



일반화된 지연 디버시티 Space-Time Code의 트렐리스 구조 및 성능 분석

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Contents



Introduction

- ❑ MIMO System
- ❑ Design Criteria
- ❑ Design Rules
- ❑ Delay Diversity



GDDC using Opt. PDC

- ❑ Product Distance Code
- ❑ Generalized Delay Diversity Code
- ❑ Trellis Structure of Opt. PDC
- ❑ Search Results of Opt. PDC



System Description



Simulation Results



Conclusion

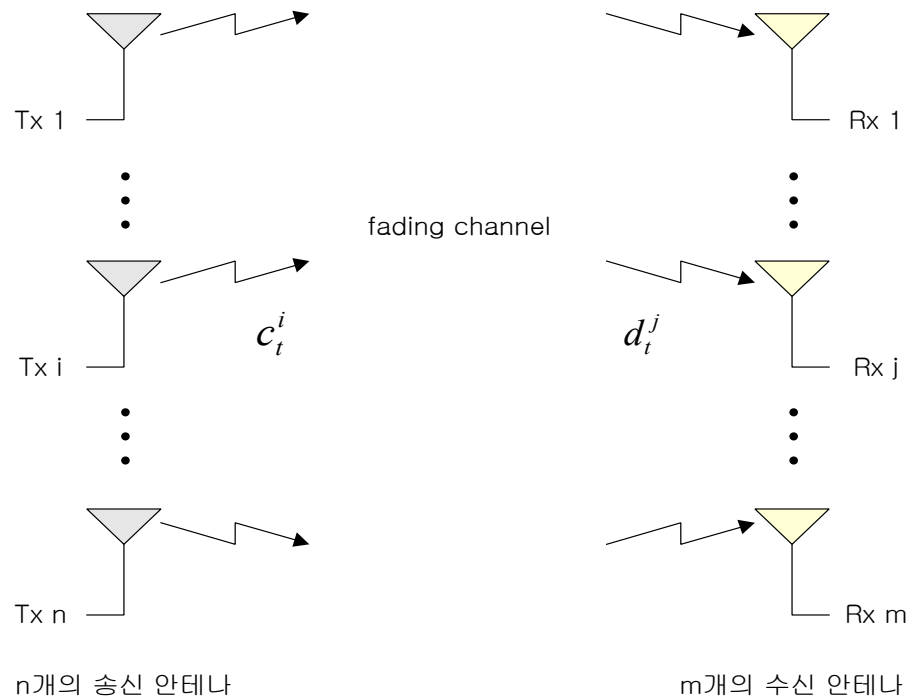
➡ **System beyond IMT-2000(4G)**

- ❑ Information data transmission rate : 2M - 150Mbps
- ❑ BER: data 10^{-6} , video 10^{-9}

➡ **Space-time Code is attractive for high data transmission.**

- ❑ Jointly combining multiple antenna and channel coding without sacrificing the BW.
 - ⇒ improve data rate and reliability in wireless channel
 - ⇒ diversity gain using higher temporal and spatial dimensions
 - ⇒ achieve coding gain
 - ⇒ Increase channel capacity

➡ MIMO System



$$\mathbf{c} = \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_l^1 \\ c_1^2 & c_2^2 & \cdots & c_l^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^n & c_2^n & \cdots & c_l^n \end{bmatrix}$$

➤ Fundamental Bound (Tarokh et. Al)

- Quasi-static channel (frame: l symbols)

$$\Pr(\mathbf{c} \rightarrow \mathbf{e}) \leq \left(\frac{\eta E_s}{4N_0} \right)^{-rm}$$

➤ Rank Criterion

□ Maximize the diversity advantage

$$r = \text{rank}(f(\mathbf{c}) - f(\mathbf{e}))$$

over all pairs of distinct codewords $\mathbf{c}, \mathbf{e} \in \mathbf{C}$

➤ Determinant Criterion

□ Maximize the coding advantage $\eta = (\lambda_1 \lambda_2 \cdots \lambda_r)^{1/r}$

over all pairs of distinct codewords $\mathbf{c}, \mathbf{e} \in \mathbf{C}$

Where η is the geometric mean of nonzero eigenvalues of

$$\mathbf{A} = (f(\mathbf{c}) - f(\mathbf{e}))(f(\mathbf{c}) - f(\mathbf{e}))^H$$

➡ Tarokh's Simple Design Rule ($n = 2$)

- Rule1: Transitions departing from the same state differ in the second symbol
- Rule2: Transitions merging into the same state differ in the first symbol

