On ZCZ sequences and its Application to MC-DS-CDMA

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ZCZ Sequences : GO(N, M, Z_0)

- ZCZ / Generalized Orthogonal Sequences : *GO*(*N*, *M*, Z_0)

  (*N* : Sequence length, *M* : family size, Z_0 : ZCZ length)

- are defined as Sequences with Zero Correlation Zone (ZCZ) Z_0

$$\phi_{r,s}(\tau) = \sum_{n=0}^{N-1} a_n^{(r)} a_{n+\tau}^{*(s)} = \begin{cases} = \eta N, & \text{for } \tau = 0,\ \mathrm{r} = \mathrm{s} \\ = 0, & \text{for } \tau = 0,\ \mathrm{r} \neq \mathrm{s} \\ = 0, & \text{for } 0 < |\tau| \leq Z_o, \text{ for all r, s} \end{cases}$$

Orthogonal

Where $\eta = \dfrac{1}{N}\sum_{n=0}^{N-1} \left| a_n^{(r)} \right| \leq 1, \quad a_n^{(r)}$: Complex number

- Well Known bound for binary ZCZ sequences $MZ_0 \leq N/2$

  for ternary ZCZ sequences $MZ_0 \leq N$ (accurately, $MZ_0 \leq N+Z_0-1$)

Fan's ZCZ Sequences GO(32, 4, 4) (N=32, M=4, Z_o=4)

$$a_n^{(1)} = \{++-+++-+---+---+--+-++-++++----+\}$$

$$a_n^{(2)} = \{--+-++-++++----+++-+++-+---+---+\}$$

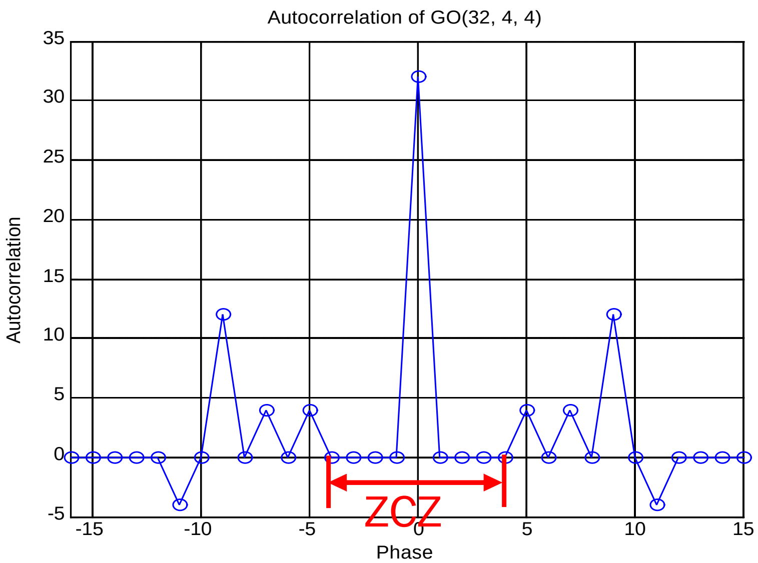
$$a_n^{(3)} = \{+---+----+---+---++++---+-++-+--\}$$

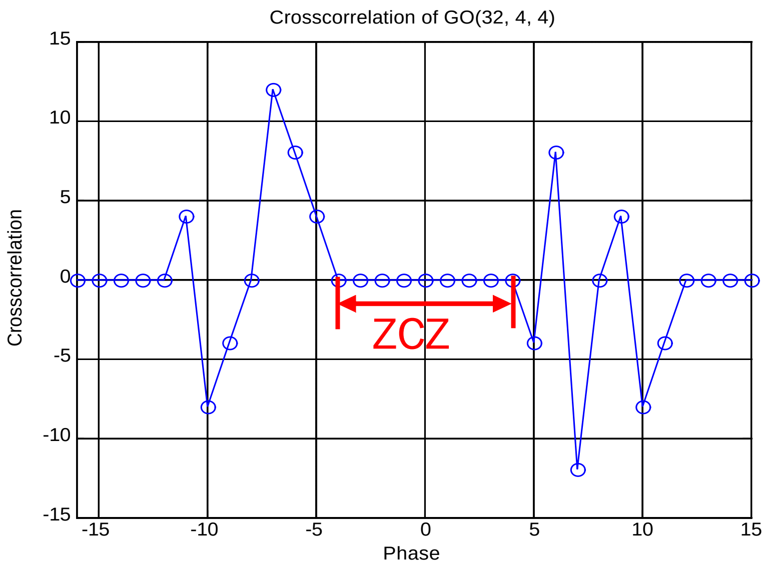
$$a_n^{(4)} = \{-++++---+-++-+--+---+----+---+--\}$$

$$\phi_{r,r} = \{\times\times\times\times\times\times\times\times\times\times\times\times 0000 \boxed{32}\ 0000 \times\times\times\times\times\times\times\times\times\times\times\times\}$$

$$\phi_{r,s} = \{\times\times\times\times\times\times\times\times\times\times\times\times 0000 \boxed{0}\ 0000 \times\times\times\times\times\times\times\times\times\times\times\times\}$$

In-phase





Sequences, Correlation & Interferences

■ Interferences and Spreading Sequences

- Autocorrelation of a sequences $\rightarrow$ MPI
- Crosscorrelation between sequences $\rightarrow$ MAI, MUI

