

# A study on the minimum distance of linear binary codes with generator matrix containing all weight-3 column vectors

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Distributed Storage System (DSS)



Huge file

node #1 node #2 node #3 node #4 node #5

✓ A huge file is partitioned into several parts and stored in networked nodes.

### Reliability of DSS



- ✓ A node failure occurs frequently due to hardware problems, network problems, and so on.
- ✓ Introducing an redundancy for reliability
- ✓ Erasure-correcting codes



### Code with locality 2

**Theorem:** Let  $C_2$  be a binary systematic  $\left[\frac{k(k+1)}{2}, k, d\right]_2$  code, where its generator matrix contains all weight-2 column vector of length k. Then, the minimum distance of  $C_2$  is k + 1.

\* Jung-Hyun Kim, Mi-Young Nam, and Hong-Yeop Song, "Binary locally repairable codes from complete multipartite graphs," International Conference on ICT Convergence 2015 (ICTC2015), Jeju Island, Korea, Oct. 2015.

#### Code with locality 3 (our contribution)

**Theorem 1:** Let  $C_3$  be a binary systematic

 $\begin{bmatrix} \frac{k(k^2-3k+8)}{6}, k, d \end{bmatrix}_2 \text{ code for } k \ge 3, \text{ where its}$ generator matrix contains all weight-3 column vector of length k. Then, the minimum distance

of  $\mathcal{C}_3$  is  $\binom{k-1}{2} + 1$ .

### Repair Problem in DSS

- ✓ To maintain the same level of reliability of DSS where frequent node failures are imposed, a code should repair a part of a codeword efficiently.
- ✓ A node repair from other surviving nodes



✓ Repair bandwidth:

the amount of data that should be communicated for a node repair

✓ Locality:

the number of nodes should be contacted for a node repair

## Locality in a Generator Matrix (systematic form)



#### Comparisons

k	<i>n</i> <sub>2</sub>	<i>d</i> <sub>2</sub>	$n_3$	<i>d</i> <sub>3</sub>
3	6	4	4	2
4	10	5	8	4
5	15	6	15	7
6	21	7	26	11
7	28	8	42	16
8	36	9	64	22
9	45	10	93	29
10	55	11	130	37
:	:	:	:	:
20	210	21	1160	172

n<sub>2</sub>: code length of C<sub>2</sub>
n<sub>3</sub>: code length of C<sub>3</sub>

➢  $d_2$ : minimum distance of  $C_2$ 

### Concluding Remarks

- A simple explicit construction of LRC with locality 3
- The exact minimum distance of the proposed code
- How can we improve the rate performance of the proposed code?

