On the Merit Factor Analysis and Linear Complexity of Polyphase Sequences

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Introduction

- Application of Polyphase Sequences
  - Reduction of MAI in DS-CDMA or SSMA
  - Reduction of Crest factors in OFDM and CDMA systems

- The Merit Factors and Linear Complexity of Polyphase Sequences

Construction of Polyphase Sequences

- All of polyphase sequences are classified as to whether the length of each sequence is square integer or not.
- **Table 1** : Classification according to Construction of Polyphase Sequences

Length N	***Construction A*** $N = M^2$	***Construction B*** N = positi
Polyphase	Frank, P_1, P_X, the proposed	Golomb, Chu, P_3, P_4

- The proposed sequence will be called Moon sequence in this thesis.

Construction of Polyphase Sequences

- Construction A. $N=M^2$

Polyphase sequence $S(s_1, s_2, \ldots , s_N)$

$$\Rightarrow \quad s_{nM+k+1} = e^{i\varphi_{n,k}} , \qquad 0 \le n \le M-1 \;\; and \;\; 0 \le k \le M-1$$

1) Frank sequence : $\varphi_{n,k} = 2\pi nk/M$

2) P1 sequence : $\varphi_{n,k} = -(\pi/M)(M-2n-1)(nM+k)$

3) Px sequence : $\varphi_{n,k} = \begin{cases} (\pi/M)[(M-1)/2-k)(M-2n-1)], & M \text{ even} \\ (\pi/M)[(M-2)/2-k)(M-2n-1)], & M \text{ odd} \end{cases}$

4) The proposed sequence : $\varphi_{n,k} = \begin{cases} (\pi/M)[(M-1)/2-k)(M-2n-1)], & M \text{ even} \\ (\pi/M)[(M-1-k)(M-2n-1)], & M \text{ odd} \end{cases}$

- **Example 1.** Phase representation of ***Construction A*** for N=9

1) *Frank*

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 2\pi/3 & 4\pi/3 \\ 0 & 4\pi/3 & 2\pi/3 \end{matrix}$$

2) P_1

$$\begin{matrix} 0 & 4\pi/3 & 2\pi/3 \\ 0 & 0 & 0 \\ 0 & 2\pi/3 & 4\pi/3 \end{matrix}$$

3) P_X

$$\begin{matrix} \pi/3 & 5\pi/3 & \pi \\ 0 & 0 & 0 \\ 5\pi/3 & \pi/3 & \pi \end{matrix}$$

4) *Moon*

$$\begin{matrix} 4\pi/3 & 2\pi/3 & 0 \\ 0 & 0 & 0 \\ 2\pi/3 & 4\pi/3 & 0 \end{matrix}$$

Construction of Polyphase Sequences

- ***Construction B.*** a positive integer N

  Polyphase sequence $S(s_1, s_2, \dots, s_N)$

➡ $$s_{k+1} = e^{i\varphi_{k+1}}, \qquad 0 \le k \le N-1$$

1) Golomb sequence : $$\varphi_{k+1} = \frac{\pi(k+1)k}{N}$$

2) P3 sequence : $$\varphi_{k+1} = \frac{\pi k^2}{N}$$

3) P4 sequence : $$\varphi_{k+1} = \frac{\pi(k-N)k}{N}$$

4) Chu sequence : $$\varphi_{k+1} = \begin{cases} \pi(k+2q)k/N, & N \ \ even \\ \pi(k+2q+1)k/N, & N \ \ odd \end{cases}, \quad for \ \ an \text{ integer } q$$

- **Example 2.** Phase representation of ***Construction B*** for N=9

1) Golomb

$$\left\{0 \quad \frac{2\pi}{9} \quad \frac{6\pi}{9} \quad \frac{12\pi}{9} \quad \frac{2\pi}{9} \quad \frac{12\pi}{9} \quad \frac{6\pi}{9} \quad \frac{2\pi}{9} \quad 0\right\}$$