■ Periodic and Aperiodic Correlation

- No ISI, Synch. among different symbols $\rightarrow$ Full correlation of spreading signals
  $\rightarrow$ Only Periodic correlation of sequences
- ISI, Asynch. among different symbols $\rightarrow$ Partial correlation of spreading signals
  $\rightarrow$ Aperiodic correlation of sequences
  $\rightarrow$ Partial Interference

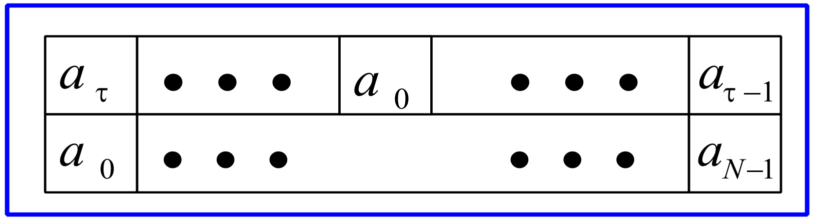
■ Spreading Sequences Design Issues

- Sequences families with impulsive ideal autocorrelation and zero crosscorrelation between sequences are IMPOSSIBLE to design : by Well known Welch' bound

Definition of Correlation

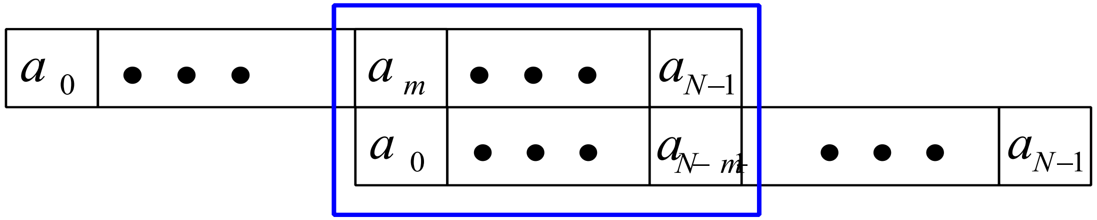
Definition 1 Periodic AutoCorrelation Function (ACF) & CrossCorrealtion Function (CCF)

$$\phi_{A_r, A_s}(\tau) = \sum_{n=0}^{N-1} a_n^{(r)} a_{n+\tau}^{*(s)} = \begin{cases} \text{ACF}, & \text{if } r = s \\ \text{CCF}, & \text{if } r \neq s \end{cases}$$



Definition 2 Aperiodic ACF & CCF

$$R_{A_r, A_s}(m) = \sum_{n=0}^{N-1-m} a_n^{(r)} a_{n+m}^{*(s)} = \begin{cases} \text{ACF}, & \text{if } r = s \\ \text{CCF}, & \text{if } r \neq s \end{cases}$$



Proposed Ternary ZCZ Sequences

Let any row of matrix $F^{(n)}$ be ZCZ sequences with $GO(N_n, M_n, Z_n)$ based on complementary sequences [*]

[*] X.M. Deng and P.Z. Fan, "Spreading Sequences sets with Zero Correlation Zone," *Electronics Letters*, Vol.36, No.11, 25th May 2000

$$F^{(n)} = \begin{bmatrix} F^n_{1,1} & F^n_{1,2} & \cdots & F^n_{1,2N_k} \\ F^n_{2,1} & F^n_{2,2} & \cdots & F^n_{2,2N_k} \\ \vdots & \vdots & \ddots & \vdots \\ F^n_{M_n,1} & F^n_{M_n,2} & \cdots & F^n_{M_n,N_k} \end{bmatrix}$$

Theorem 1 Any row of matrix $F'^{(n)}$ made from padding N_k Zs of Zeros with length L as below is ZCZ sequences with (N_n+2N_kL, M_n, Z_n+L)

$$F'^{(n)} = \begin{bmatrix} F^n_{1,1} & Z & F^n_{1,2} & Z & \cdots & F^n_{1,2N_k} & Z \\ F^n_{2,1} & Z & F^n_{2,2} & Z & \cdots & F^n_{2,2N_k} & Z \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ F^n_{M_n,1} & Z & F^n_{M_n,2} & Z & \cdots & F^n_{M_n,N_k} & Z \end{bmatrix}$$

Proposed Ternary ZCZ Sequences

- EX) Binary ZCZ sequences GO(8, 4, 1)

$$F^{(1)} = \begin{bmatrix} F_{11}^{1} & F_{12}^{1} & F_{13}^{1} & F_{14}^{1} \\ F_{21}^{1} & F_{22}^{1} & F_{23}^{1} & F_{24}^{1} \\ F_{31}^{1} & F_{32}^{1} & F_{33}^{1} & F_{34}^{1} \\ F_{41}^{1} & F_{42}^{1} & F_{43}^{1} & F_{44}^{1} \end{bmatrix} = \begin{bmatrix} -- & ++ & +- & -+ \\ -- & -- & +- & +- \\ +- & -+ & -- & ++ \\ +- & +- & -- & -- \end{bmatrix}$$

