + \sum_{j=1}^{m} e_j.
\]

2.5 Decimation and Constant Multiple

Taking decimation and/or constant multiple is a well-known method for constructing a new sequence from the given sequence [5], [7].

**Definition 3** Let \( a(t) \) be an \( M \)-ary sequence of period \( L \). Then (1) the \( d \)-decimation sequence \( b(t) \) of \( a(t) \) is \( b(t) = a(dt) \) for \( t = 0, 1, \ldots \); (2) the \( d \)-multiple sequence \( c(t) \) of \( a(t) \) is \( c(t) = d \cdot a(t) \) for \( t = 0, 1, \ldots \).

When we apply the \( d \)-decimation to the periodic sequence of period \( L \), we see easily that the period of decimated sequence becomes \( \frac{L}{d} \) where \( k = \gcd(d, L) \). Therefore, we have to choose \( d \) such that \( \gcd(d, L) = 1 \) in order to maintain the original period.

When we apply the \( d \)-multiple to the \( M \)-ary sequence, we see easily that the number of distinct values of the sequence becomes \( \frac{M}{d} \) where \( k = \gcd(d, M) \). Therefore, we have to choose \( d \) such that \( \gcd(d, M) = 1 \) in order to keep the resultant sequence to have truly \( M \) distinct values, i.e., to be an \( M \)-ary sequence.

3. Main Result

The Sidelnikov sequence is constructed using a primitive element of a finite field. It is well-known that if \( \beta \) is a primitive element of \( GF(q) \), then \( \beta^d \) is also a primitive element of \( GF(q) \) whenever \( \gcd(q, d-1) = 1 \). It is interesting to note that the \( d \)-decimation of a Sidelnikov sequence defined by a primitive element \( \beta \) is a \( d \)-multiple of another Sidelnikov sequence defined by the primitive element \( \beta^d \). Following has been essentially proved as Lemma 2 in [7].

**Theorem 2** (Lemma 2 in [7]) Let \( q = p^m \) and \( \{s(t)\} \) be an \( M \)-ary Sidelnikov sequence of period \( q-1 \). Let \( d \) be relatively prime to \( q-1 \). Then \( s(dt) \equiv d \cdot s'(t) \mod M \) where \( \{s'(t)\} \) given by \( s'(t) \equiv \log_q(\gamma^t + 1) \mod M \) is another \( M \)-ary Sidelnikov sequence of period \( q-1 \) defined by the primitive element \( \gamma = \beta^d \).

**Example 1** Table 1 shows some decimation sequences of a 10-ary Sidelnikov sequence of period \( q-1 = 10 \). \( GF(11) \) has 4 primitive elements: \( 2, 6 \equiv 2^6, 7 \equiv 2^7, 8 \equiv 2^8 \). We can show that \( 9 \cdot s_1(t) \equiv s(9t) \mod 6 \equiv 2^9 \). Similarly, \( 7 \cdot s_2(t) \equiv s(7t) \mod 3 \) and \( 3 \cdot s_3(t) \equiv s(3t) \mod 3 \).

**Corollary 1** Let \( q = p^m \) and \( \{s(t)\} \) be an \( M \)-ary Sidelnikov sequence of period \( q-1 \). If \( d = p^k \) with \( \gcd(p, k) = 1 \) and a nonnegative integer \( l \), then \( s(pt^l) \equiv p^l s(t) \mod M \) for all \( t \). In particular, when \( k = 1 \), this implies \( s(p^l t^k) = p^l s(t) \mod M \) for all \( t \).

The above corollary is quite straightforward since \( \beta \)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Decimation sequences of a Sidelnikov sequence in Example 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( \beta = 2 )</td>
<td>( s(t) )</td>
</tr>
<tr>
<td>( \gamma = 6 \equiv 2^6 )</td>
<td>( s_1(t) )</td>
</tr>
<tr>
<td>( \gamma = 7 \equiv 2^7 )</td>
<td>( s_2(t) )</td>
</tr>
<tr>
<td>( \gamma = 8 \equiv 2^8 )</td>
<td>( s_3(t) )</td>
</tr>
</tbody>
</table>
and $\beta^d$ generate the same Sidelnikov sequence, but its converse is not at all trivial. We prove the converse for the case of $M = q - 1$ and $k = 1$. We guess that it is also true for any divisor $M$ of $q - 1$ and $k = 1$, but leave this as an open problem.

**Theorem 3** Let $q = p^m$, and $(s(t))$ be a $(q - 1)$-ary Sidelnikov sequence of period $q - 1$. If there exists a $d$ such that $s(dt) = d \cdot s(t)$ for all $t$, then $d = p^l$ for some nonnegative integer $l$.