2) P_3

$$\left\{0 \quad \frac{\pi}{9} \quad \frac{4\pi}{9} \quad \pi \quad \frac{16\pi}{9} \quad \frac{7\pi}{9} \quad 0 \quad \frac{13\pi}{9} \quad \frac{10\pi}{9}\right\}$$

3) P_4

$$\left\{0 \quad \frac{10\pi}{9} \quad \frac{4\pi}{9} \quad 0 \quad \frac{16\pi}{9} \quad \frac{16\pi}{9} \quad 0 \quad \frac{4\pi}{9} \quad \frac{10\pi}{9}\right\}$$

4) Chu

$$\left\{0 \quad \frac{2\pi}{9} \quad \frac{6\pi}{9} \quad \frac{12\pi}{9} \quad \frac{2\pi}{9} \quad \frac{12\pi}{9} \quad \frac{6\pi}{9} \quad \frac{2\pi}{9} \quad 0\right\}$$

- Nonperiodic Autocorrelation Function

$$\varphi_{ss}(\tau) = \sum_{n=1}^{N-\tau} s_n \cdot s_{n+\tau}^*$$

- Merit Factor F and R [3]

$$F = \frac{\varphi_{ss}(0)^2}{2\sum_{\tau=1}^{N-1} \left|\varphi_{ss}(\tau)\right|^2}$$

$$R = \frac{\varphi_{ss}(0)}{\max_{1\le\tau\le N-1} \left|\varphi_{ss}(\tau)\right|}$$