- EX) Proposed Ternary ZCZ sequencesGO(16, 4, 3)

$$F^{(1)} = \begin{bmatrix} F_{11}^{1} & Z & F_{12}^{1} & Z & F_{13}^{1} & Z & F_{14}^{1} & Z \\ F_{21}^{1} & Z & F_{22}^{1} & Z & F_{23}^{1} & Z & F_{24}^{1} & Z \\ F_{31}^{1} & Z & F_{32}^{1} & Z & F_{33}^{1} & Z & F_{34}^{1} & Z \\ F_{41}^{1} & Z & F_{42}^{1} & Z & F_{43}^{1} & Z & F_{44}^{1} & Z \end{bmatrix} = \begin{bmatrix} -- & 00 & ++ & 00 & +- & 00 & -+ & 00 \\ -- & 00 & -- & 00 & +- & 00 & +- & 00 \\ +- & 00 & -+ & 00 & -- & 00 & ++ & 00 \\ +- & 00 & +- & 00 & -- & 00 & -- & 00 \end{bmatrix}$$

Proposed Ternary ZCZ Sequences

- EX) Cha's Ternary ZCZ GO(16, 6, 1)

$$F = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{bmatrix} = \begin{bmatrix} + + 0\ 0 + -\ 0\ 0 + +\ 0\ 0 - +\ 0\ 0 \\ + - 0\ 0 + +\ 0\ 0 + -\ 0\ 0 - -\ 0\ 0 \\ 0\ 0 + -\ 0\ 0 + +\ 0\ 0 - +\ 0\ 0\ + + \\ 0\ 0 + +\ 0\ 0 + -\ 0\ 0 - -\ 0\ 0\ + - \\ + -\ 0\ 0 + +\ 0\ 0 - +\ 0\ 0\ + +\ 0\ 0 \\ + +\ 0\ 0 + -\ 0\ 0 - -\ 0\ 0\ + -\ 0\ 0 \end{bmatrix}$$

- Comparison of bounds to Cha's ternary ZCZ
  - Cha's ternary sequences bounds : $MZ_0 \le 3N/4$
  - Proposed ternary sequences bounds : $MZ_0 \approx N$, if $L \to \infty$

MC-DS-CDMA

- Various propagation environments

  - Diverse delay spread (indoor, open rural, suburban, urban)

    ; distributed over [0.1us, 3us]

  - More flexible for diverse channel → More capacity and Better performance

- MC-DS-CDMA

  ☑ Higher degree of freedom by using Time-Frequency spreading

  → More flexible for the diverse environments

  ☑ Optimum diversity gain through path (time) diversity and frequency diversity

  ☑ Total spreading factor : $SF_{Total} = SF_{time} \times SF_{frequency}$

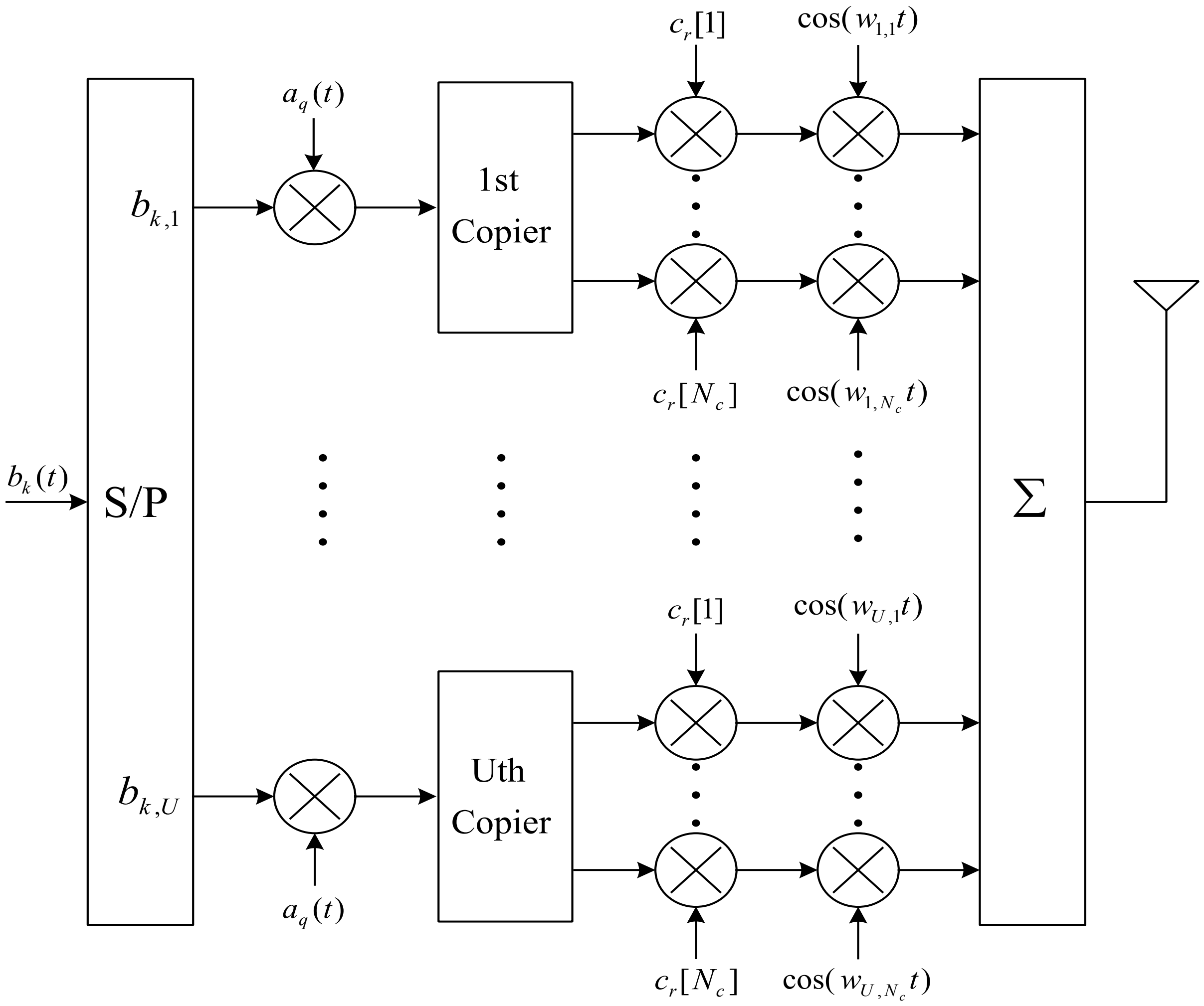
  ☑ # of TD spreading sequences : Max. M_t user groups