⇒ Satisfying full rank for the case of $n = 2$

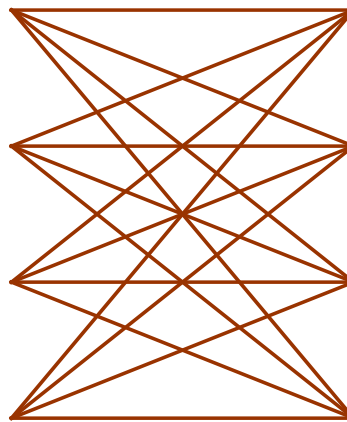
➡ Example : Delay Diversity

00 01 02 03

10 11 12 13

20 21 22 23

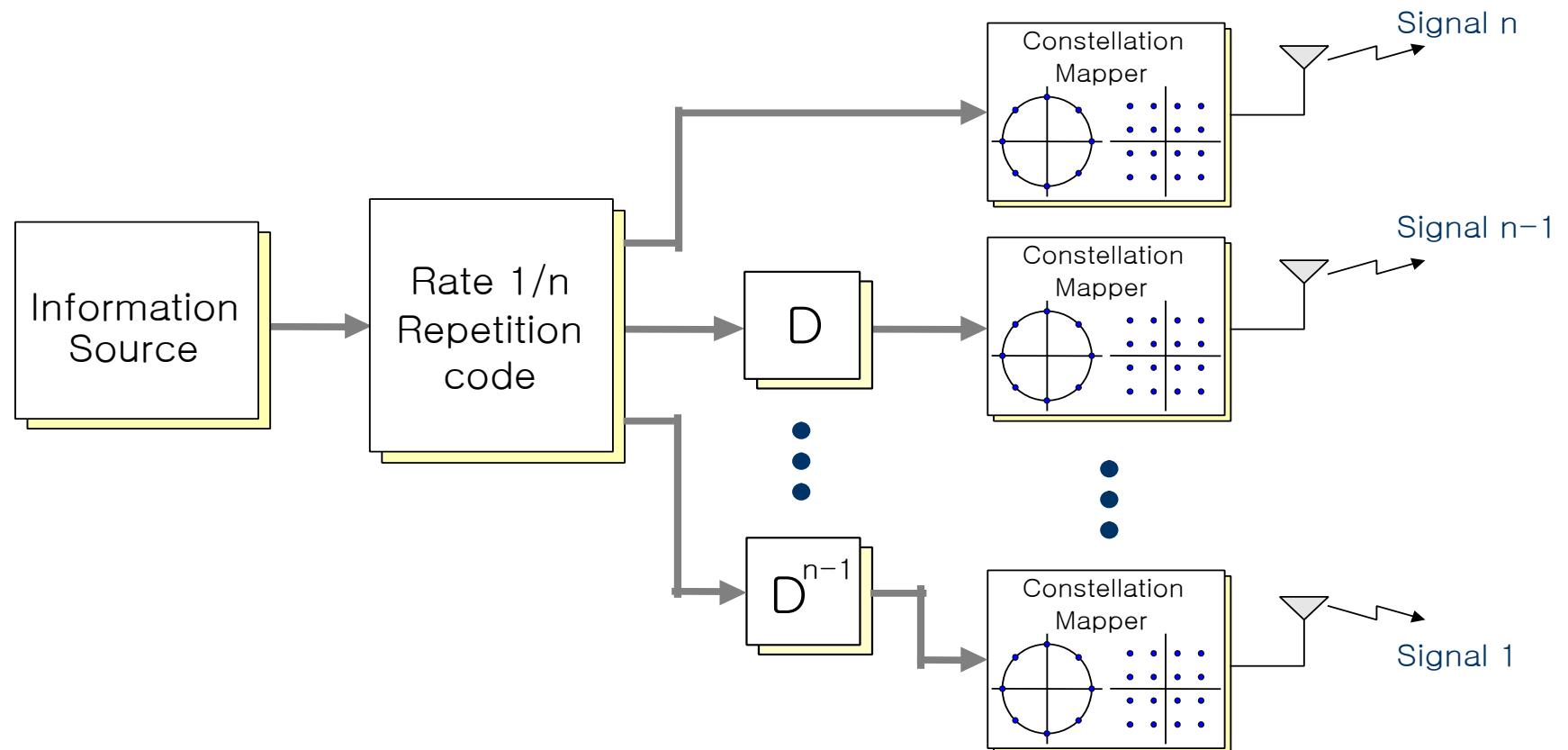
30 31 32 33

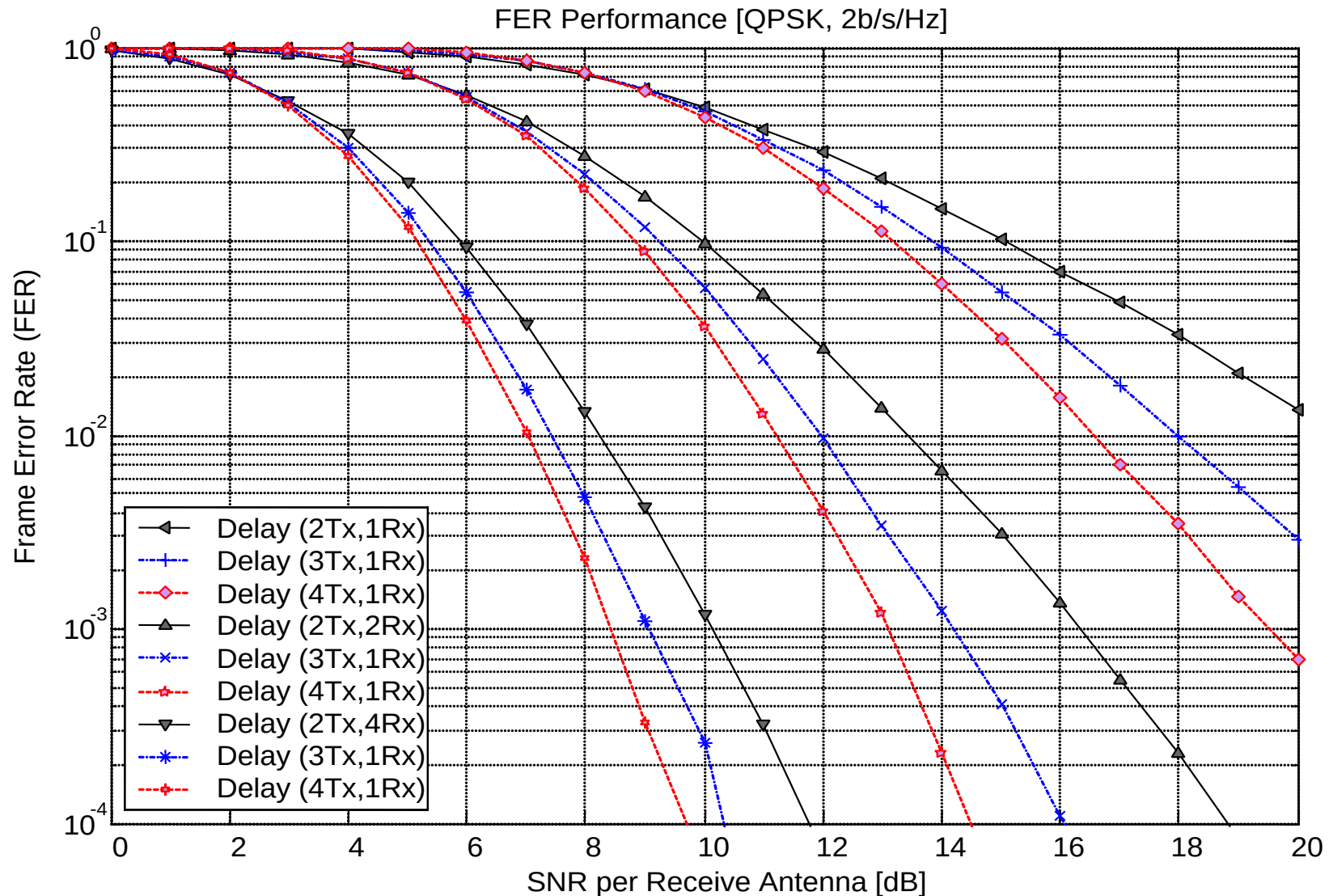


Input	0	1	3	2	1	0	3	1
Tx 1	0	0	1	3	2	1	0	3
Tx 2	0	1	3	2	1	0	3	1

Delay Diversity

➡ **Delay Diversity = Repetition code + Delay element between multiple Tx Antennas**





Product Distance Code

- **More Efficient Block Code than Repetition Code?**
 → **Opt. Product Distance Code**
 (Originally best in case of TCM, Divsalar1988)

- **Definition : Product Distance**

$$D_{(p,q)} = \prod_{i=1}^n \left| f(a_p^{(i)}) - f(a_q^{(i)}) \right|^2$$

$$\begin{pmatrix} 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ M-1 & M-1 & \cdots & M-1 \end{pmatrix} \xrightarrow{\text{yellow arrow}} A = \begin{pmatrix} A_0 \\ A_1 \\ \vdots \\ A_{M-1} \end{pmatrix} = \begin{pmatrix} a_0^{(1)} & a_0^{(2)} & \cdots & a_0^{(n)} \\ a_1^{(1)} & a_1^{(2)} & \cdots & a_1^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1}^{(1)} & a_{M-1}^{(2)} & \cdots & a_{M-1}^{(n)} \end{pmatrix}$$

