New Design of High-Rate Generalized Root Protograph LDPC Codes for Nonergodic Block Interference

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***Abstract*— In this letter, we propose high-rate generalized root protograph (GRP) low-density parity-check (LDPC) codes for a nonergodic two-state binary symmetric channel with block interference with and without block fading. In these channels, the proposed GRP LDPC codes have asymptotic and finite-length performance improvement approaching to channel outage probability.**

***Index Terms*— Block fading, block interference, low-density parity-check (LDPC) codes, protograph LDPC codes, root protograph (RP) LDPC codes.**

I. Introduction

A CHANNEL with block interference (BI) has been researched in wireless communication systems [1], in which interference occurs in the block units, known as hops. If the length of each hop is not short compared to the length of the code, BI cannot be averaged over other hops and the nature of the BI channel is said to be nonergodic.

A two-state binary symmetric channel with block interference (TS-BSC-BI) [2] is a BSC channel with two states in each hop according to the existence of BI. It was adopted in the Gilbert-Elliot channel [3] and a channel with jamming [4]. In this letter, we propose a new class of flexible high-rate low-density parity check (LDPC) codes, referred to as generalized root protograph (GRP) LDPC codes for channels with one BI hop. For the proposed construction, the low-rate root [5] and the root protograph (RP) LDPC codes [6] for a block fading (BF) channel are used. Recently, design methods of protograph LDPC codes for BF channels have also been researched in many literatures [7]–[10].

This letter is organized as follows. In Section II, the channel models and analysis are introduced. In Section III, the new design method for GRP LDPC codes is proposed. In Section IV, the results of an asymptotic analysis are presented and the finite-length performances of the proposed LDPC codes are compared with full-diversity LDPC codes and channel outage probability in a numerical analysis. Finally, in Section V, the letter is concluded.

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II. System Model

In this section, the channel model and analysis are introduced. Several basic mathematical expressions are presented as follows. Let $\mathbf{v}$ be a row vector and v_i be the i-element of $\mathbf{v}$. Let $\mathbf{1}$ and $\mathbf{0}$ be all-one and all-zero vectors, respectively. The support set of $\mathbf{v}$ is denoted by $\text{supp}(\mathbf{v}) = \{i; v_i \neq 0, 1 \leq i \leq N\}$ and the Hamming weight of $\mathbf{v}$ by $\text{wt}(\mathbf{v}) = |\text{supp}(\mathbf{v})|$. Let $[a, b] = \{i \in \mathbb{N};\ a \leq i \leq b\}$ and $[i] = [1, i]$ for the set of positive integers $\mathbb{N}$.

The channel model is divided into two cases of TS-BSC-BIs with and without BF. Here, K information bits are encoded by an (N, K) binary protograph LDPC code $\mathcal{C}_P$ with $N - K$ check nodes (CNs), N variable nodes (VNs), and code rate $R = K/N$, where the codeword is indexed as $\mathbf{c} = (\mathbf{c}_1, \ldots, \mathbf{c}_L) = (c_{1,1}, \ldots, c_{1,h}, c_{2,1}, \ldots, c_{L,1}, \ldots, c_{L,h})$, where $\mathbf{c}_i$ denotes the partial codeword in the i-th hop and h denotes the length of the hop with $h|N$ and the number of hops $L = \frac{N}{h}$. Binary phase shift keying (BPSK) modulation converts $\mathbf{c}$ to $\mathbf{x} = (x_{1,1}, \ldots, x_{1,h}, x_{2,1}, \ldots, x_{L,1}, \ldots, x_{L,h})$ with the elements $x_{i,j} = (-1)^{c_{i,j}}$ for $i \in [L]$ and $j \in [h]$. The received signal can be expressed as

$$y_{i,j} = \alpha_i(1 - \beta_i)x_{i,j} + n_{i,j}, \tag{1}$$

where α_i denotes the fading coefficient, $\beta_i \in \{0, 1\}$ denotes the indicator of BI with probability $\rho = \Pr(\beta_i = 1)$, and $n_{i,j}$ denotes the additive white Gaussian noise with $\mathcal{N}(0, \sigma^2)$. Note that the existence of BI is often considered in a binomial distribution independent of the BF and noise [3], [4]. In this model, BI hops only contain additive white Gaussian noise independent of the desired signal.