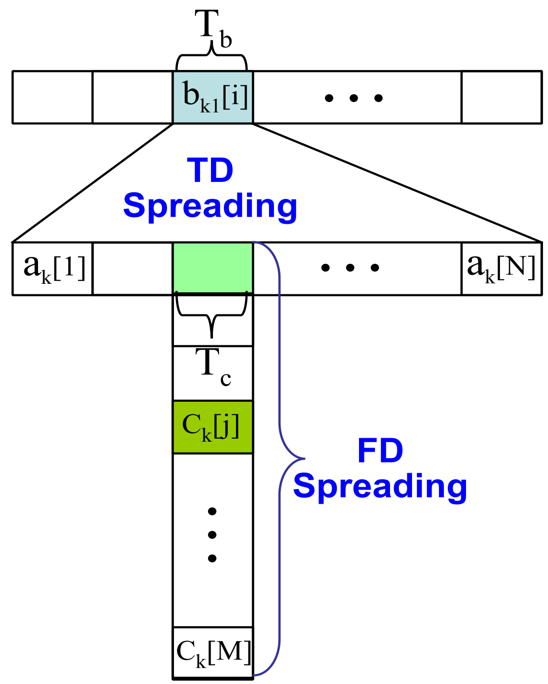
  ☑ # of FD spreading sequences : Max. M_f users per group

