compared to the equiprobable one, the channel capacity limit is 10.2 dB, and we achieve $P_b(e) = 10^{-5}$ within 1.2 dB of this limit.

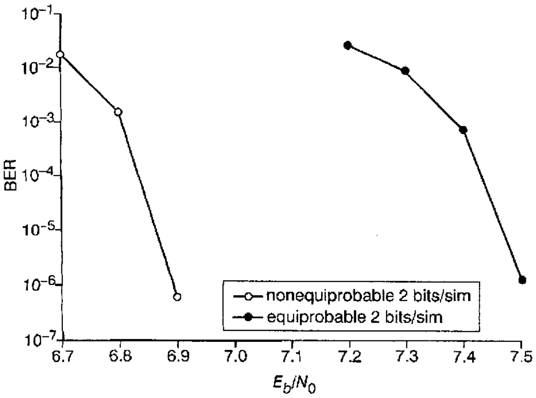

Fig. 3 *Performance comparison of two schemes of non-equiprobable and equiprobable signalling at rate 2 bits/dim using pragmatic binary turbo coded modulation with 18 iterations*

Block length $N = 32768$ bit channel capacity limit 5.74 dB

Conclusions: We have presented a new scheme for improving the performance of pragmatic binary turbo coded modulation by using non-equiprobable signalling. We have described a non-equiprobable signalling technique that makes it possible to approach the maximum capacity gain of a finite constellation AWGN channel. Our non-uniform signalling scheme is very easy to implement and adds negligible load on the turbo decoder. We have shown for an example of 6 bits/QAM symbol, a gain of 0.93 dB out of the available shaping gain of 1.07 dB, and transmission within 1.2 dB of the Shannon limit.

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Product distance profile of product distance codes for STTC with delay diversity

S.-E. Park, M.-H. Shin, H.-Y. Song and D.K. Kim

Space-time trellis codes (STTC) using product distance codes with delay diversity structure are studied and the optimal condition of product distance codes is proposed. The upper bound of the optimal product distance of codes on 8-PSK modulation is derived. The performance of the STTC using various product distance codes found through a systematic code search is compared with that of delay diversity.

Introduction: We consider the delay diversity scheme as proposed by Wittneben [1] which can also be thought of as the combination of a repetition code with a delay element. Tarokh's space-time trellis codes (STTCs) contains this delay diversity structure [2].

Fig. 1 shows a labelling of the QPSK constellation and trellis description of STTC with two transmit antennas, using the block code {00, 11, 23, 32}. Delay diversity is also an STTC using the repetition code {00, 11, 22, 33}. Each branch has the label s_1s_2, which indicates that symbol s_1 is transmitted over the first antenna and that symbol s_2 is transmitted over the second antenna. Both of these codes have the largest minimum product distance over the QPSK alphabet, but their performances are not at all the same [simulation demonstrates this] as shown in Fig. 2 which is the main topic of this Letter.

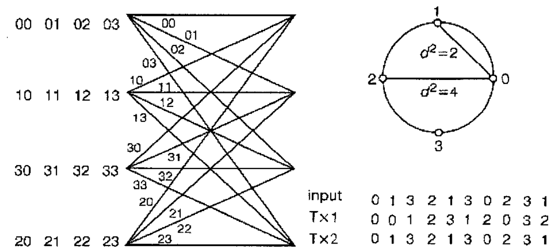

Fig. 1 *STTC with {00, 11, 23, 32}, QPSK, Tx = 2*

We note that the number of codeword pairs with each product distances are different between these codes, and we define N_{min} as the number of codeword pairs with the minimum product distance. The former has $N_{min} = 4$ and the latter $N_{min} = 2$.