M-ary Delay Diversity(Tx Ant: n) M-ary Opt. PDC (Tx Ant: n)

➡ Example(QPSK, Tx Ant.: 3)

□ Opt. Product Distance Code {000, 112, 231, 323}

Delay Diversity
{000, 111, 222, 333}

8 8 8

64 64

8

Minimum Dist. = 8

of Minimum = 4

Avg. Dist. = 26.67

Opt. PDC
{000, 112, 231, 323}

16 16 16

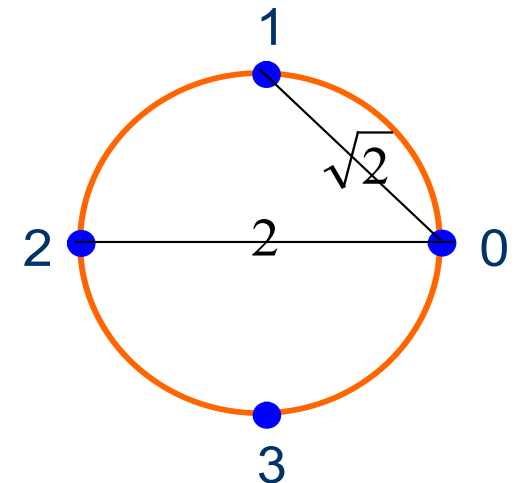
16 16

16

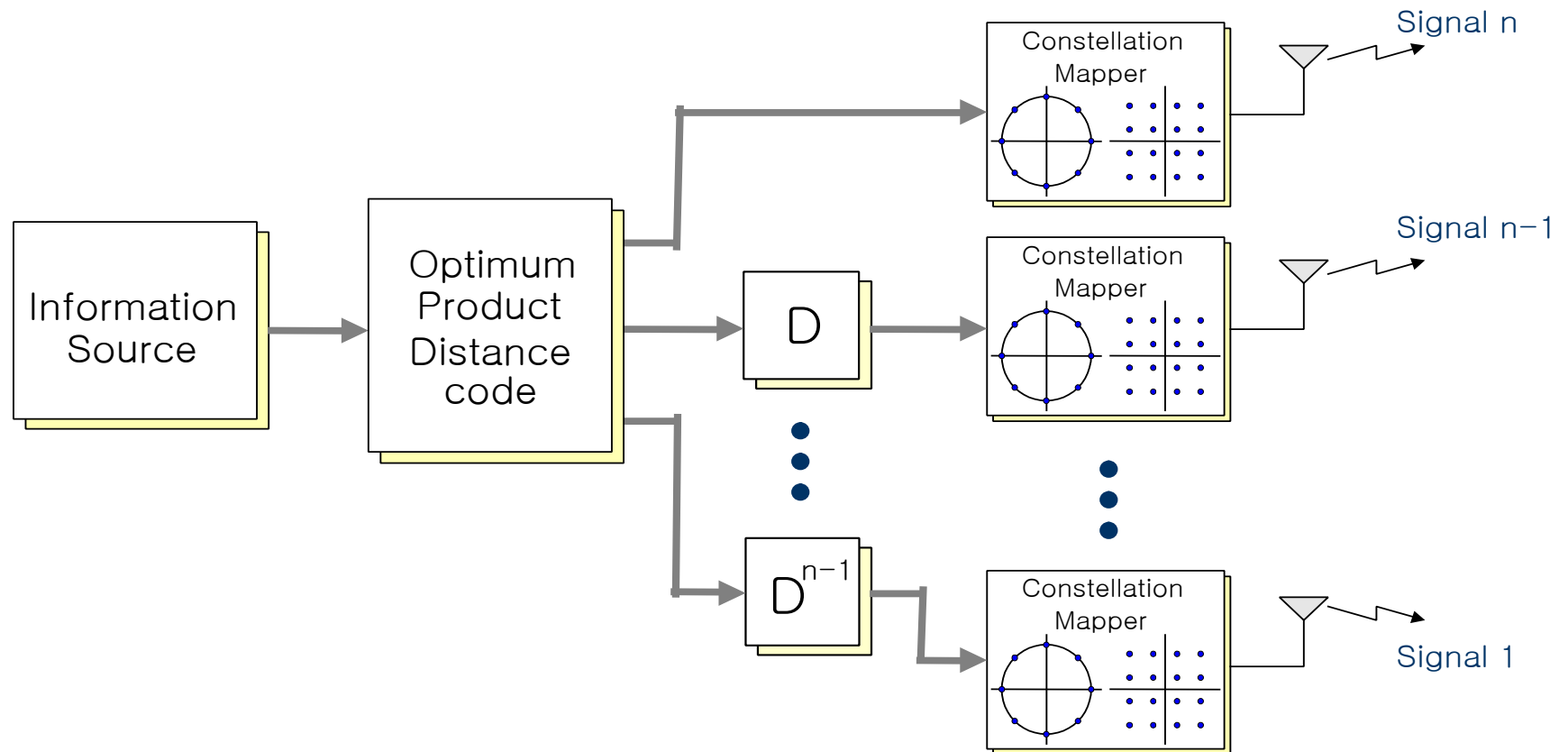
Minimum Dist. = 16

of Minimum = 6

Avg. Dist. = 16.00



Transmitted Signal with Opt. PDC

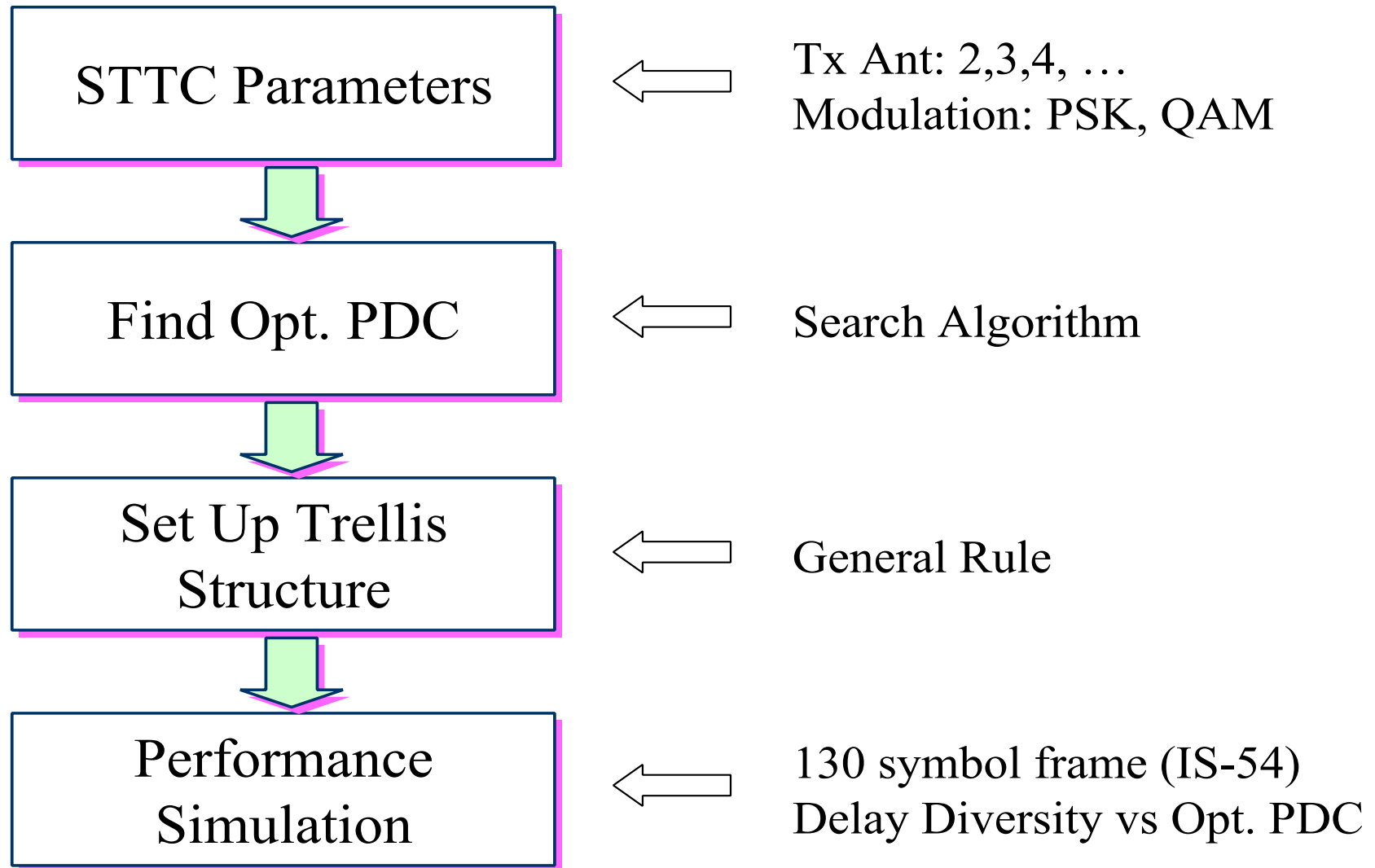


➡ Example : QPSK, Tx Ant. = 3

Block Code	Delay Diversity										
0 0 0	Input	0	1	3	2	3	0	2	1	1	2
1 1 1	Tx 1	0	0	0	1	3	2	3	0	2	1
2 2 2	Tx 2	0	0	1	3	2	3	0	2	1	1
3 3 3	Tx 3	0	1	3	2	3	0	2	1	1	2

Block Code	Delay Diversity										