The value of α_i is divided into the following two cases:

1) Without BF, $\alpha_i = 1$;
2) With BF, α_i is Rayleigh distributed with $E(\alpha_i^2) = 1$; i.e., α_i follows the pdf of $p_{\alpha_i}(\xi) = 2\xi e^{-\xi^2}$,

for all $i \in [L]$. At the receiver, the LDPC decoder uses the belief propagation (BP) algorithm with perfect channel state information (CSI) of the noise variance and fading coefficients. Hence, the initial log-likelihood ratio (LLR) message value of the BP decoding algorithm is written as

$$m_{i,j} = \begin{cases} \dfrac{2\alpha_i(1 - \beta_i)y_{i,j}}{\sigma^2}, & \text{without BI in the } i\text{-th hop} \\ 0, & \text{with BI in the } i\text{-th hop.} \end{cases} \tag{2}$$

Outage analyses of channels with BF were widely used for performance evaluations in previous works [5], [6]. These analyses can also be useful for channels with nonergodic BI. Here, the instantaneous input-output mutual information (MI) of each hop in the BI channel is expressed as $I_G\left(\frac{\alpha_i^2(1-\beta_i)^2 RE_b}{N_0}\right)$, where E_b/N_0 denotes the signal-to-noise ratio per information bit and

$$I_G(x) = -\int_{-\infty}^{\infty} \phi(\tau, x)\log_2[\phi(\tau, x)]d\tau - \frac{1}{2}\log_2 \frac{\pi e}{x} \quad (3)$$

with $\phi(\tau, x) = \frac{1}{2\sqrt{\pi/x}}(e^{-x(\tau+1)^2} + e^{-x(\tau-1)^2})$ [6]. The channel capacity is the expectation of MI of each hop, expressed as

$$I_A = \sum_{i\in[L]} \frac{1}{L} I_G\left(\frac{\alpha_i^2(1-\beta_i)^2 RE_b}{N_0}\right). \quad (4)$$

The channel threshold is then defined as the minimum E_b/N_0 satisfying $P(I_A < R) = 0$ in the channel without BF. Moreover, the channel outage probability is defined as $P(I_A < R)$ in the channel with BF. This is the well-known lower bound for the frame error rate (FER) of any coding scheme for the given channel and R [6]. In addition, it is possible to derive the channel outage probability by generating α_i and β_i randomly and checking $I_A < R$. Channel outage analyses will be used in asymptotic and numerical analyses in the next section.

III. Generalized Root Protograph LDPC Codes for Nonergodic BI

In this section, we propose a new GRP LDPC code for TS-BSC-BIs with and without BF. First, we explain the motivation behind the new code design using the minimum blockwise Hamming weight.

A. Minimum Blockwise Hamming Weight

The minimum blockwise Hamming weight of the code [5] is defined as follows.

Definition 1: The minimum blockwise Hamming weight d_c of code $\mathcal{C}$ is defined as

$$d_c = \min_{\mathbf{c}\in\mathcal{C}\setminus\{\mathbf{0}\}} \left(\sum_{i\in[L]} u(\mathrm{wt}(\mathbf{c}_i))\right), \quad (5)$$

where $u(x)$ returns 1 if $x > 0$ and 0, otherwise.

