Construction of 2-dimensional Arrays with Ideal Crosscorrelation

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최적상관특성을 갖는 2차원 신호집합의 생성

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◉ Objective

To construct **a set of sonar arrays** of the same size with an additional constraint that the non-periodic two-dimensional cross-correlation value of any two arrays from the set is limited to either 1 or 0.(an **ideal cross-correlation**)

◉ Application

Application in various **multiuser communication systems** such as multiuser radar and sonar systems and/or fiberoptic CDMA networks.

◉ Sonar array

- Pattern of n dots with **one dot per column** having the property that any horizontal and vertical shifting would result in at most one dot position agreement.
- sonar sequence problem is **to maximize** m **in an** $n \times m$ **pattern** where n is given.
- It is easy to prove that with n rows, the maximum number of columns cannot be more than $2n$.

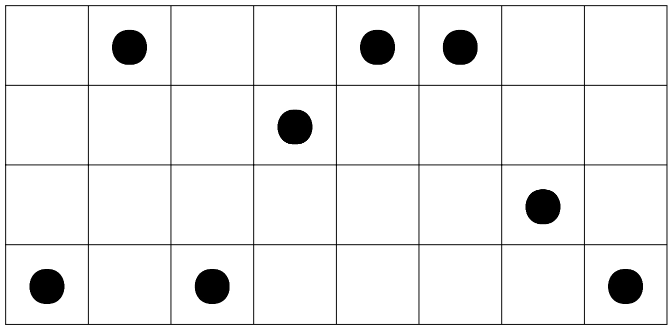

FIG. 4×8 sonar array

◉ $(n, 2)$-sequence

- Let $a_1, a_2, \cdots, a_{2n}$ be a permutation of $0, 1, 2, \cdots, 2n-1$.

- Let (a_i, a_j) be a **comparable pair** if $0 \le a_i, a_j \le n-1$ or $n \le a_i, a_j \le 2n-1$,

- $a_1, a_2, \cdots, a_{2n}$ is called $(n, 2)$-**sequence** ,
  if $a_{s+d} - a_s \neq a_{t+d} - a_t \pmod{n}$ for every $s, t,$ and d
  such that $1 \le s < t < t+d \le kn$ and
  such that (a_{s+d}, a_s) and (a_{t+d}, a_t) are comparable pairs

◉ Construction of $(n, 2)$-sequence

- Let α be a primitive root modulo $p = 2n + 1\ (> 2)$ where p is a prime.

- For $i = 1, 2, \cdots, 2n$, take the value of j to be between 0 and $2n - 1$ such that $\log_\alpha i = j$ if $\alpha^j = i$.

- Let q_i **and** r_i **be the quotient and remainder**, respectively, **when** $\log_\alpha i$ **is divided by 2**;
  that is , $\log_\alpha i = 2q_i + r_i$, where $r_i \in \{0, 1\}$.

- Then $a_i = q_i + r_i n$ for $i = 1, 2, \cdots, 2n$ is an $(n, 2)$-sequence.

◉ Construction of 2-dimensional Arrays with Ideal Crosscorrelation

- Generally, a set of k sonar arrays of size $kn \times n$ having ideal cross-c
o
  rrelation functions can be **constructed from** (n, k)-**sequences** of length kn.

- **Whenever** $p = kn + 1$ **is an odd prime**, the construction can best be described as follows:
  - Let α be a primitive root mod p .
  - Then each A_j for $j = 0, 1, 2, 3, \cdots, k-1$ has a dot in row α^{ki+j}

for every column $i = 1, 2, \cdots, n$.

◉ Optimization of a pair of sonar arrays

- optimal pair
  - a pair of sonar arrays having **<u>maximal number of columns</u>**, given number of rows

- **<u>apply above technique to "modular sonar array" and then take modularity-preserving transformations</u>**.

- for any integers a, b, c_A, c_B, two arrays given by

  $g_A(i) = af(2i) + b(2i) + c_A$ and

  $g_B(i) = af(2i-1) + b(2i-1) + c_B$ where $f(i)$ is modular sonar array, are sonar arrays having an ideal cross-correlation function.

◉ Conclusion

① sonar arrays
② $(n, 2)$-sequences
③ a pair of sonar arrays from $(n, 2)$-sequences
④ optimal pair of sonar arrays from modular sonar arrays

when above optimization method is applied to **every modular sonar array construction**, we can guess that better pair of sonar arrays can be found.

0		6		2		1		10		8		11		7		4		5		9		3
	*		*		5		*		4		3		2		*		1		*		*	
		2		*		*		*		1		5		*		*		*		4		
			1		4		*		*		3		*		*		2		5			
				*		2		*		*		*		*		4		*				
					*		5		*		3		*		1		*					
						*		1		2		4		5		*						
							*		*		3		*		*							
								4		*		*		2								
									5		3		1									
										*		*										
											3											

FIG. (6,2)-sequence & difference triangle mod 6