0 0 0	Input	0	1	3	2	3	0	2	1	1	2
1 1 2	Tx 1	0	0	0	2	3	1	3	0	1	2
2 3 1	Tx 2	0	0	1	2	3	2	0	3	1	1
3 2 3	Tx 3	0	1	3	2	3	0	2	1	1	2

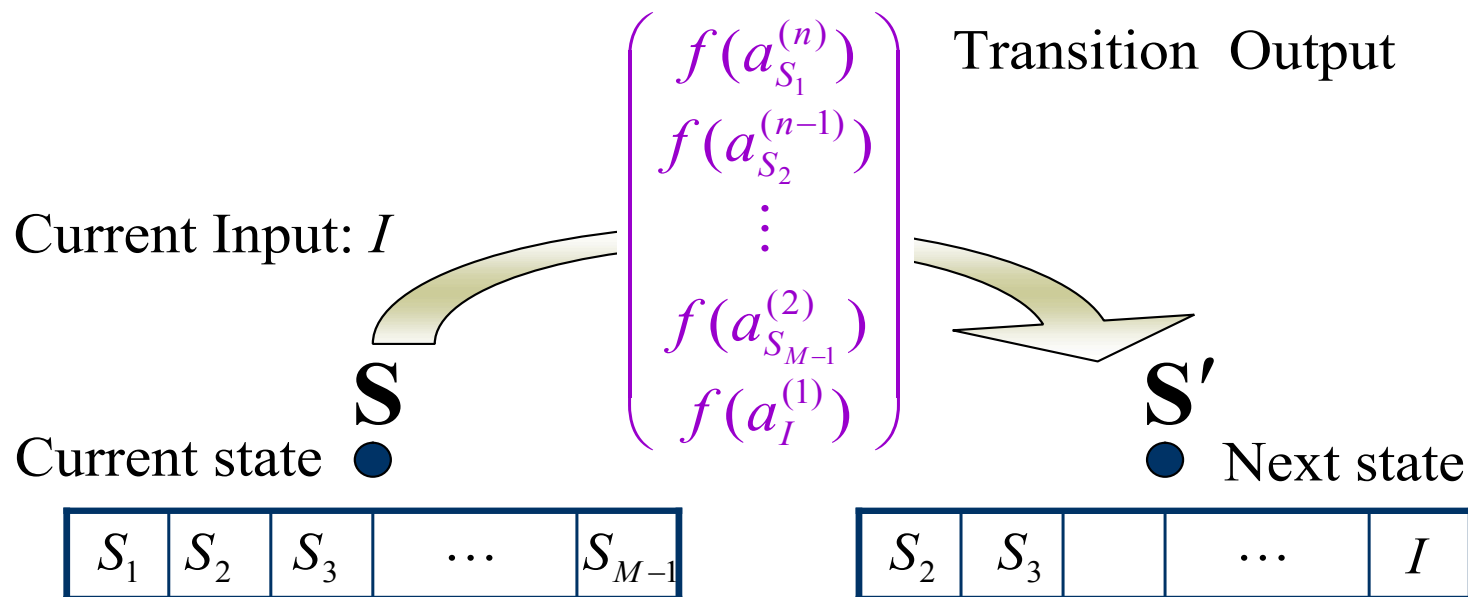
Work Procedure



➡ Given Opt. PDC (M-ary, n Tx Ant)

$$\begin{pmatrix} 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ M-1 & M-1 & \cdots & M-1 \end{pmatrix} \longrightarrow \begin{pmatrix} a_0^{(1)} & a_0^{(2)} & \cdots & a_0^{(n)} \\ a_1^{(1)} & a_1^{(2)} & \cdots & a_1^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1}^{(1)} & a_{M-1}^{(2)} & \cdots & a_{M-1}^{(n)} \end{pmatrix}$$

➡ Trellis Structure



➡ Definition : N_{pd}

- The number of minimum product distance of a given product distance code.

➡ Example(QPSK, Tx Ant.: 2)

- 3! Cases: all such codes have $D_{min}=4$.