  → Max. M_t x M_f users

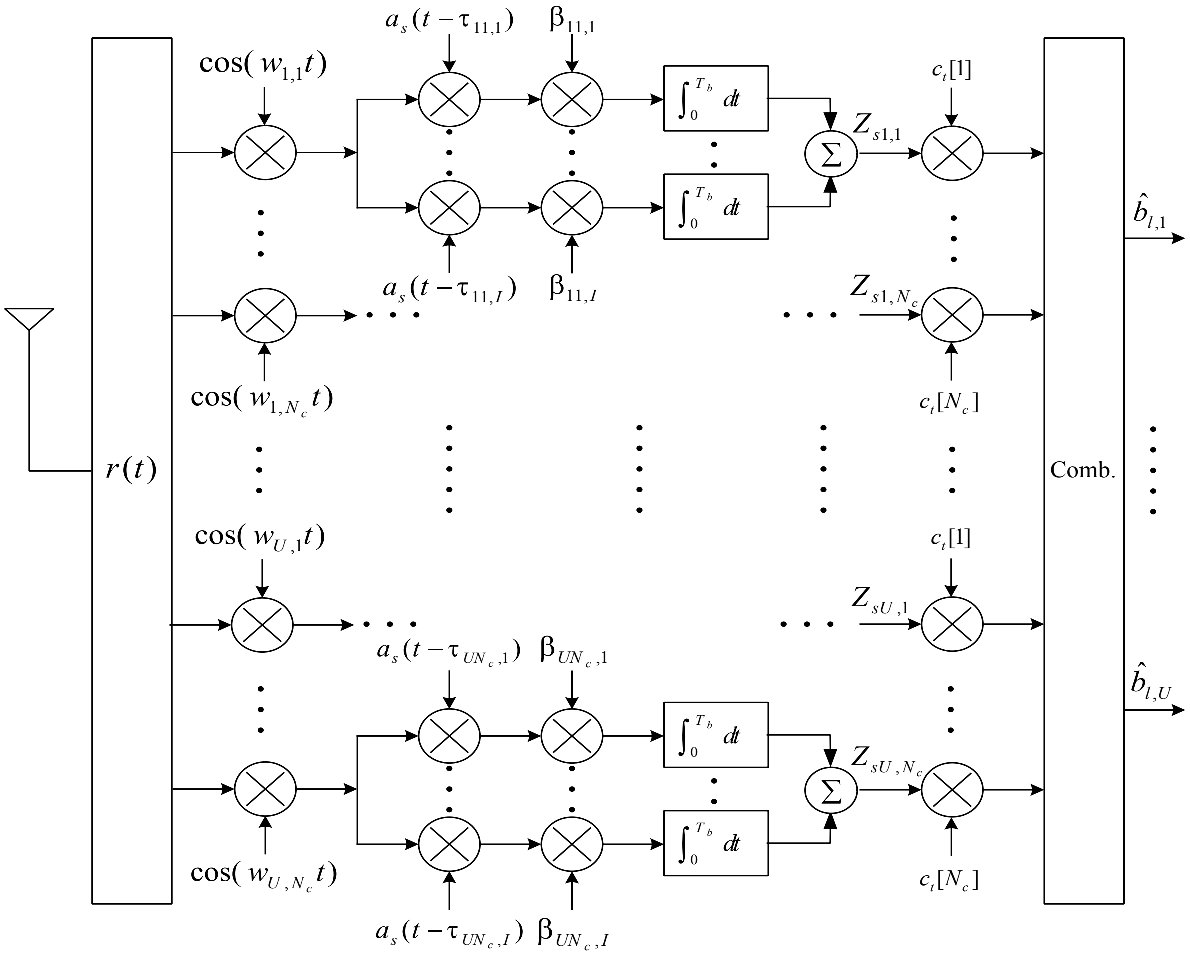
MC-DS-CDMA System Model

- Transmitter Block





■ Proposed Receiver Block based on Rake Receiver



BER Analysis

- BER of Receiver based on Rake receiver over multipath non-fading channel

$$P_e = Q\left(\sqrt{\frac{(E[z])^2}{\mathrm{var}[z]}}\right) = Q\left(\sqrt{\frac{N_c^2}{N_o N_c^2 / 2E_b}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

- BER of Receiver based on Rake receiver over multipath Rayleigh fading channel (MRC in time domain & EGC in frequency domain)

$$P_e = \int_0^\infty P(error \mid \alpha) f_\alpha(\alpha) d\alpha = \int_0^\infty Q\left(\sqrt{\frac{2L\alpha^2 E_b}{2(M_f - 1)N_c E_b + N_0 N_c L\alpha}}\right) f_\alpha(\alpha) d\alpha$$

$$\text{where } f_\alpha(\alpha) = \frac{\alpha^{N_c L - 1}}{(N_c L - 1)!\,\Omega^{N_c L}} \exp\left(-\frac{\alpha}{\Omega}\right) U(\alpha)$$

BER Analysis

- BER of Receiver based on Rake receiver over multipath Rayleigh fading channel (MRC in time domain & ORC in frequecy domain)

$$P_e = \int_0^\infty \int_0^\infty \cdots \int_0^\infty P(error \mid \lambda_n) f_{\lambda_1}(\lambda_1) f_{\lambda_2}(\lambda_2) \cdots f_{\lambda_{N_c}}(\lambda_{N_c}) d\lambda_1 d\lambda_2 \cdots d\lambda_{N_c}$$

$$= \underbrace{\int_0^\infty \int_0^\infty \cdots}_{N_c \text{ fold}} \int_0^\infty Q\left(\sqrt{\frac{2E_b}{\frac{N_0}{N_c} \sum_{n=1}^{N_c} \lambda_n}} \right) f_{\lambda_1}(\lambda_1) f_{\lambda_2}(\lambda_2) \cdots f_{\lambda_{N_c}}(\lambda_{N_c}) d\lambda_1 d\lambda_2 \cdots d\lambda_{N_c}$$

$$\text{where } f_{\lambda_i}(\lambda_i) = \frac{\left(\lambda_i^{-1}\right)^{L+1}}{(L-1)!\Omega^L} \exp\left(-\frac{1}{\Omega \lambda_i} \right) U(\lambda_i) \qquad \text{for } i = 1, 2, \ldots N_c$$

BER Analysis

- BER of Receiver based on Rake receiver over multipath Nakagkmi-m fading channel (MRC in time domain & EGC in frequecy domain)

$$P_e = \int_0^\infty P(error \mid \alpha) f_\alpha(\alpha) d\alpha = \int_0^\infty Q\left(\sqrt{\frac{2mL\alpha^2 E_b}{2(M_f - 1)mN_c E_b + N_0 N_c mL\alpha}} \right) f_\alpha(\alpha) d\alpha$$

$$\text{where } f_\alpha(\alpha) = \frac{\alpha^{N_c Lm - 1}}{\Gamma(N_c Lm)\left(\Omega/m\right)^{N_c Lm}} \exp\left(-\frac{\alpha}{(\Omega/m)} \right) U(\alpha)$$

- BER of Receiver based on Rake receiver over multipath Nakagkmi-m fading channel (MRC in time domain & ORC in frequecy domain)

$$P_e = \int_0^\infty \int_0^\infty \cdots \int_0^\infty P(error \mid \lambda_n) f_{\lambda_1}(\lambda_1) f_{\lambda_2}(\lambda_2) \cdots f_{\lambda_{N_c}}(\lambda_{N_c}) d\lambda_1 d\lambda_2 \cdots d\lambda_{N_c}$$

$$= \underbrace{\int_0^\infty \int_0^\infty \cdots}_{N_c \text{ fold}} \int_0^\infty Q\left(\sqrt{\frac{2E_b}{\frac{N_0}{N_c} \sum_{n=1}^{N_c} \lambda_n}} \right) f_{\lambda_1}(\lambda_1) f_{\lambda_2}(\lambda_2) \cdots f_{\lambda_{N_c}}(\lambda_{N_c}) d\lambda_1 d\lambda_2 \cdots d\lambda_{N_c}$$

$$\text{where } f_{\lambda_n}(\lambda_n) = \frac{\left(\lambda_n^{-1}\right)^{Lm+1}}{\Gamma(Lm)\left(\Omega/m\right)^{Lm}} \exp\left(-\frac{\lambda_n^{-1}}{(\Omega/m)} \right) U(\lambda_n) \qquad \text{for } i = 1, 2, \ldots N_c$$