**proof:** Since the period of the sequence is $q - 1$, we may assume that $d \leq q - 2$. We assume that $s(dt) = d \cdot s(t)$ for all $t$, and write, on the contrary, $d = pl$, where $l$ is a nonnegative integer, $\gcd(p, r) = 1$ and $r > 1$. Then we will consider the binomial coefficient $\binom{d}{l}$ and see if it is divisible by $p$ or not. It will turn out that it is divisible by $p$ in one way, and is not in other way, giving a desired contradiction to $r > 1$.

First, we have $\log_{p}(\beta + 1) = \log_{p}(\beta^d + 1)$, and hence $(\beta + 1)^d = \beta^d + 1$ for all $0 \leq t \leq q - 2$. Therefore, $a + 1)^d = 1$ for all $a \in GF(q)$. This implies that $p \binom{d}{l}$ is divisible by $p$.

Now we use Kummer’s criterion [14]: the power of $p$ dividing $\binom{d}{l}$ is the number of carries when we add $k$ to $n - k$ in base $p$. We may divide $r$ by $p$ and write $r = pa + j$ for some $1 \leq j \leq p - 1$. By Kummer’s criterion, the power of $p$ dividing $\binom{d}{l}$ is the number of carries when adding $p^1$ to $d - pl = p^1(r - 1)$ in base $p$. Observe that the right-most non-zero digit (in base $p$) of $p^1(r - 1)$ is $(j - 1)$ when $a = 0$ and those of $p^1$ is $1$, and hence, there are no carries in the sum that is just $j$. Therefore, $\binom{d}{l}$ is not divisible by $p$. Similarly, it is obvious that $\binom{d}{l}$ is not divisible by $p$ when $a > 0$. This is a desired contradiction to $r > 1$.

Let $s(t)$ be an $M$-ary Sidelnikov sequence of period $q - 1$. We will now derive the maximum correlation bound between $c_1 \cdot s(dt)$ and $c_2 \cdot s(dt)$ where $c_1, c_2, d$ and $d'$ are some constants. If $p$ divides $d$, i.e. $d = pl$, then $s(dt) = p^l \cdot s(kt)$ by Corollary 1. Especially, if $d = p^l$ and $d' = p^l$, then the correlation between $c_1 \cdot s(dt)$ and $c_2 \cdot s(dt)$ becomes just a correlation between two different constant multiples of a Sidelnikov sequence, and it was computed earlier by Kim and Song [5] and also by Yu and Gong [13]. Consequently, we need to consider only the case where $p$ divides neither $d$ nor $d'$.

**Lemma 1** Let $d, d'$ be positive integers with $(d, q - 1) = (d', q - 1) = 1$, and let $0 \leq \tau \leq q - 2$. Then we have the following:

(a) The only root in $GF(q)$ of $x^d + 1 = 0$ is $-1$. The only root in $GF(q)$ of $x^{d'} + \beta^{-d'} = 0$ is $-\beta^{-\tau}$.

(b) If $p$ does not divide $d$, then $x^d + 1$ has no multiple roots. If $p$ does not divide $d'$, then $x^{d'} + \beta^{-d'}$ has no multiple roots.

**proof:** (a) If $\gamma$ and $\delta$ are roots of $x^d + 1$, then $(\gamma)^d = 1$. But the only root of $x^d + 1$ is $1$, as $\beta^d = 1 \Rightarrow q - 1 \mid id \Rightarrow q - 1 \mid i$, since $(d, q - 1) = 1$. So $\beta = 1$, and hence $\gamma = 0$. If $d$ is odd, then the root of $x^d + 1 = 0$ is $-1$. And if $d$ is even, then $q$ is even since $(d, q - 1) = 1$, and hence the characteristic is $2$. Hence, $-1 = +1$ is the only root of $x^d + 1$ over $GF(q)$, $q$ even. The other case is similar. 

(b) If $p$ does not divide $d$, the $x^d + 1$ and its derivative $dx^{d-1}$ are relatively prime. The other case is similar.

(c) If $\tau = 0$, then $-1$ is the common root. Conversely, let $\gamma^d + 1 = 0$ and $\gamma^{d'} + \beta^{-d'} = 0$. Then $\gamma = -\xi = -\beta^{-\tau} \eta$, with $\xi^d = 1$ and $\eta^{d'} = 1$. So $\beta = \eta^{-\tau}$. Raising $dd'$-th power of both sides, we have $\beta^{dd'} = 1$, which implies $\tau = 0$, since $(d, q - 1) = (d', q - 1) = 1$.

**Theorem 4** Assume that $(d, q - 1) = (d', q - 1) = 1$ and that $p$ divides neither $d$ nor $d'$. Let $a(t) = c_1 \cdot s(dt)$ and $b(t) = c_2 \cdot s(dt)$ are cyclically inequivalent for some $M$-ary Sidelnikov sequence $s(t)$ of period $q - 1$ and constants $1 \leq c_1, c_2 \leq M - 1$. Then we have

$$\left| \max_\tau \left( C_{a,b}(\tau) \right) \right| \leq (d + d' - 1) \sqrt{q} + 3$$

where $\tau$ runs over the integers $0 \leq \tau \leq q - 2$.