- F and R are important criteria for a nonperiodic autocorrelation function

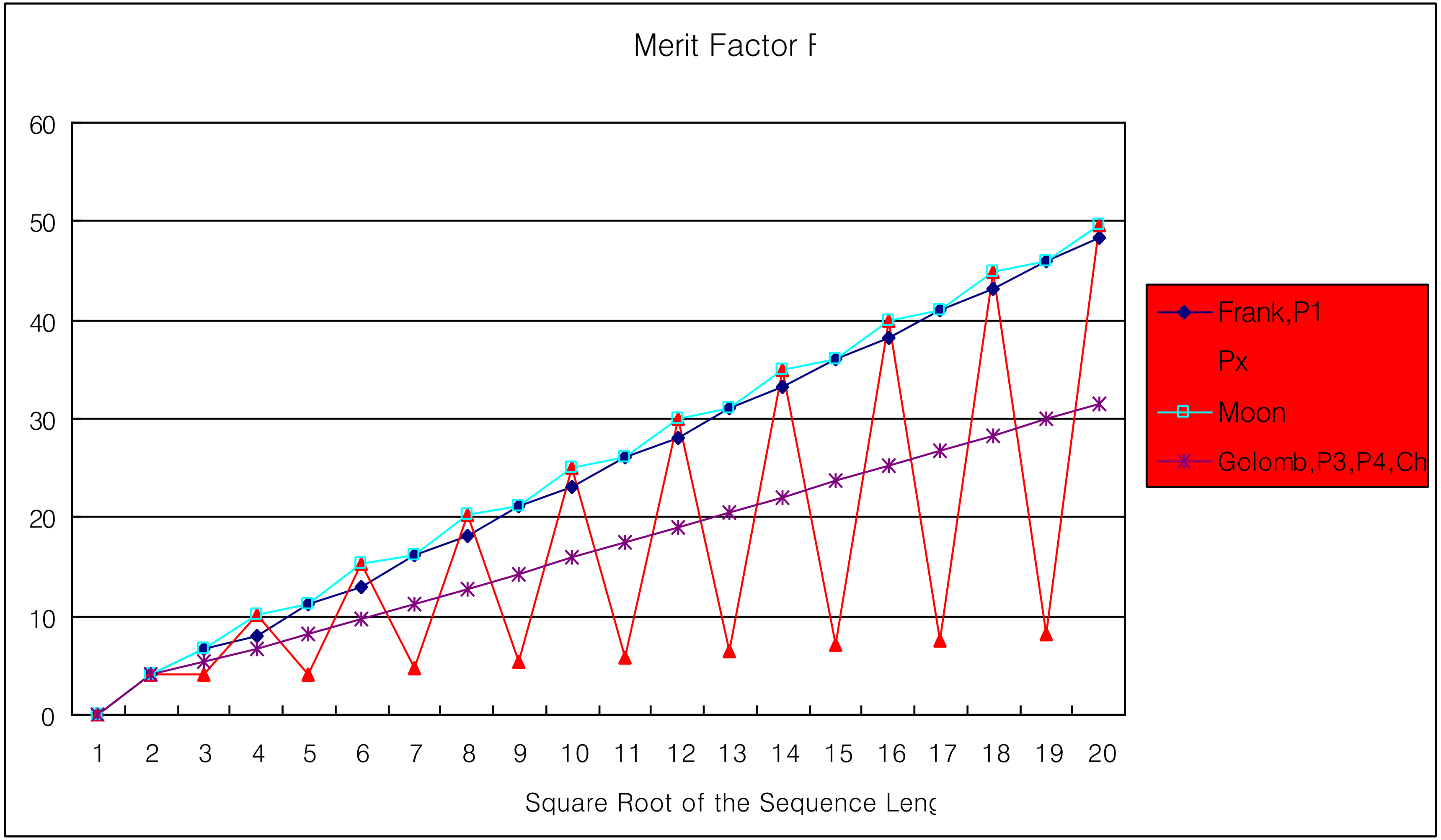


Fig 1. Merit Factor F for $\sqrt{N}$

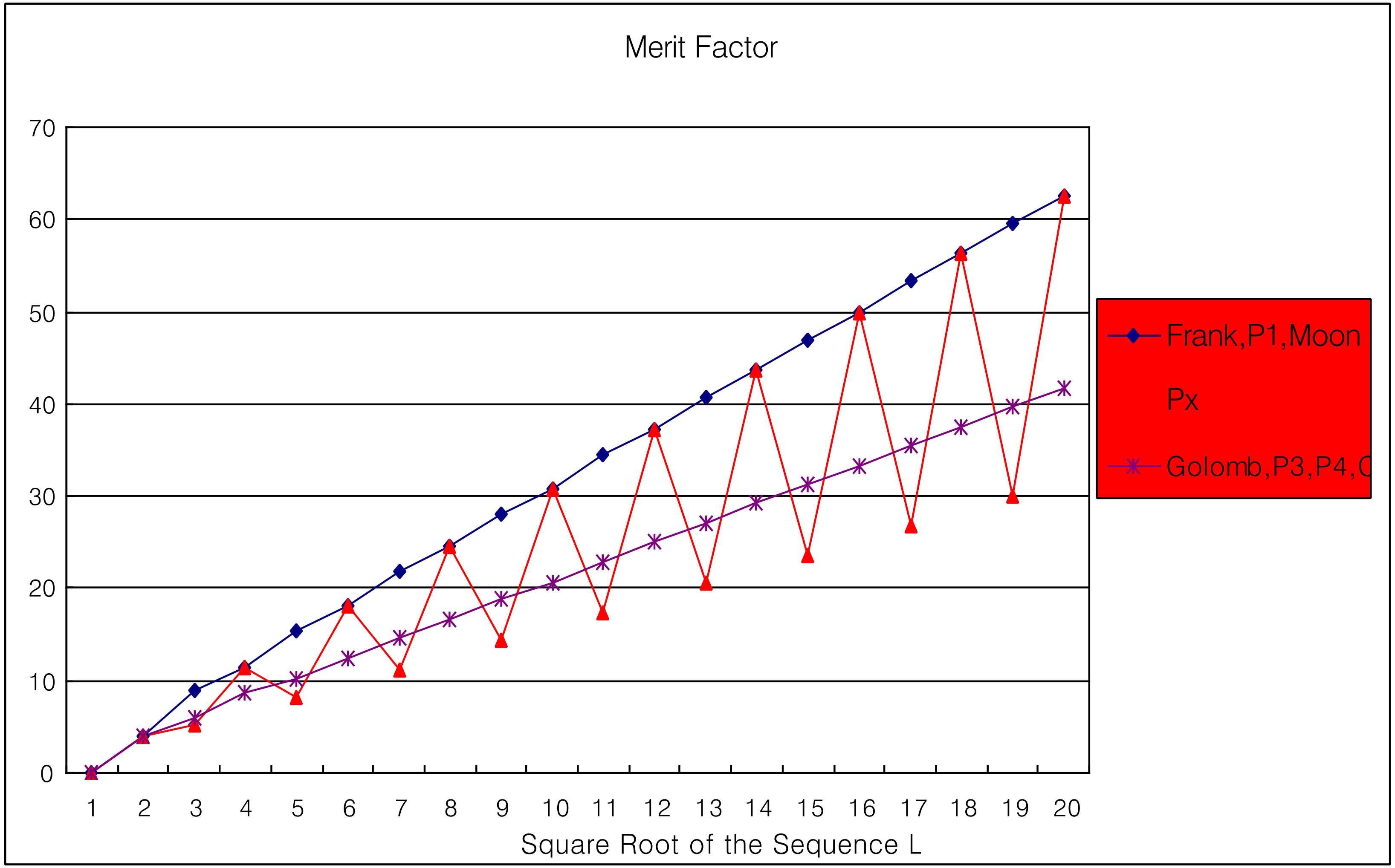


Fig 2. Merit Factor R for $\sqrt{N}$

1. LFSR Synthesis of a Polyphase Sequence over a Field

- The symbol set can't be any finite field because the set of the symbols isn't closed under addition.
- The symbol set is an complex field.
  - ➡ an occurrence of a computation error
- A small error of the inverse operation give a huge influence on the computation recursively.