ZCZ sequences in Simulation

- Binary ZCZ GO(32, 4, 4)

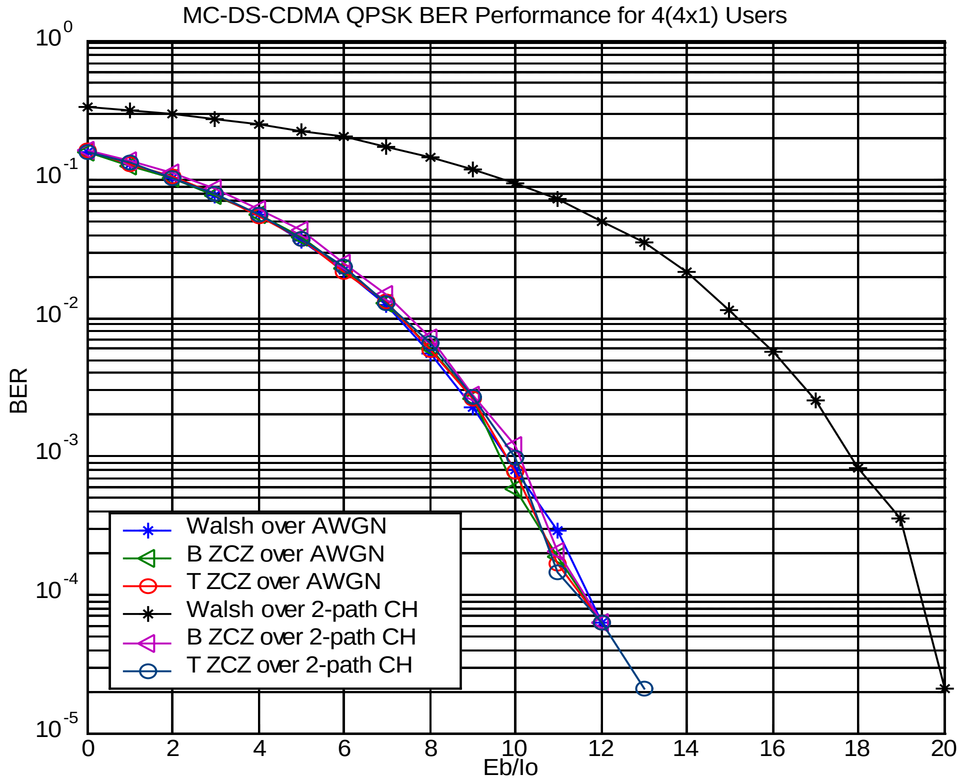
$$F = \begin{bmatrix} ++-+++-+ & ---+---+ & --+-++-+ & +++----+ \\ +---+--- & -+---+-- & -++++--- & +-++-+-- \\ --+-++-+ & +++----+ & ++-+++-+ & ---+---+ \\ -++++--- & +-++-+-- & +---+--- & -+---+-- \end{bmatrix}$$

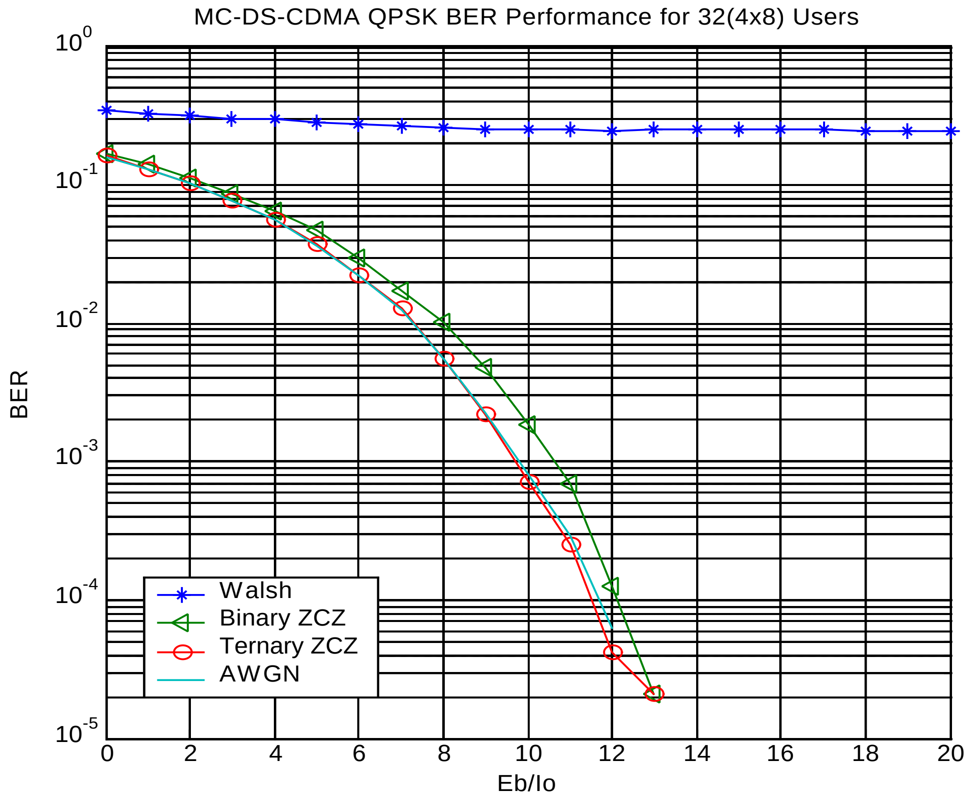
- Ternary ZCZ GO(32, 4, 4)

$$F = \begin{bmatrix} ++-+++-+ & 00000000 & ---+---+ & 00000000 \\ +---+--- & 00000000 & -+---+-- & 00000000 \\ --+-++-+ & 00000000 & +++----+ & 00000000 \\ -++++--- & 00000000 & +-++-+-- & 00000000 \end{bmatrix}$$

