A construction of QC-LDPC codes using Golomb rulers generated from sonar sequences

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KICS Summer Conference 2023

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QC-LDPC codes [1]

- A simpler structure & Good error-correcting performance.

- The multiplication table method

  $M \times N$ Exponent matrix

$$E = \begin{bmatrix} e(1,1) & e(1,2) & \cdots & e(1,N) \\ e(2,1) & e(2,2) & \cdots & e(2,N) \\ \vdots & \vdots & \ddots & \vdots \\ e(M,1) & e(M,2) & \cdots & e(M,N) \end{bmatrix}.$$

where $e(i,j) = e(i,1)e(1,j)$.

Cyclically shift the $P \times P$ circular permutation matrices (CPM) by each element of E and substitute into that element. → $MP \times NP$ H-matrix

[1] M. P. C. Fossorier, "Quasicyclic low-density parity-check codes from circulant permutation matrices," IEEE Transactions on Information Theory, vol. 50, pp. 1788–1793, 2004.

Golomb ruler

Definition (Golomb ruler [2])

A set of integers $G = \{g_1, g_2, \dots, g_N\}$ where $g_1 < g_2 < \cdots < g_N$ is an ***N*-mark Golomb ruler** iff

$$g_i - g_j \text{ are all different}$$

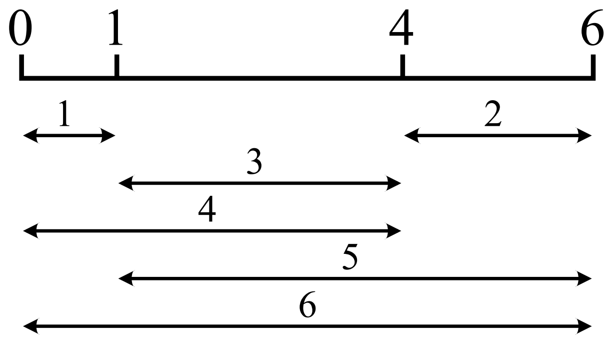
for $i, j \in [1,2, \dots, N]$ such that $i \neq j$.

$L = g_N - g_1$ is called the **length** of a Golomb ruler.

[2] A. Dimitromanolakis, "Analysis of the Golomb ruler and the Sidon set problems, and determination of large, near-optimal Golomb rulers," Master's Thesis, Department of Electronic and Computer Engineering, Technical University of Crete, June 2002.

- Example

  $\{0, 1, 4, 6\}$ is a 4-mark Golomb ruler.



Sonar sequence

Definition (Sonar sequence [5])

A sequence $S = [s_1, s_2, \dots, s_n]$ is an $(\boldsymbol{m}, \boldsymbol{n})$ **sonar sequence** of length n over the set of integers $\{1,2, \dots, m\}$ iff

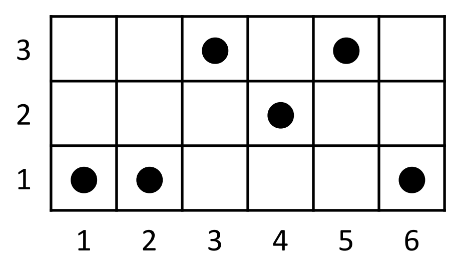
$$s_{u+r} - s_u \text{ are all different for the same } r$$

where $r \in [1,2, \dots, n-1]$, $u \in [1,2, \dots, n-r]$.

[5] O. Moreno, R. A. Games, and H. Taylor, “Sonar Sequences from Costas Arrays and the Best Known Sonar Sequences with up to 100 Symbols,” IEEE Transactions on Information Theory, vol. 39, no. 6, pp. 1985-1987, 1993.

- Example

  $[1, 1, 3, 2, 3, 1]$ is a $(3, 6)$ sonar sequence.



A construction of QC-LDPC codes using Golomb rulers [3]

- The construction of exponent matrix

$$E = \begin{bmatrix} e(1,1) & e(1,2) & \cdots & e(1,N) \\ e(2,1) & e(2,2) & \cdots & e(2,N) \\ e(3,1) & e(3,2) & \cdots & e(3,N) \end{bmatrix}$$

where $e(i,j) = e(i,1)e(1,j)$ such that $\{e(i,1) = i | i = 1,2,3\}$, $e(1,j)$ is a Golomb ruler.

Theorem [3]

The QC-LDPC codes from the construction have girth-8 if $P > 2L$, where P is the size of CPM and L is the length the Golomb ruler.

[3] I. Kim and H.-Y. Song, "A construction for girth-8 QC-LDPC codes using Golomb rulers," Electronics Letters, vol. 58, no. 15, pp. 582-584, July 2022.