0 0	0 0	0 0	0 0	0 0	0 0
1 1	1 3	1 1	1 2	1 2	1 3
2 2	2 2	2 3	2 1	2 3	2 2
3 3	3 1	3 2	3 3	3 1	3 1
$N_{pd} = 4$		$N_{pd} = 2$			

▶ Minimum Product Distance of Delay Diversity

QPSK

Tx	D_{min}	N_{pd}
2	2^2	4
3	2^3	4
4	2^4	4
5	2^5	4

8PSK

Tx	D_{min}	N_{pd}
2	$(2 - \sqrt{2})^2$	8
3	$(2 - \sqrt{2})^3$	8
4	$(2 - \sqrt{2})^4$	8
5	$(2 - \sqrt{2})^5$	8

16QAM

Tx	D_{min}	N_{pd}
2	4^2	24
3	4^3	24
4	4^4	24
5	4^5	24

➡ QPSK

T _x	PDC	D_{opt}	N_{pd}	D_{avg}
2	0 1 2 3 0 1 3 2	4	2	6.667
	0 1 2 3 0 1 2 3		4	8
3	0 1 2 3 0 1 3 2 0 2 1 3	16	6	16
4	0 1 2 3 0 1 2 3 0 1 3 2 0 2 1 3	32	4	42.667
5	0 1 2 3 0 1 2 3 0 1 3 2 0 1 3 2 0 2 1 3	64	2	106.667
	0 1 2 3 0 1 2 3 0 1 2 3 0 1 3 2 0 2 1 3		4	128

Search Results (8PSK, 16QAM)

8PSK

Tx	PDC	D_{opt}	N_{pd}	D_{avg}
2	$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 3 & 6 & 1 & 4 & 7 & 2 & 5 \end{matrix}$	2	16	4.57143
3	$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 2 & 5 & 7 & 3 & 1 & 6 & 4 \\ 0 & 3 & 7 & 4 & 1 & 6 & 2 & 5 \end{matrix}$	4	12	6.28571
	$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 2 & 4 & 7 & 1 & 6 & 3 & 5 \\ 0 & 3 & 6 & 2 & 5 & 7 & 1 & 4 \end{matrix}$		18	8.28571
4	$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 5 & 3 & 7 & 2 & 6 & 4 \\ 0 & 3 & 2 & 5 & 1 & 7 & 6 & 4 \\ 0 & 3 & 7 & 5 & 4 & 2 & 6 & 1 \end{matrix}$	4	2	25.5714
	$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 3 & 6 & 1 & 5 & 2 & 7 & 4 \\ 0 & 3 & 7 & 4 & 1 & 6 & 2 & 5 \end{matrix}$		16	17.2857

16QAM

2	$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 6 & 13 & 11 & 9 & 15 & 4 & 2 & 7 & 11 & 0 & 12 & 14 & 8 & 3 & 5 \end{matrix}$	64	4	426.667
	$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 1 & 3 & 5 & 7 & 9 & 11 & 13 & 15 \end{matrix}$		33	494.933

➡ Channel Model

- ❑ Quasi-static flat fading channel
- ❑ 1 frame = 130 symbols ($l=130$)
- ❑ No delay spread (No multipath)
- ❑ Complex multiplicative channel

$$\mathbf{r} = \mathbf{H}\mathbf{c} + \mathbf{n}$$

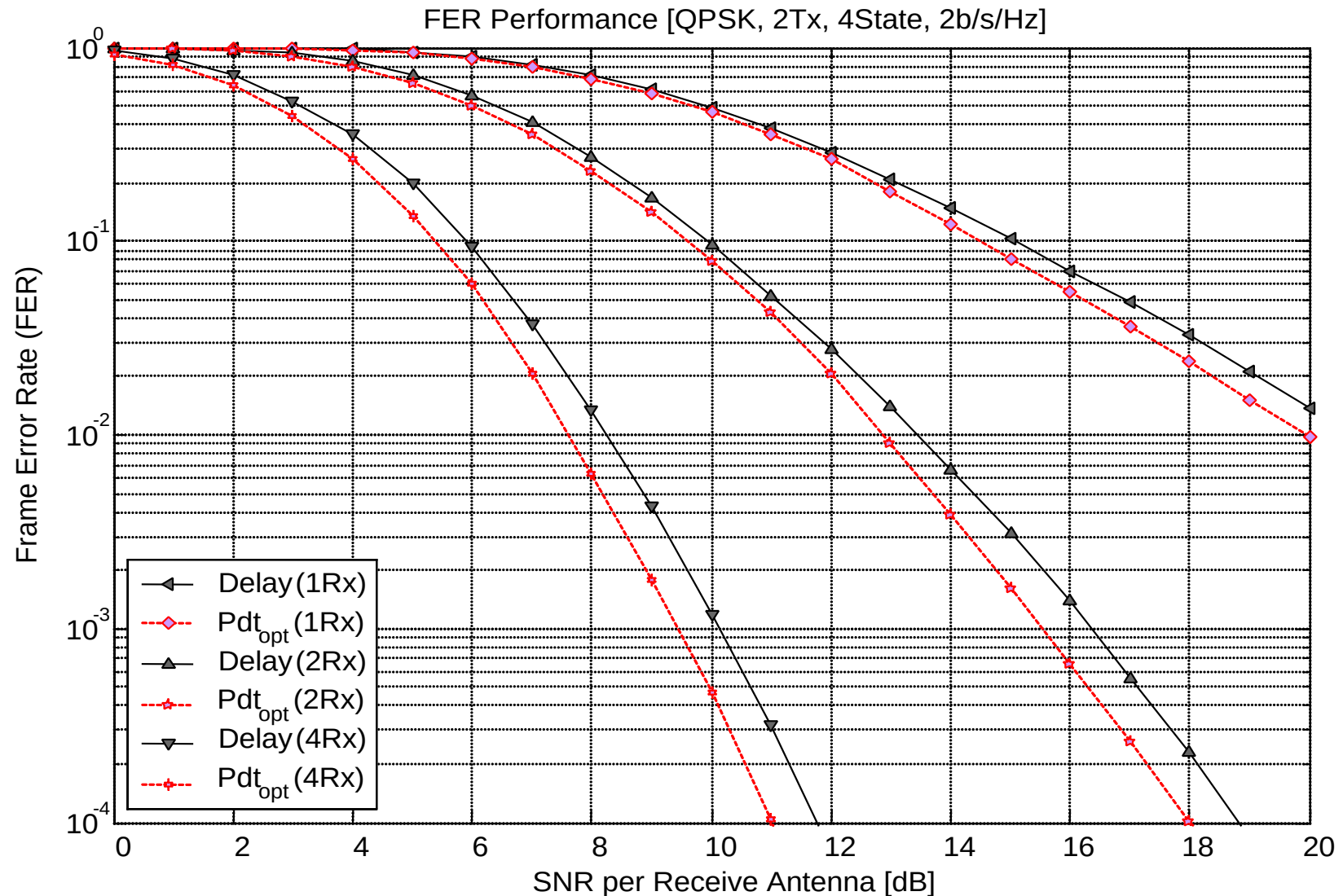
$$\mathbf{H} = [\alpha_{ij}] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix}$$