Simulation Result I

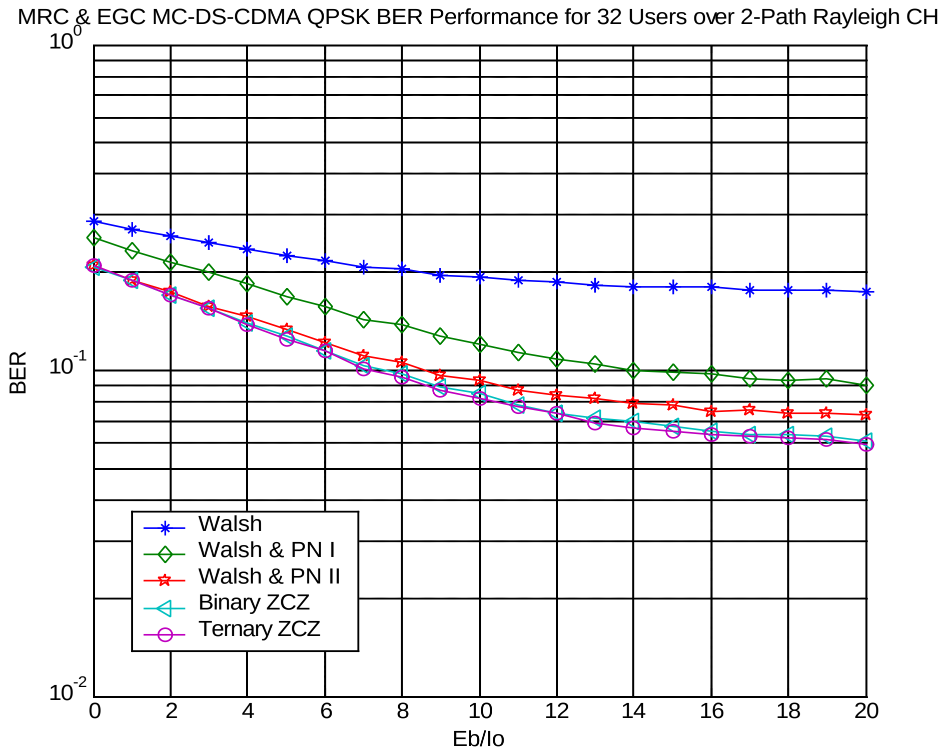
- MC-DS-CDMA using ZCZ sequence over AWGN and multipath non-fading channel

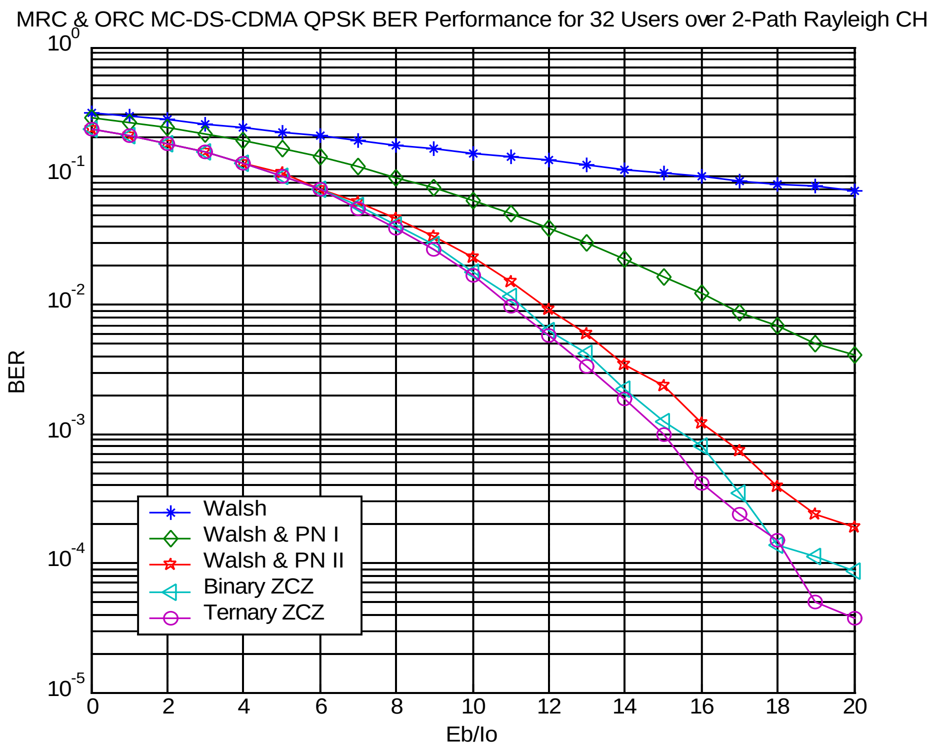




Simulation Result II

- MC-DS-CDMA using ZCZ sequence for 32 users over multipath Rayleigh fading channel