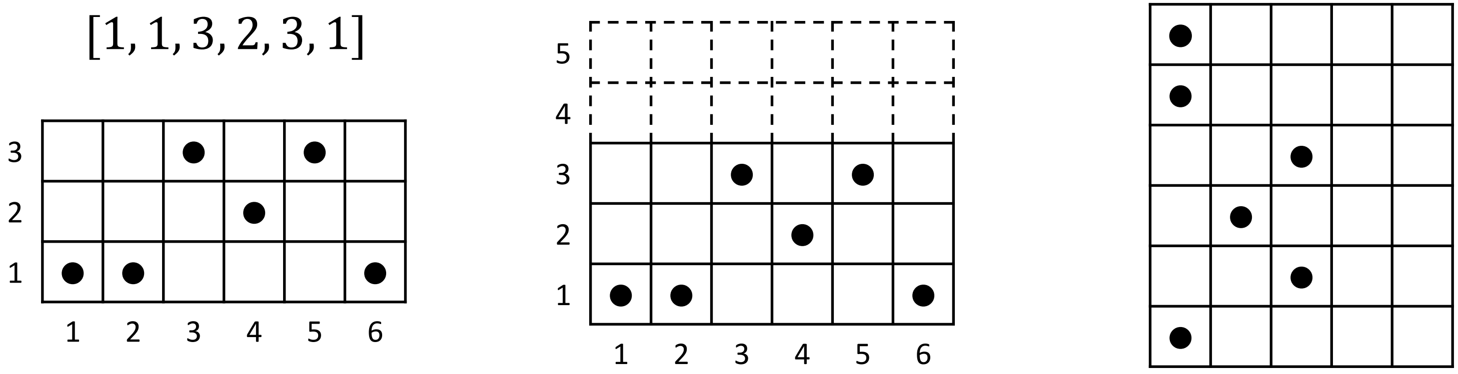
- This paper presents a QC-LDPC code constructed using Golomb rulers generated from sonar sequences.

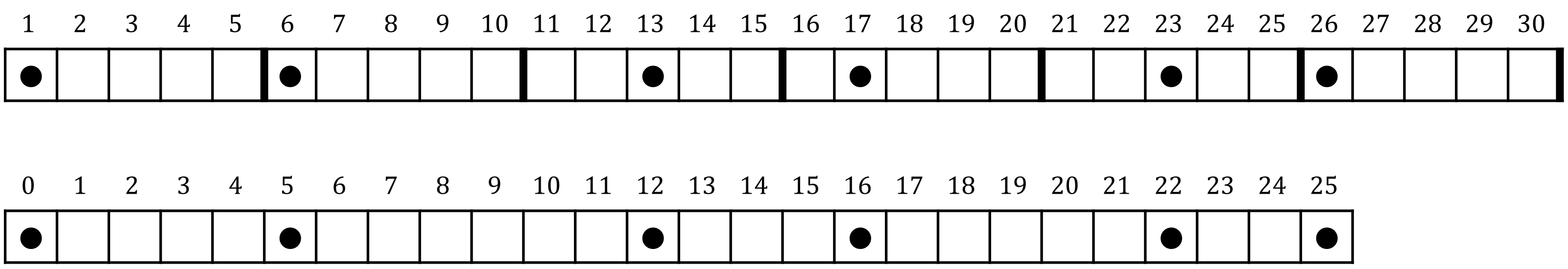
Contents

Proposed method

- A method for transforming the sonar sequence into the Golomb ruler.

$[1, 1, 3, 2, 3, 1]$





$(0, 5, 12, 16, 22, 25)$ is a 6-mark Golomb ruler.

Proposed method

Theorem 1

Let $S = [s_1, s_2, \dots, s_n]$ be an (m, n) sonar sequence. If the integer $t \geq 2m - 1$, then $G = \{g_i | g_i = (i-1)t + s_i\}$ is a n-mark Golomb ruler for $i = 1, 2, \dots, n$.

A (3,6) sonar sequence $[1,1,3,2,3,1]$, take $t = 2m - 1 = 5$.

$g_1 = (1-1) \times 5 + 1 = 1$, $g_2 = (2-1) \times 5 + 1 = 6$,

$g_3 = (3-1) \times 5 + 3 = 13$, $g_4 = (4-1) \times 5 + 2 = 17$,

$g_5 = (5-1) \times 5 + 3 = 23$, $g_6 = (6-1) \times 5 + 1 = 26$.

$\Longrightarrow$ A 6-mark Golomb ruler $(0, 5, 12, 16, 22, 25)$.