- **Example 3.** Let the system be possible for us to calculate down to six decimal places.

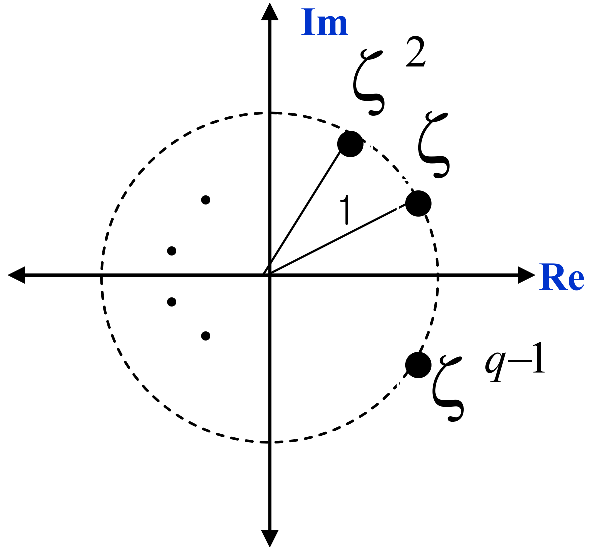
- F_9 : Frank sequence S with length 9
- F_9^{new} : New sequence which is generated by LFSR

F_9 : {1, 1, 1,···, 1 , -0.5-0.866025i , - 0.5+0.866025i }

F_9^{new} : {1, 1, 1,···, 1+0.000002i, -0.500003-0.866026i, -0.499998+0.866024i}

2. LFSR Synthesis of a Polyphase Sequence over a Ring

ζ_q: complex primitive q-th root of unity



$$V = \{1, \zeta^1, \cdots, \zeta^{q-1}\} \longrightarrow S(s_1, s_2, \cdots, s_N)$$

$$\updownarrow \quad \updownarrow \quad \cdots \quad \updownarrow \qquad\qquad \updownarrow$$