- ❑ Ideal channel estimation

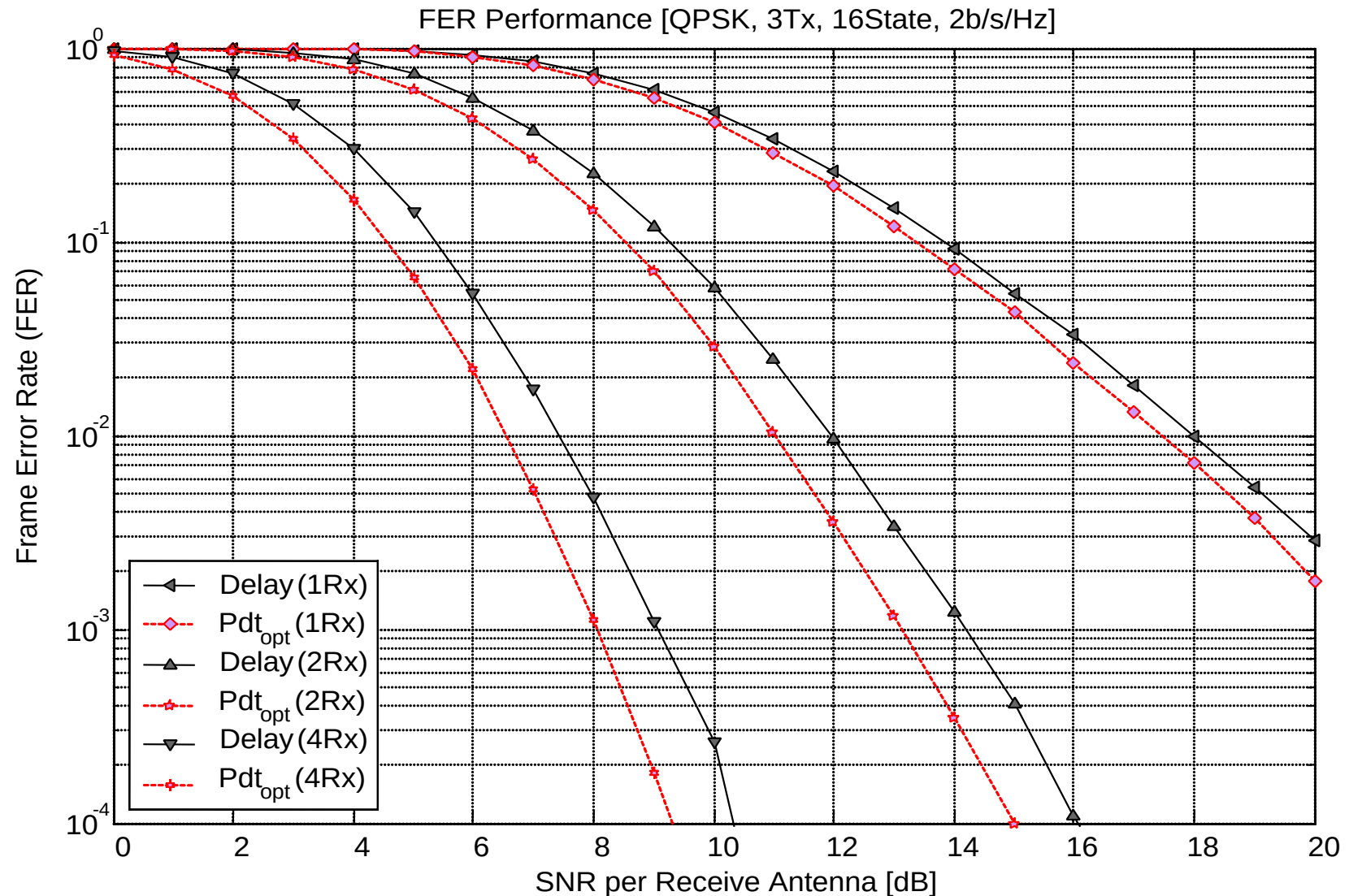
➡ System model

- ❑ Modulation: QPSK, 8PSK, 16QAM
- ❑ Viterbi Decoder (ML)
- ❑ Unquantized Soft Decision

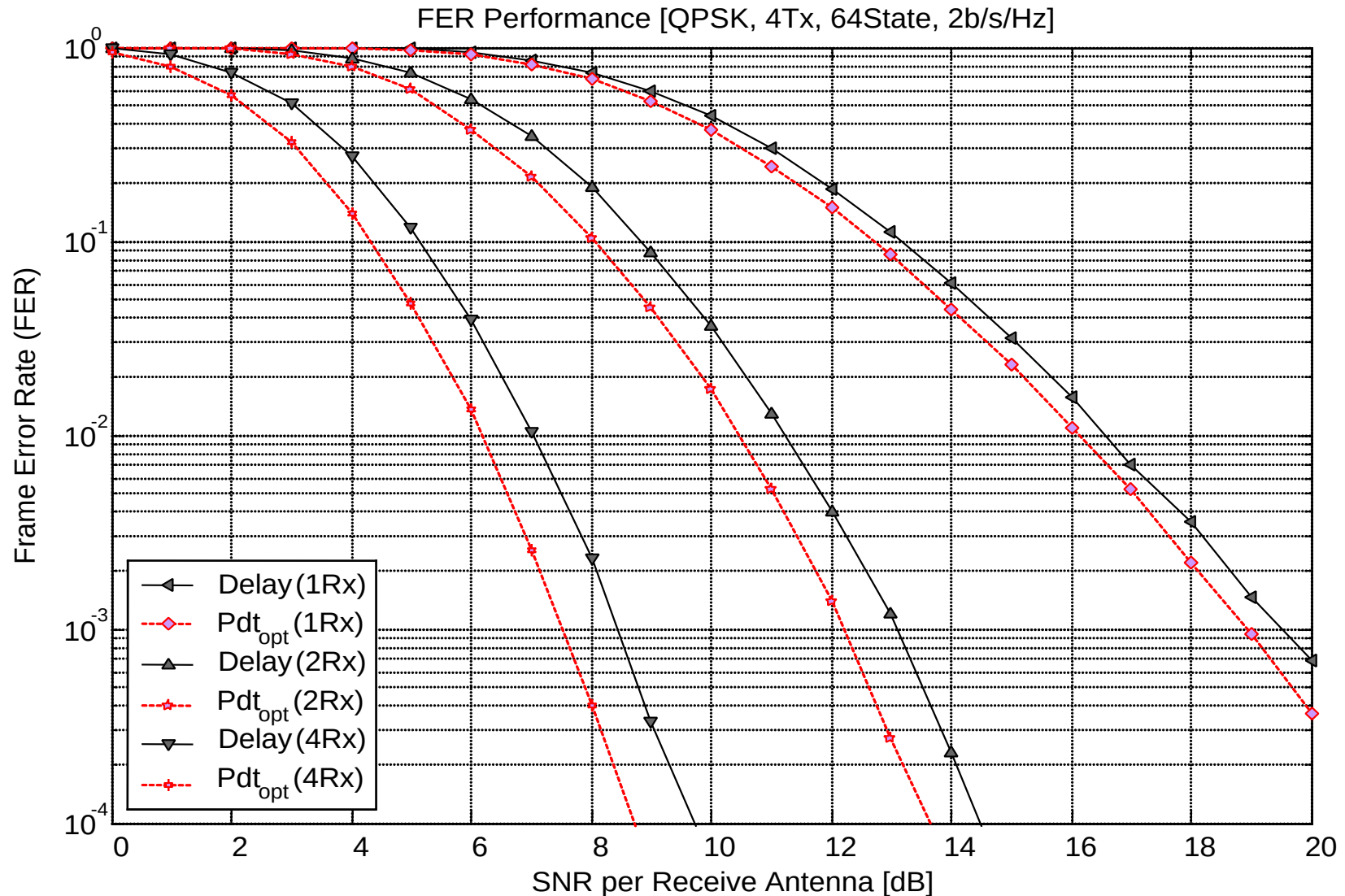
Delay VS Opt. PDC [QPSK, 2Tx]