Note that the minimum blockwise Hamming weight d_c is also referred to as the diversity order in the BF channel, which corresponds to the slope of the FER curve in the coded performance. In addition, the codes with the minimum block Hamming weight d_c are designed to be robust against $d_c - 1$ deep fades. It is known that the minimum blockwise Hamming weight of codes with L hops is upper bounded by the Singleton-like bound given by earlier work [5],

$$d_c \leq 1 + \lfloor L(1-R)\rfloor, \quad (6)$$

which implies a trade-off between R and d_c. If the code of $R \leq \frac{1}{L}$ has a minimum blockwise Hamming weight of $d_c = L$ in the BF channel, the code is said to achieve full diversity. Among LDPC codes, low-rate root LDPC codes [5] and RP LDPC codes [6] with $R = \frac{1}{L}$ are designed to have full diversity with $d_c = L$. In contrast, the maximum value of d_c can be determined to be two from (6) if the code is such that $R \in (\frac{L-2}{L}, \frac{L-1}{L}]$. For example, a high-rate turbo code with $R = \frac{L-1}{L}$ and $d_c = 2$ was proposed in a BF channel [11].

In fact, the codes that are robust against deep fade are also advantageous for BI channels because BI can be considered as a deep fade. In addition, BI channels in the high E_b/N_0 region are considered to be block-erasure channels as explained in [5, Sec. III.A]. However, a high-rate code with $R = \frac{L-1}{L}$ and $d_c = 2$ does not work well in the BI channel of (1) because it should be always the case that $I_A < R$ when BI exists and $E_b/N_0 < \infty$. Thus, we propose a new high-rate GRP LDPC code with $R \in [\frac{L-2}{L-1}, \frac{L-1}{L})$ and $d_c = 2$ in the next subsection.

B. Construction of GRP LDPC Codes

In this subsection, we propose a GRP LDPC code with L hops and $R = \frac{b(L-2)+\beta}{b(L-1)}$, $0 \leq \beta < b$. First, the structure of its base matrix is proposed to enhance the performance in the high E_b/N_0 region.

Construction 1 (GRP LDPC Codes): Let T_i be a $b(L-1) \times b(L-1)$ upper or lower triangular low-density matrix with diagonal elements 1, which is equally row-partitioned with row size b as $T_i = [T_{i,1}^T, T_{i,2}^T, \ldots, T_{i,(L-1)}^T]^T$, $i \in [L]$. Let D_i be a $(b-\beta) \times b(L-1)$ nonnegative integer matrix. Then, the GRP LDPC code of $R = \frac{bL(L-2)+\beta}{bL(L-1)}$ has a parity check matrix lifted from the $(bL-\beta) \times b(L-1)L$ base matrix $B = [B_1, B_2, \ldots, B_L]$ with $B_i = [T_{i,1}^T, T_{i,2}^T, \ldots, T_{i,i-1}^T, D_i^T, T_{i,i}^T, \ldots, T_{i,(L-1)}^T]^T$.

Recall that the BI channel can be considered as a block-erasure channel in the high E_b/N_0 region. In the erasure channel, it is well-known that the existence of a stopping set can degrade the performance of a BP decoder. The proposed code is designed to avoid a stopping set in a hop while retaining two as the maximum value of d_c for the high E_b/N_0 region.

Theorem 1: The minimum blockwise Hamming weight of the proposed GRP LDPC code is two and the proposed code does not have a stopping set in one hop.

Proof: Suppose that there exists binary vector $\mathbf{v} = [\mathbf{v}_1, \ldots, \mathbf{v}_L]$, where the indices of all nonzero elements are in the i-th hop; i.e., $\mathrm{wt}(\mathbf{v}) = \mathrm{wt}(\mathbf{v}_i) > 0$. Note that stopping set $\mathcal{S}$ is defined as a subset of VNs, where every CN connected to them has at least two edges emanating from the VNs in $\mathcal{S}$. To prove that there is no stopping set $\mathcal{S}$ in the i-th hop, we will show that some CNs have only one edge emanating from the VNs in $\mathcal{S}$.