$$Z_q = \{0, 1, \cdots, q-1\} \longrightarrow T(t_1, t_2, \cdots, t_N)$$

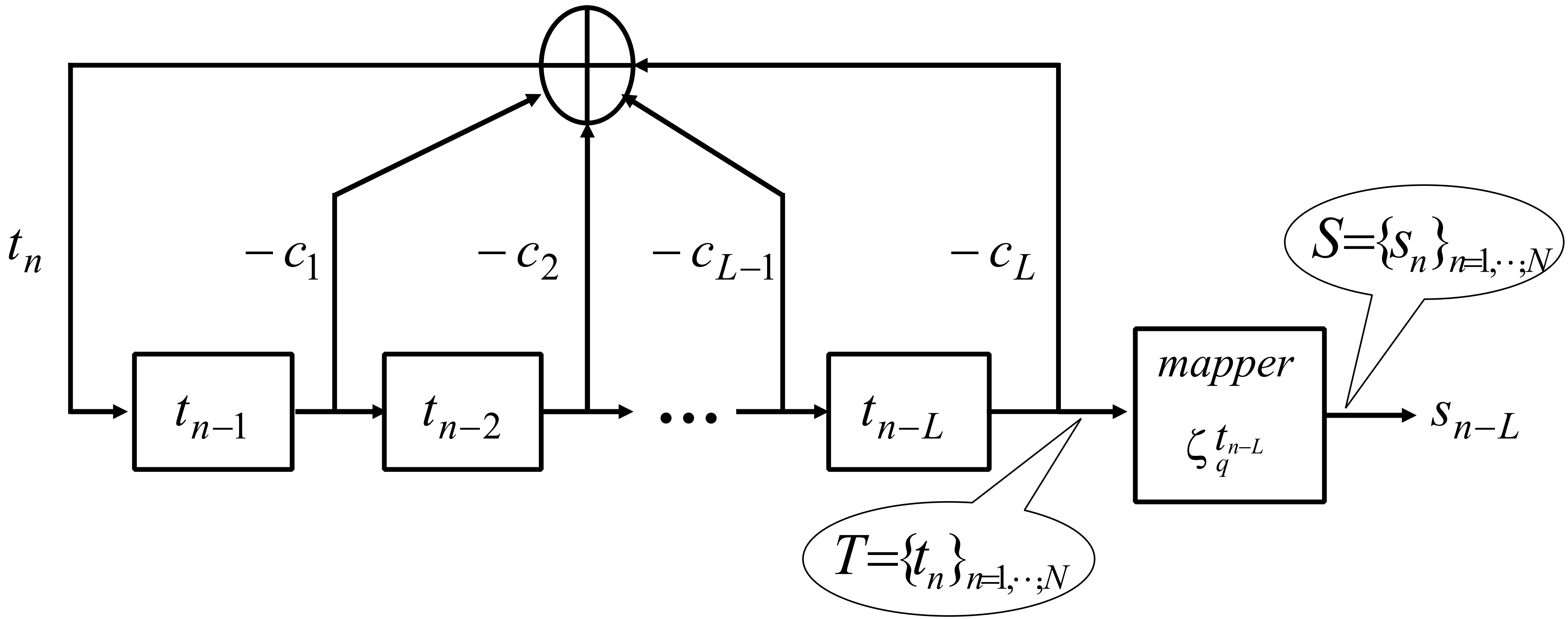
$$s_n = \zeta_q^{t_n}$$

- **Table 2** : The transformed symbol set for length $N = M^2$

Original sequ	Frank	P_X	P_1	Moon
New symbol	Z_M	Z_{2M} for N even	Z_{4M} for N even	Z_{4M} for N eve
		Z_M for N odd	Z_{2M} for N odd	Z_M for N odd

Original sequ	Golomb	P_3	P_4	Chu
New symbol	Z_N	Z_{2N}	Z_{2M} for N ev	Z_{2N} for N ev
			Z_N for N odd	Z_N for N od

- Polyphase sequence generator



- The linear complexity of a polyphase sequence S can be regarded as the linear complexity of a new sequence T.