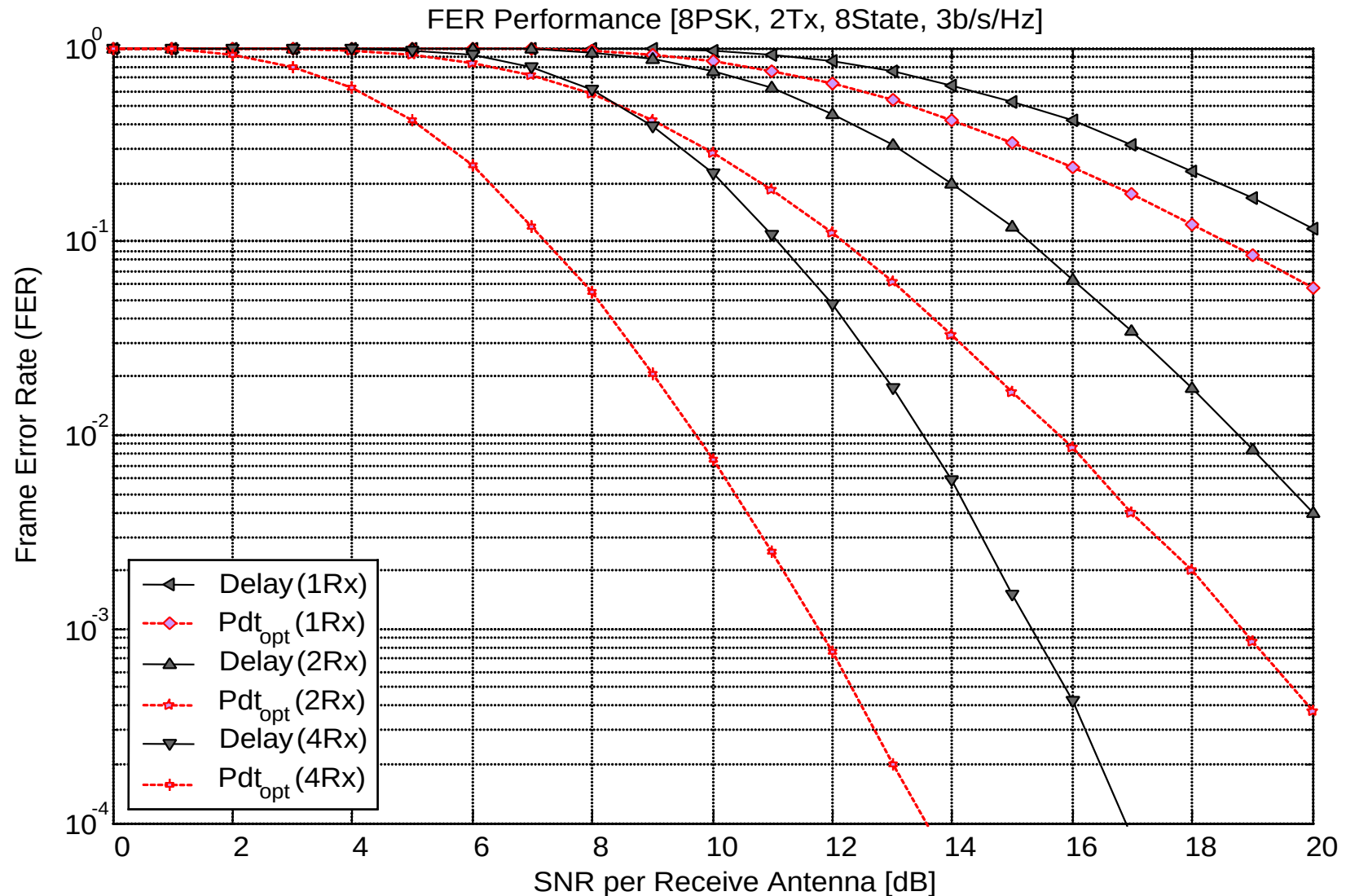
Delay VS Opt. PDC [QPSK, 3Tx]