For an upper triangular low-density matrix T_i, suppose that the last index of the nonzero element in $\mathbf{v}_i$ is the one lifted from the j-th column of B_i, $j \in [b(L-1)]$. It is easy to prove this because there is a CN with one edge emanating from the VNs with indices in $\mathrm{supp}(\mathbf{v}_i)$ by the permutation matrix P_I lifted from the diagonal elements 1 of $T_{i,\lceil \frac{j}{b}\rceil}^T$ in the base matrix. Therefore, the code has no stopping set in the i-th hop and $d_c = 2$ because $\mathbf{v}$ is not a codeword if the set of

VNs with indices in supp($\mathbf{v}$) does not contain $\mathcal{S}$. Thus, $d_c \geq 2$ according to (5). It is easily checked that the maximum value of d_c is two for code rates larger than or equal to $\frac{L-2}{L-1}$ from (6). For a lower triangular low-density matrix T_i, the proof can also be done similarly. ■

Some integers of the base matrix remain undetermined in Construction 1. In order to find the best base matrix operating well even in the low E_b/N_0 region, the modified PEXIT algorithm [6] is applied to the proposed GRP LDPC codes. The detailed method using the PEXIT algorithm is identical to that in [6] except for the initialization. In order to enhance the performance in a channel with a BI hop, the initial fading coefficient in the l-th hop is set to 0; i.e., $\alpha_l = 0$ and $\alpha_i = 1$ for $i \in [L] \setminus \{l\}$. Then, we find the best base matrix such that the corresponding BP threshold $(E_b/N_0)_{BP,th}$ has the minimum value. According to Construction 1 and the modified PEXIT algorithm, a GRP LDPC code with $L = 2$, $b = 3$, $\beta = 2$, and $R = \frac{1}{3}$ is constructed as

$$B_{GRP1} = \left(\begin{array}{c|c} 021 & 100 \\ 110 & 110 \\ 011 & 011 \\ 001 & 120 \end{array}\right). \quad (7)$$

Similarly, two GRP LDPC codes with $L = 3$ can be constructed as shown below. The first one is constructed for $b = 2$, $\beta = 1$, and $R = \frac{7}{12}$ as

$$B_{GRP2} = \left(\begin{array}{c|c|c} 2111 & 1000 & 1000 \\ 1000 & 0100 & 0103 \\ 0100 & 2112 & 0010 \\ 3010 & 0010 & 0001 \\ 0001 & 0001 & 1112 \end{array}\right). \quad (8)$$

The second one is constructed for $b = 2$, $\beta = 0$, and $R = \frac{1}{2}$ as

$$B_{GRP3} = \left(\begin{array}{c|c|c} 0012 & 1000 & 1000 \\ 1120 & 0100 & 0100 \\ 1000 & 0012 & 0010 \\ 0100 & 1120 & 0001 \\ 0010 & 0010 & 0012 \\ 0001 & 0001 & 1120 \end{array}\right). \quad (9)$$

In the next subsection, the proposed codes of (8) and (9) will be compared with other low-rate full-diversity LDPC codes with $L = 2, 3$ and the channel outage probability in (4).

IV. Asymptotic and Numerical Analyses of GRP LDPC Codes

In this section, the proposed GRP LDPC codes are compared with the full-diversity rate-compatible RP (RCRP) LDPC codes designed for $L = 2$ and $R = \frac{1}{3}$ in [8, eq. (4)] and the irregular RP2 (IRP2) LDPC code designed for $L = 3$ and $R = \frac{1}{3}$ in [10, eq. (18)]. Note that it is not possible to make the same code rate as both the proposed codes of $R \geq \frac{L-2}{L-1}$ and the full-diversity codes of $R \leq \frac{1}{L}$ by definitions, if $L \geq 3$. For an additional comparison, we also simulate the regular protograph LDPC code of $R = 1/3$ and $d_c = 1$, whose base matrix is a 4×6 all-one matrix. In order to obtain asymptotic performance, the BP and fading thresholds are shown in the next subsection.