Optimal and super-optimal product distance codes: Consider a block code C given by

$$C = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_M\} \tag{1}$$

C consists of M codewords and each codeword has length N. The ith codeword is $\mathbf{c}_i = c_i^1 c_i^2 \cdots c_i^N$, where $c_i^m \in \{0, 1, 2, \ldots, M-1\}$, $m = 1, 2, \ldots, N$. With M-ary modulation constellation, we define the product distance $D_{(\mathbf{c}_i,\mathbf{c}_j)}$ between pairs of distinct codewords $(\mathbf{c}_i, \mathbf{c}_j)$ as follows [3]:

$$D_{(\mathbf{c}_i,\mathbf{c}_j)} = \prod_{m=1}^{N} |f(c_i^m) - f(c_j^m)|^2 \tag{2}$$

where $f(c_i^m)$ denotes the modulation constellation point corresponding to the symbol element c_i^m and has complex value. We assume that the modulation mapping function f is a bijection $(1-1$ mapping) from $\{0, 1, 2, \ldots, M-1\}$ onto the set of constellation points S. In the case of QPSK modulation, c_i^m and $f(c_i^m)$ belong to $\{0, 1, 2, 3\}$ and $\{e^{j(2\pi/4)k}$, $k = 0, 1, 2, 3\}$, respectively. In the case of 8-PSK, c_i^m and $f(c_i^m)$ belong to $\{0, 1, 2, \ldots, 7\}$ and $e^{j(2\pi/8)k}$, $k = 0, 1, 2, \ldots, 7\}$, respectively.

We only consider two-dimensional signal space in this Letter. Then an optimal product distance code is defined as a block code where minimum of the product distance between pairs of distinct codeworks is maximum among all such block codes. Formally, we use the product distances between pairs of distinct codewords of C, and use the notation given by

$$D_{min} = \min_{i \neq j} D_{(\mathbf{c}_i,\mathbf{c}_j)} = \min_{i \neq j} \prod_{m=1}^{N} |f(c_i^m) - f(c_j^m)|^2 \tag{3}$$

and we denote D_{min} of an optimal product distance code by D_{opt}. Using this optimal product distance code with delay elements we can construct a space-time transmit system.

To further classify the optimal product distance codes of given parameters, we define N_{min} as the number of distinct codeword pairs $(\mathbf{c}_i, \mathbf{c}_j)$ with $D_{(\mathbf{c}_i,\mathbf{c}_j)} = D_{min}$. From the relation between the free distance and the number of paths achieving free distance in convolutional codes, we can suppose that the space-time trellis code using the optimal product distance code performs better as N_{min} is minimised. This prompts a definition of a super-optimal product distance codes as the optimal code with minimum N_{min}.

It is proved that the set $\{c_1^n, c_2^n, \ldots, c_M^n\}$ should be a permutation of $\{0, 1, 2, \ldots, M-1\}$ for each $n = 1, 2, \ldots, N$ if and only if D_{min} is maximised [4]. We can assume that C has an all-zero codeword without loss of generality to reduce the searching complexity. So we let $\mathbf{c}_1 = c_1^1 c_1^2 \cdots c_1^N$ be the all-zero codeword.

Upper bound of optimal product distance on 8-PSK: Li *et al.* [4] found the value of D_{opt} on QPSK modulation as follows:

$$D_{opt} = 2^{N+\lfloor N/3 \rfloor} \qquad (4)$$

where N is the number of transmit antennas. Using a similar argument, we can derive the upper bound of D_{opt} on 8-PSK modulation as

$$D_{opt} \leq 2^{6/7N} \qquad (5)$$

As the number of transmit antennas increases, the complexity of searching for the exact value of D_{opt} also increases. Thus, the derived upper bound will help estimate D_{opt} when the number of transmit antennas is large.

Table 1: Product distance profiles (QPSK)

Tx	Product distance profile									Multiplicity	Total
	N_4	N_8	N_{16}	N_{32}	N_{64}	N_{128}	N_{256}	N_{512}	N_{1024}		
2	2	4	0							4	6
	4	0	2							2	
3		0	6	0	0					8	6^2
		2	2	2	0					24	
		4	0	0	2					4	
4			0	4	2	0	0			96	6^3
			2	0	4	0	0			48	
			2	2	0	2	0			64	
			4	0	0	0	2			8	
5				0	2	4	0	0	0	480	6^4
				0	4	0	2	0	0	320	
				2	0	2	2	0	0	320	
				2	2	0	0	2	0	160	
				4	0	0	0	0	2	16	