Cyclic Extension of ZCZ sequences

- Nonzero Partial Correlation in Decision value for nth subcarrier
  - Asynch., different value between consecutive symbols $\rightarrow$ Aperiodic correlation
  - Causing some interferences called Partial Interferences in MC-DS-CDMA receiver

$$\boxed{\int_{\tau_{n,j}}^{T_b+\tau_{n,j}} b_k(t-\tau_{n,i})a_q(t-\tau_{n,i})a_s(t-\tau_{n,j})dt} \longrightarrow \text{Nonzero Partial Correlation}$$

$$= \begin{cases} (if\,\tau_{n,i} > \tau_{n,j}) \\ \dfrac{T_b\Delta}{NT_c}\left(b_k^{(-1)}R_{A_s,A_q}(N-m+1)+b_k^{(0)}R_{A_q,A_s}(m-1)\right)+\dfrac{T_b(T_c-\Delta)}{NT_c}\left(b_k^{(-1)}R_{A_s,A_q}(N-m)+b_k^{(0)}R_{A_q,A_s}(m)\right), \\ (if\,\tau_{n,i} < \tau_{n,j}) \\ \dfrac{T_b\Delta}{NT_c}\left(b_k^{(0)}R_{A_s,A_q}(m+1)+b_k^{(+1)}R_{A_q,A_s}(N-m-1)\right)+\dfrac{T_b(T_c-\Delta)}{NT_c}\left(b_k^{(0)}R_{A_s,A_q}(m)+b_k^{(+1)}R_{A_q,A_s}(N-m)\right) \end{cases}$$

$$\left(N=\frac{T_b}{T_c},\ m=\frac{\left\lfloor\left|\tau_{n,i}-\tau_{n,j}\right|\right\rfloor}{T_c},\ mT_c \le \Delta \le (m+1)T_c \right)$$

Cyclic Extension of ZCZ sequences

■ Cyclic Extension of ZCZ sequences

- $A=(a_1a_2...a_{N-1})$ be ZCZ sequence with length N, ZCZ=Z
- τ is defined as follows if maximum delay among multipaths is d.

$$\tau = \frac{\lfloor d \rfloor}{T_c}$$

- The concatenation of cyclically repeated sequence with length $\tau+1$ to the front and the end of original ZCZ sequence ($\tau+1 \leq Z$)

$a_{N-1-\tau}$	• • •	a_{N-1}	a_0	• • • • • •	a_{N-1}	a_0	• • •	a_τ

- Practically concatenation of the cyclically repeated version with length $\tau+1$ of original spreading symbol to the front and the end of the original spreading symbol similar to Cyclic Prefix in OFDM
- But may be some difficult in practical implementation.
- Increased overhead : 0 $\rightarrow$ $2(\tau+1)$, low spectral efficiency : $1/N \rightarrow 1/\{N+2(\tau+1)/T_c\}$

Cyclic Extension of ZCZ sequences

■ Resulted Partial Correlation function after Cyclic Extension of ZCZ sequences

$$\int_{\tau_{n,j}}^{T_b+\tau_{n,j}} b_k(t-\tau_{n,i})a_q(t-\tau_{n,i})a_s(t-\tau_{n,j})dt$$

$$= \begin{cases} \frac{T_b\Delta}{NT_c}\left(b_k^{(0)}\phi_{A_s,A_q}(-m+1)\right)+\frac{T_b(T_c-\Delta)}{NT_c}\left(b_k^{(0)}\phi_{A_s,A_q}(-m)\right), & \text{if } \tau_{n,i}>\tau_{n,j} \\ \frac{T_b\Delta}{NT_c}\left(b_k^{(0)}\phi_{A_s,A_q}(m+1)\right)+\frac{T_b(T_c-\Delta)}{NT_c}\left(b_k^{(0)}\phi_{A_s,A_q}(m)\right), & \text{if } \tau_{n,i}<\tau_{n,j} \end{cases}$$

$$\boxed{=0}$$

$$\left(\text{Since } \phi_{A_q,A_s}(n)=\sum_{k=0}^{N-1} a_k^{(q)}a_{k+n}^{(s)}=0, \ \text{for } 0<|n|\le Z, \ (\tau+1\le Z)\right)$$

$$\left(N=\frac{T_b}{T_c}, \ m=\frac{\lfloor \tau_{n,d} \rfloor}{T_c}, \ \tau=\max(\tau_{n,d}), \ mT_c\le\Delta\le(m+1)T_c\right)$$

- Resulted BER after Cyclic Extension of ZCZ Sequences

$$\boxed{Pe_l=Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

Concluding Remarks

- Proposed ternary ZCZ Sequences
  - Flexible ZCZ length
  - Not optimal but better bound than Cha's ternary ZCZ sequences

- MC-DS-CDMA using ZCZ sequences
  - Effective elimination of interferences inside ZCZ

- Cyclic Extension of ZCZ sequences
  - Theoretically Perfect elimination of all interferences if every MPD is inside ZCZ
  - Same BER performance as AWGN channel even in multipath non-fading channel
  - Increased overhead, low spectral efficiency