Proposed method

- The minimum length L_{min} of Golomb ruler in Theorem 1:

$$\begin{aligned} L &= g_n - g_1 \\ &= (n-1)t + s_n - s_1 \\ &\geq (n-1)(2m-1) + 1 - m = L_{min} \end{aligned}$$

$\Longrightarrow$ In order to have girth-8, $P > 2L$,

code length $N_c = nP > 2nL_{min}$.

- Using the given sonar sequence, obtain the Golomb ruler from Theorem 1 and construct a QC-LDPC code of length N_c.

Contents

Simulation

We chose several $(m, 6)$ sonar sequences where $m \leq 6$ all possible values, According to [5].

(m, n)	sonar sequence	Golomb ruler
$(3, 6)$	$[1,1,3,2,3,1]$	$(0,5,12,16,22,25)$
$(4, 6)$	$[1,3,4,4,2,1]$	$(0,9,17,24,29,35)$
$(5, 6)$	$[1,3,4,2,5,4]$	$(0,11,21,28,40,48)$
$(6, 6)$	$[1,5,4,6,2,3]$	$(0,15,25,38,45,57)$

Compare with the best performing $(0,1,8,12,14,91)$ [4] using optimal Golomb ruler $(0,1,8,12,14,17)$ when $N_c = 1200, \;\; \text{rate} = 0.5$.

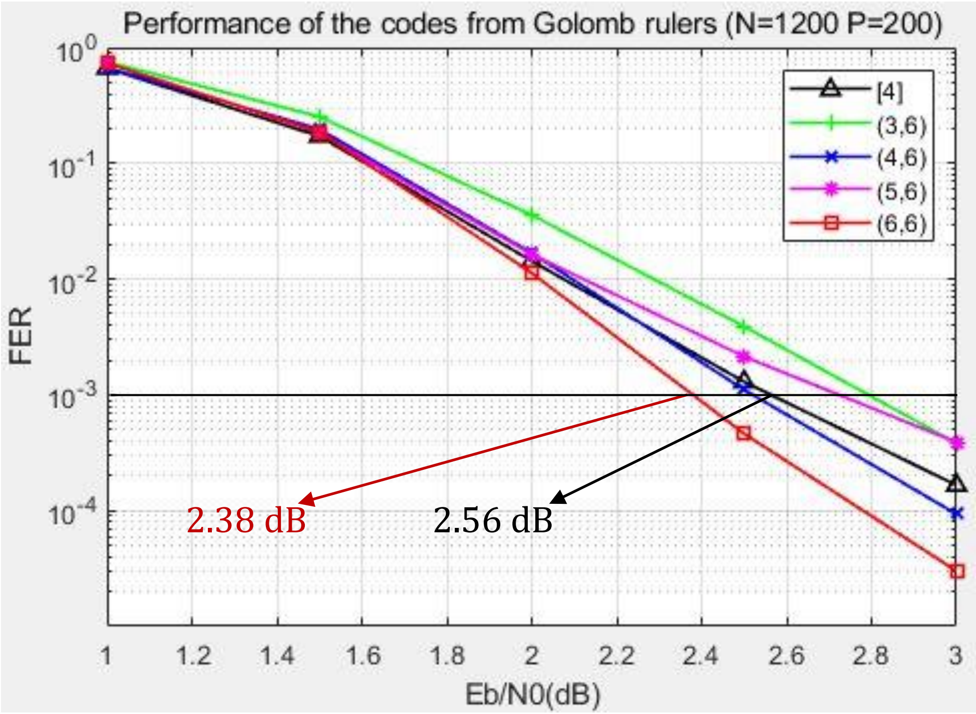
[4] D. Kim, I. Kim, H. Cho, H. Choi, and H.-Y. Song, “Performance Analysis of QC-LDPC codes constructed by Some New Golomb Rulers,” The 27th Asia-Pacific Conference on Communications (APCC 2022), Oct. 2022.

[5] O. Moreno, R. A. Games, and H. Taylor, “Sonar Sequences from Costas Arrays and the Best Known Sonar Sequences with up to 100 Symbols,” IEEE Transactions on Information Theory, vol. 39, no. 6, pp. 1985-1987, 1993.

Simulation

$N_c = 1200$, rate $= 0.5$.

AWGN channel, BPSK modulation, Sum-product decoding.



[4] D. Kim, I. Kim, H. Cho, H. Choi, and H.-Y. Song, "Performance Analysis of QC-LDPC codes constructed by Some New Golomb Rulers," The 27th Asia-Pacific Conference on Communications (APCC 2022), Oct. 2022.

Contents

Conclusion

- A method for generating a Golomb ruler from a sonar sequence was presented.

- Performance simulations were performed on QC-LDPC codes generated using sonar sequences.

- Further research:

Construct QC-LDPC codes with more diverse sonar sequences and analyze their performance.