**proof:** By the definition of Sidelnikov sequences and its decimation, we see that $a(t) = c_1 \log_{d^2}(\beta^d + 1)$ and $b(t) = c_2 \cdot \log_{d^2}(\beta^{d'}) + 1)$. Then, their correlation becomes as follows:

$$C_{a,b}(\tau) = \sum_{t=0}^{q-2} \omega_M a(t - b(t + \tau))$$

$$= \sum_{t=0}^{q-2} \omega_M c_1 \log_{d^2}(\beta^d + 1) - c_2 \log_{d^2}(\beta^{d'} + 1)$$

$$= \sum_{x \in GF(q)} \psi(x(x^d + 1) - \psi^{M-c_1}(\beta^{d'}) (x^{d'} + 1) - 1$$

$$= \sum_{x \in GF(q)} \psi(x(x^d + 1) - \psi^{M-c_1}(\beta^{d'}) (x^{d'} + \beta^{-d'}) - 1$$

**Case 1.** $\tau \neq 0$.

Lemma 1 says that, for any $\tau$ with $1 \leq \tau \leq q - 2$, $f_1(x) = x^\tau + 1$ and $f_2(x) = x^{d'} + \beta^{-d'}$ are relatively prime (cf. Lemma 1(c)) with the respective number $e, e'$ of distinct roots in $GF(q)$ equal to $e = e' = 1$. (cf. Lemma 1(a)). Since $M$ does not divide $c_1$ and $M - c_2, \psi_1 = \psi^{c_1}$ and $\psi_2 = \psi^{M-c_2}$ are not trivial. Therefore we have

$$\psi_1(x^\tau + 1) \cdot \psi_2(\beta^{d'}(x^{d'} + \beta^{-d'}))$$

$$= \psi_1(x + 1) \cdot \psi_2(h_1(x)) \cdot \psi_2(h_2(x))$$

where $x^\tau + 1 = (x + 1)h_1(x)$ and $x^{d'} + \beta^{-d'} = (x + \beta^{-d'})h_2(x)$ for some polynomials $h_1(x)$ and $h_2(x)$. By Lemma 1, $h_1(x)$ and
and hence, therefore, we can apply Weil bound in Theorem 1, and hence Corollary 1.

\[ |C_{\alpha, \beta}(\tau)| \leq (d + d' - 1) \sqrt{q} + 3. \]

Case 2. \( \tau = 0 \)

In this case, we have

\[ C_{\alpha, \beta}(\tau = 0) = \sum \psi^{c_1}(x^{d} + 1) \cdot \psi^{c_2}(x^{d'} + 1) - 1. \]

Assume that \( d = d' \). Then (1) becomes \( \sum \psi^{c_1}(x^{d} + 1) \cdot \psi^{c_2}(x^{d'} + 1) - 1 \). If \( c_1 = c_2 \), then two sequences are the same. Otherwise,

\[ |C_{\alpha, \beta}(\tau = 0)| \leq (d - 1) \sqrt{q} + 2 \]

by Weil bound and Lemma 1.

Otherwise, assume that \( d \neq d' \). Then (1) becomes \( \sum \psi^{c_1}(x + 1) \cdot \psi^{c_2}(x^{d-1} - x^{d-2} - \cdots - x + 1) \cdot \psi^{c_3}(x^{d'-1} - x^{d'-2} - \cdots - x + 1) - 1 \). Second and third polynomials of degree \( d - 1 \) and \( d' - 1 \) inside the character are products of distinct monic irreducible polynomials over \( GF(q) \) by Lemma 1. Therefore, we can apply Weil bound, and hence

\[ |C_{\alpha, \beta}(\tau = 0)| \leq (d + d' - 2) \sqrt{q} + 2. \]

Corollary 2 Assume that \( \gcd(d, q - 1) = 1 \), and that \( p \) does not divide \( d \). Let \( s(t) \) be an \( M \)-ary Sidelnikov sequence of period \( q - 1 \). Let \( a(t) = c \cdot s(t) \) and \( b(t) = s(dt) \). Then we have

\[ \max_{\tau} |C_{a, b}(\tau)| \leq d \sqrt{q} + 3. \]

Example 2 Table 2 shows the difference between the exact maximal correlation magnitude and our correlation bound given in the above corollary for some \( q, d, c = 1 \) and \( M \).