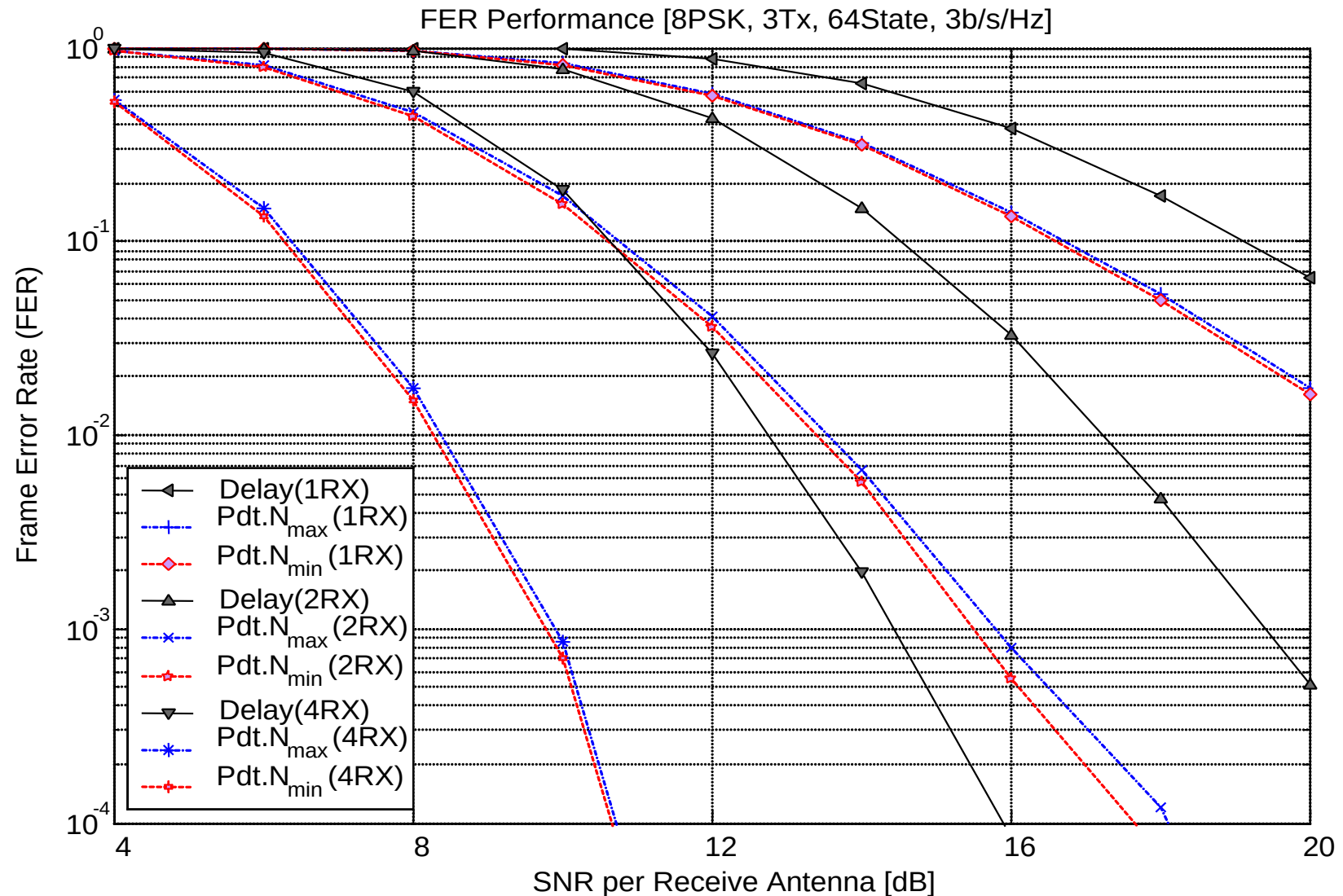
Delay VS Opt. PDC [QPSK, 4Tx]



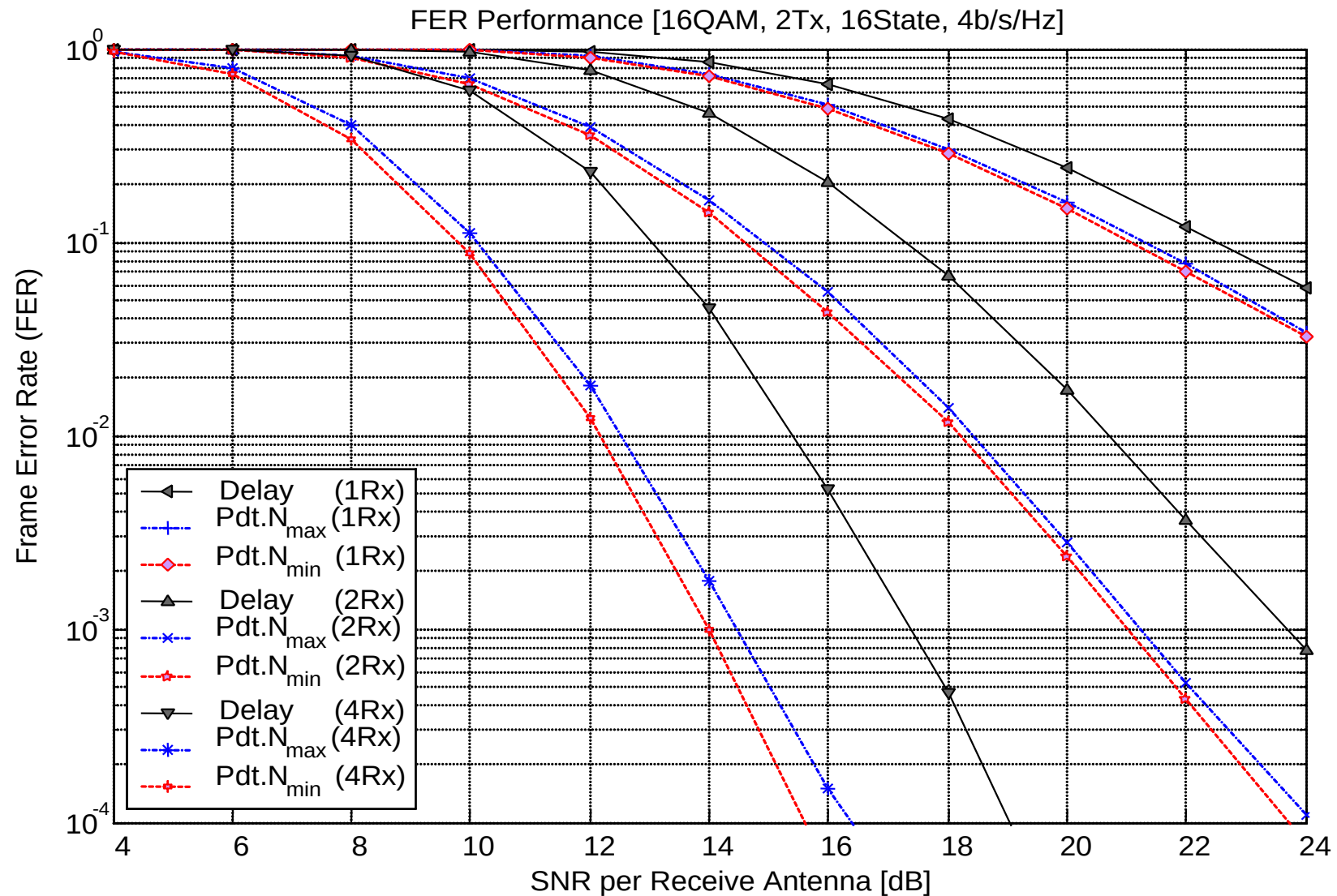
Delay VS Opt. PDC [8PSK, 2Tx]



Delay VS Opt. PDC [8PSK, 3Tx]



Delay VS Opt. PDC [16QAM, 2Tx]



Conclusion

➤ **Systematic Design(Construction)**

- ❑ From the Optimum Product Distance Codes
 - Originally known to best in case of TCM
- ❑ Applicable to the case of the number of Tx Ant. is 2,3,4...

➤ **Search Optimal Product Distance Code**

- ❑ 4PSK, 8PSK, 16QAM

➤ **Set up Trellis structure from Opt. PDC**

- ❑ For ML decoding

➤ **Performance Simulation**

- ❑ Comparison with Delay Diversity (←Baseline Performance)
- ❑ Analysis