TABLE I

Channel and BP Thresholds of the GRP, Regular, and Full-Diversity LDPC Codes in Channels With $L = 2, 3$

L	2			3		
BP/Chan. thr.	GRP1	RCRP	Reg.	GRP2	GRP3	IRP2
BP thr. w. BI	4.364	4.736	∞	4.745	3.630	3.149
Chan. thr. w. BI	4.070			4.606	3.387	1.948
BP thr. w.o. BI	1.716	1.946	1.730	1.606	1.266	1.691
Chan. thr. w.o. BI	-0.496			0.590	0.158	-0.496

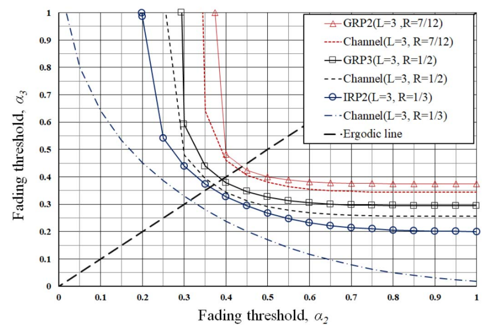

Fig. 1. Fading threshold vectors (α_2, α_3) in the channels of $E_b/N_0 = 12$[dB] and $L = 3$, where the first hop is with BI for the GRP and IRP2 LDPC codes.

A. Asymptotic Analysis of LDPC Codes

An asymptotic analysis of the proposed and full-diversity LDPC codes can be conducted using a fading threshold [6] with the PEXIT algorithm and the channel threshold by (4). In the initialization of the PEXIT algorithm, we consider the case in which the first hop is with BI; i.e., $\alpha_1 = 0$.

For a channel with a BI hop, Table I shows that the proposed GRP LDPC codes have smaller gaps between the channel and the BP thresholds with BI than the full-diversity codes for both $L = 2$ and $L = 3$. For $L = 2$ and $R = \frac{1}{3}$, the GRP1 LDPC codes have a lower BP threshold than the RCRP LDPC codes and the regular LDPC code does not have a BP threshold for any finite value of E_b/N_0 due to the small value of $d_c = 1$. Table I also includes the thresholds for $L = 3$.

Fig. 1 shows the fading threshold vectors (α_2, α_3) in the channels of $E_b/N_0 = 12$[dB] and $L = 3$ for the GRP LDPC codes with $R = \frac{7}{12}$ and $R = \frac{1}{2}$ and for the IRP2 LDPC code with $R = \frac{1}{3}$, where the first hop is with BI. For a channel with a BI hop, it is shown that the proposed GRP LDPC codes have good performances approaching the channel fading thresholds, whereas the IRP2 and RCRP LDPC codes have larger gaps with regard to the channel fading thresholds.

B. Numerical Analysis of Finite-Length LDPC Codes

In this subsection, we show that the finite-length FER performances of the proposed GRP and full-diversity LDPC codes match the aforementioned asymptotic analyses. All of the finite-length LDPC codes have a codelength of 2304 with two or three hops lifted by circulant permutation matrices. In addition, we generate the shift values for the parity check matrix to avoid girth 6. Furthermore, the following channel environments are assumed. First, the existence of BI for each hop is assumed to follow a binomial distribution with $\rho = 0.01$. For the BP decoder, the maximum number of

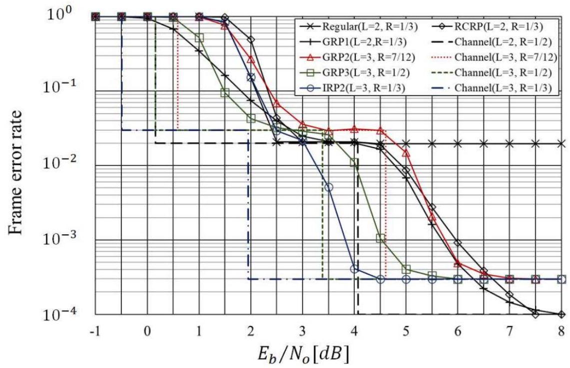

Fig. 2. FER performances of the finite-length regular, GRP1, and RCRP LDPC codes with $L = 2$ and the GRP2, GRP3, and IRP2 LDPC codes with $L = 3$ and $n = 2304$ in a channel with BI of $\rho = 0.01$ and without BF.