Product distance profile and super-optimality: Let $i_1, i_2, \ldots, i_k$ be all possible product distance of the pairs of all possible codes such that $i_1 < i_2 < \cdots < i_k$ when the parameters, e.g. the type of modulation and the number of transmit antennas are given. k is a finite number depending on above two parameters. In general, we use the notation N_i as the number of codeword pairs where product distance of the pair is i. We define the list $(N_{i_1}, N_{i_2}, \ldots, N_{i_k})$ as the product distance profile of the code. Given a code C, let N_{i_j} be the first nonzero number of the product distance profile i.e. all the $N_{i_1}, N_{i_2}, \ldots, N_{i_j - 1}$ are zero and $N_{i_j} \neq 0$. Then $D_{min} = i_j$ and the optimal product distance code has the largest i_j among all codes of given parameters. The super-optimal code is the optimal product distance code whose N_{i_j} is minimum among all the optimal ones.

Table 1 provides the product distance profiles of all possible block codes using QPSK constellation shown in Fig. 1 in which the number of transmit antennas ranges from 2 to 5.

Simulation results: Performance evaluation in terms of frame error rate (FER) is conducted using computer simulation. Quasi-static flat fading and perfect channel estimation are assumed. Maximum likelihood decoding with unquantised soft decision employing a space-time Viterbi decoder is applied. A frame consists of 130 successive symbols as IS-54 standard. We present simulation results of space-time trellis codes found through systematic search with various parameters compared with delay diversity as baseline performance. Fig. 2 shows that, in case of QPSK modulation with two transmit antennas, both delay diversity and STTC with an optimal product distance code have the same $D_{min} = 4$ but different N_{min}, which are 4 and 2, respectively. The latter code outperforms by about 0.7 dB over the former at 10^{-3} FER when using 4 receive antennas. In the 8-PSK modulation format as shown in Fig. 3 with 3 transmit antennas, STTC with an optimal product distance code outperforms by about 4 dB over delay diversity.

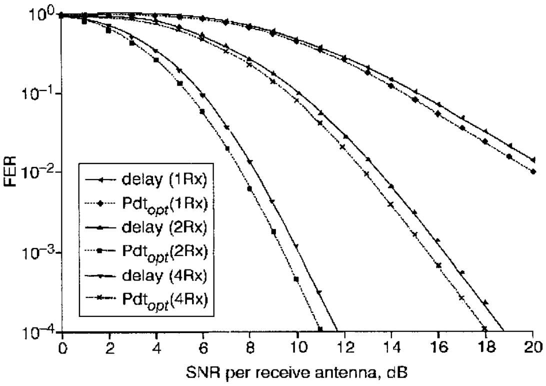

Fig. 2 *Performance simulation*
QPSK, Tx = 2, 4 State, 2 bit/s/Hz

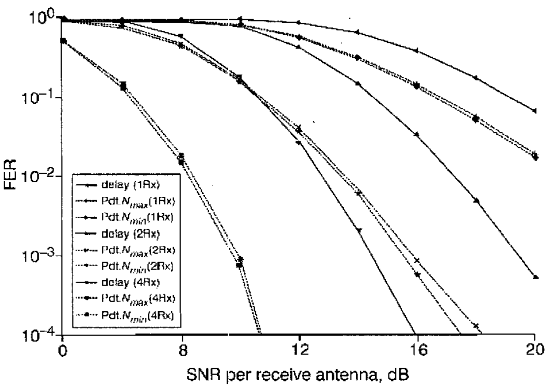

Fig. 3 *Performance simulation*
8-PSK, Tx = 3, 64 State, 3 bit/s/Hz

Conclusion: We have investigated a design of STTC applicable to various number of transmit antennas for both 4-PSK and 8-PSK. We considered existing optimal product distance codes in terms of the minimum product distance and derived the upper bound of optimal product distance of codes on 8-PSK modulation. We suggested the product distance profile and defined a super-optimal product distance code in order to classify further the optimal ones. Monte-Carlo simulations have been conducted for performance analysis. Simulation results show that higher coding gain over delay diversity is attainable as modulation constellation expands.

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