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q = d' - 1 )</th>
<th>( d )</th>
<th>( M )</th>
<th>True Max</th>
<th>Bound = ( d \sqrt{q} + 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>64 = ( 2^6 )</td>
<td>5</td>
<td>7</td>
<td>17.62</td>
<td>43.00</td>
</tr>
<tr>
<td>3</td>
<td>243 = ( 3^5 )</td>
<td>5</td>
<td>11</td>
<td>41.78</td>
<td>80.94</td>
</tr>
<tr>
<td>2</td>
<td>256 = ( 2^8 )</td>
<td>7</td>
<td>15</td>
<td>49.79</td>
<td>115.00</td>
</tr>
<tr>
<td>17</td>
<td>289 = ( 17^2 )</td>
<td>5</td>
<td>8</td>
<td>45.96</td>
<td>88.00</td>
</tr>
<tr>
<td>7</td>
<td>343 = ( 7^3 )</td>
<td>5</td>
<td>9</td>
<td>47.78</td>
<td>95.60</td>
</tr>
<tr>
<td>2</td>
<td>512 = ( 2^9 )</td>
<td>5</td>
<td>7</td>
<td>56.58</td>
<td>68.88</td>
</tr>
</tbody>
</table>

\( h_2(x) \) are products of some distinct monic irreducible polynomials over \( GF(q) \) of degrees greater than 1 (cf. Lemma 1(b)). Therefore, we can apply Weil bound in Theorem 1, and hence

\[ |C_{\alpha, \beta}(\tau)| \leq (d + d' - 1) \sqrt{q} + 3. \]

Case 2. \( \tau = 0 \)

In this case, we have

\[ C_{\alpha, \beta}(\tau = 0) = \sum \psi^{c_1}(x^{d} + 1) \cdot \psi^{c_2}(x^{d'} + 1) - 1. \]

Assume that \( d = d' \). Then (1) becomes \( \sum \psi^{c_1}(x^{d} + 1) \cdot \psi^{c_2}(x^{d'} + 1) - 1 \). If \( c_1 = c_2 \), then two sequences are the same. Otherwise,

\[ |C_{\alpha, \beta}(\tau = 0)| \leq (d - 1) \sqrt{q} + 2 \]

by Weil bound and Lemma 1.

Otherwise, assume that \( d \neq d' \). Then (1) becomes \( \sum \psi^{c_1}(x + 1) \cdot \psi^{c_2}(x^{d-1} - x^{d-2} - \cdots - x + 1) \cdot \psi^{c_3}(x^{d'-1} - x^{d'-2} - \cdots - x + 1) - 1 \). Second and third polynomials of degree \( d - 1 \) and \( d' - 1 \) inside the character are products of distinct monic irreducible polynomials over \( GF(q) \) by Lemma 1. Therefore, we can apply Weil bound, and hence

\[ |C_{\alpha, \beta}(\tau = 0)| \leq (d + d' - 2) \sqrt{q} + 2. \]

Table 2 Comparison between true max and bound in Cor. 2 for \( c = 1, d' = 1, \gcd(d, q - 1) = 1 \) and \( p \) not dividing \( d \).

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\( p \) & \( q \) & \( d \) & \( M \) & True Max & Bound = \( d \sqrt{q} + 3 \) \\
\hline
2 & 64 = \( 2^6 \) & 5 & 7 & 17.62 & 43.00 \\
\hline
3 & 243 = \( 3^5 \) & 5 & 11 & 41.78 & 80.94 \\
\hline
2 & 256 = \( 2^8 \) & 7 & 15 & 49.79 & 115.00 \\
\hline
17 & 289 = \( 17^2 \) & 5 & 8 & 45.96 & 88.00 \\
\hline
7 & 343 = \( 7^3 \) & 5 & 9 & 47.78 & 95.60 \\
\hline
2 & 512 = \( 2^9 \) & 5 & 7 & 56.58 & 68.88 \\
\hline
\end{tabular}

in Table 2, which is about 81. The true max turns out to be about 42, showing some gap between the two numbers. Figure 2 shows the correlation function of the 9-ary Sidelnikov sequence of period \( 7^3 - 1 = 342 \) and its 5-decimation. In this case, the true max turns out to be about 48.

4. Concluding Remark

Both figures in the above example show that there is a gap between the true max and the bound calculated using the Weil bound. This gap will increase when the constant multiple \( c \) is not relatively prime to \( M \). For example, when \( q = 343 \) and \( M = 9 \) (cf. Example for Fig. 2), the true max between \( 3 \cdot s(t) \) and \( s(5t) \) is around 20 while the bound in Cor. 2 is also 95.6. We guess that these gaps can be reduced by some direct calculation of the correlation between \( c \cdot s(t) \) and \( s(dt) \) for \( 1 \leq c \leq M - 1 \) and \( \gcd(d, q - 1) = 1 \). It would be interesting in the future to see and compare these gaps between those from the direct calculation and from the Weil bound.

References

[1] V.M. Sidelnikov, “Some \( k \)-valued pseudo-random sequences and

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