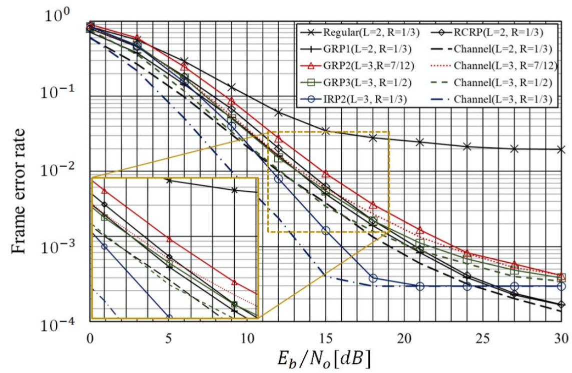

Fig. 3. FER performances of the finite-length regular, GRP1, and RCRP LDPC codes with $L = 2$ and the GRP2, GRP3, and IRP2 LDPC codes with $L = 3$ and $n = 2304$ in a channel with BI of $\rho = 0.01$ and Rayleigh BF.

iterations is set to 100. For FER, the GRP LDPC code declares an error if at least one of the coded bits is erroneous, but the IRP2 and RCRP LDPC codes declare an error if at least one of the information bits is erroneous. Note that the locations of information bits are known in the RCRP and IRP2 LDPC codes but not explicitly known in the proposed GRP LDPC codes.

The finite-length FER performances of the regular, GRP1, and RCRP LDPC codes with $L = 2$ and the GRP2, GRP3, and IRP2 LDPC codes with $L = 3$ in a channel with BI of $\rho = 0.01$ and without BF are shown in Fig. 2. First, the FER of the LDPC codes has the shape of stairs with two levels, which shows that the codes cannot be decoded for cases without BI and with a BI hop if E_b/N_0 is lower than the channel thresholds. The proposed GRP LDPC codes have good performance approaching the channel threshold and their gaps are smaller at the lower stair, showing the case of a channel with a BI hop. For the full-diversity RCRP and IRP2 LDPC codes, they can correct BI for a high E_b/N_0, but the gap between the FER and the channel threshold is larger than in the GRP LDPC code because it has a large BP threshold, as indicated in Table I. For the regular LDPC code, the FER does not approach the channel outage probability even for a high E_b/N_0 due to the small value of $d_c = 1$.

The finite-length FER performances of the regular, GRP1, and RCRP LDPC codes with $L = 2$ and the GRP2, GRP3, and IRP2 LDPC codes with $L = 3$ in a channel with BI of $\rho = 0.01$ and Rayleigh BF are shown in Fig. 3. In this case, the FER curves of the GRP and IRP2 LDPC codes show a high error-floor for a high E_b/N_0 due to the existence of BI. The GRP LDPC codes have good performance approaching the channel outage probability but the IRP2 and RCRP LDPC codes have a larger gap between the FER and the channel outage probability than the proposed GRP LDPC codes. For the regular LDPC codes, the FER does not approach the channel outage probability even for a high E_b/N_0 due to the small value of $d_c = 1$.

V. Conclusion

In this letter, we proposed high-rate GRP LDPC codes for channels with BI using the concept of the minimum blockwise Hamming weight. The design and asymptotic analyses of the proposed GRP LDPC codes were conducted by the modified PEXIT algorithm. The finite-length GRP LDPC codes show good performance approaching the channel outage probability. As a future work, designs and analyses of the LDPC codes with arbitrary values of d_c for channels with BI and BF can also be researched.

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