Linear Complexity Analysis of Polyphase Sequences

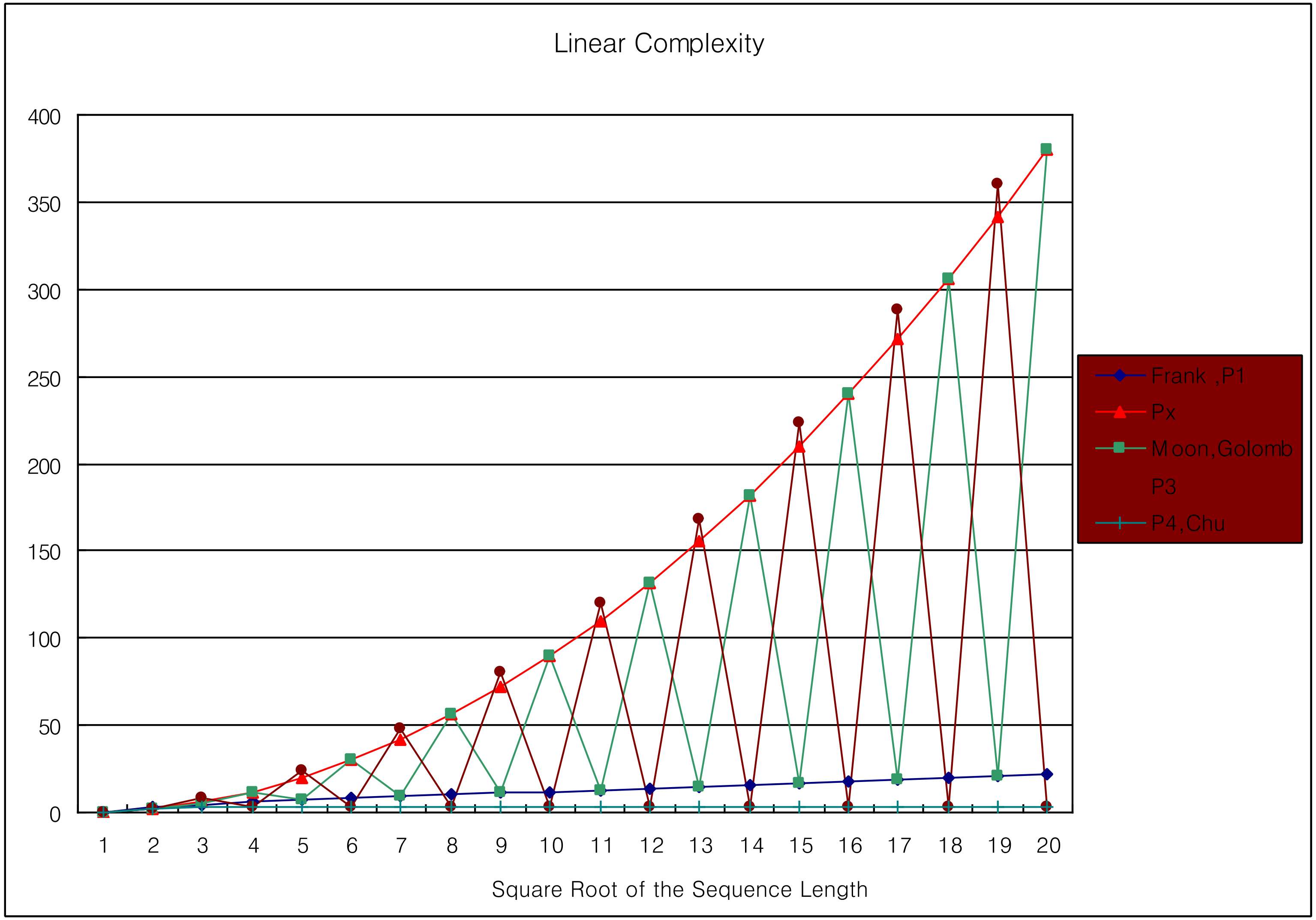


Fig 3. Linear complexities of polyphase sequences for $\sqrt{N}$

Concluding Remark

- **Conclusion**
  - Merit factors analysis of each polyphase sequence
  - BM-algorithm can't synthesize the shortest LFSR to generate a polyphase sequence over any field.
  - Implementation of a polyphase sequence generator over a ring by the symbol set transformation
  - Linear complexity analysis of each polyphase sequence by Reeds-Sloane algorithm
- **Future Works**
  - Algebraic analysis of linear complexity for each polyphase sequence
  - Find good polyphase sequences continuously