QC-LDPC codes from Various Golomb Rulers

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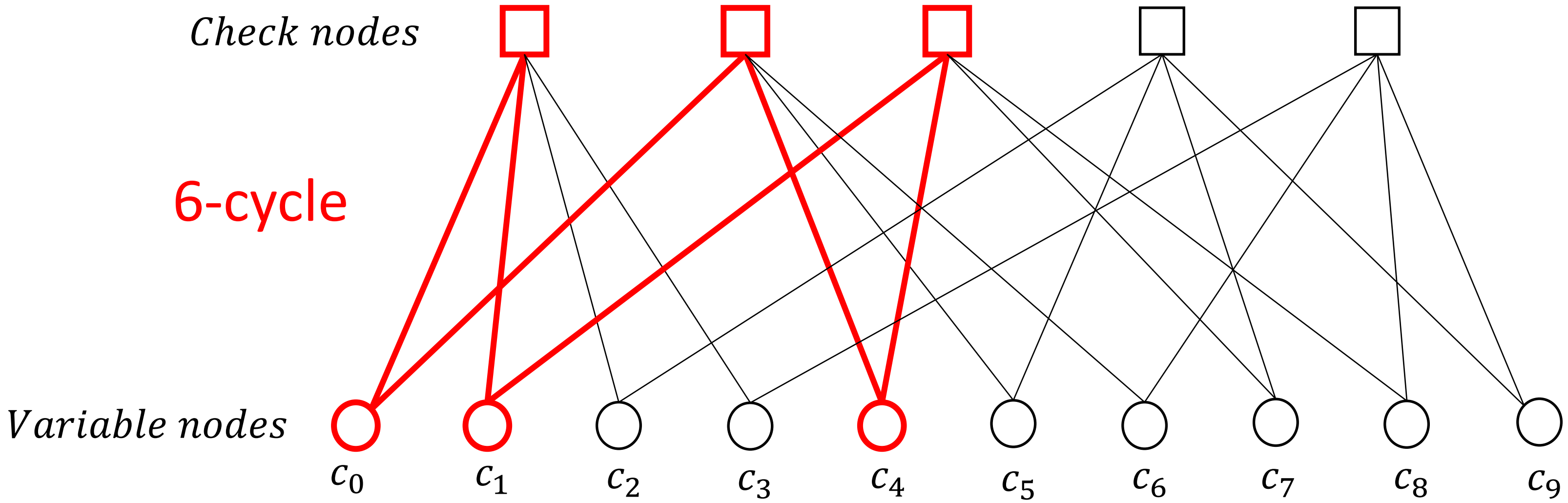
Motivation

- Kim in [1] proposed the construction of girth-8 QC-LDPC codes using Golomb ruler in multiplication table algorithm.
- In this paper, we construct QC-LDPC codes using a special type of Golomb ruler called B_h sequence and compare the number of cycle and performance of the resulting codes with the codes from Golomb ruler.

[1] I. Kim and H.-Y. Song, "A construction for girth-8 QC-LDPC codes using Golomb rulers," Electronic Letters, vol. 58, no. 15, pp. 582-584, July 2022.

QC-LDPC codes

- Widely used because of its simple encoding and parallel decoding (WIMAX, WIFI, 5GNR, …).
- Avoid short length cycles and increase the length of shortest cycle (Girth) in Tanner graph for decoding performance



Example of 6-cycle in Tanner graph

Golomb ruler

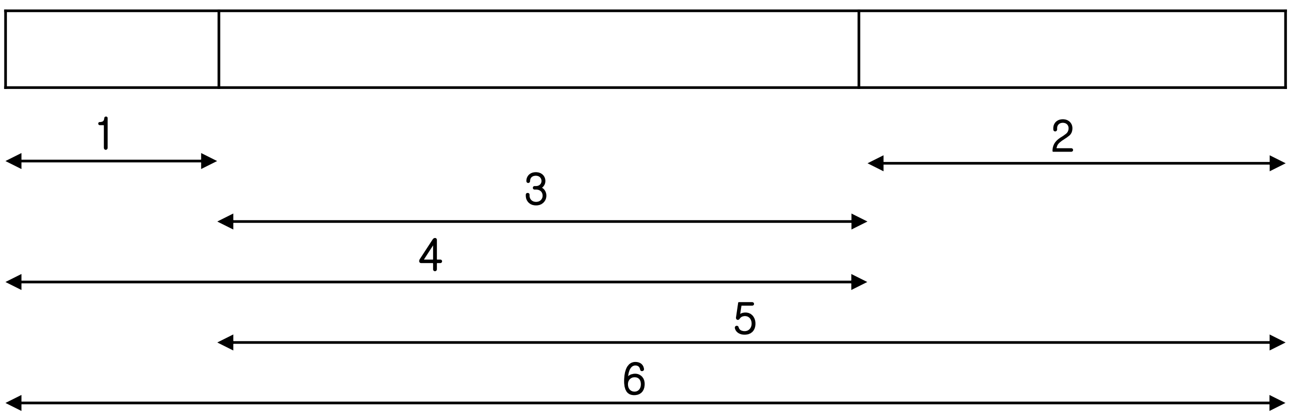
- A Golomb ruler [2] is a set of s marks of integers $\{g_1, g_2, \dots, g_s\}$ with $g_1 < g_2 < \dots < g_s$ such that

$$g_j - g_i$$

are all distinct for all $i < j$.

- The distance $L = g_n - g_1$ is called the length of the above s-mark Golomb ruler.

- Example of 4-mark Golomb ruler $\{0, 1, 4, 6\}$



Distances are all distinct

B_h sequence

- A sequence $a_1 < a_2 < \dots < a_n$ is called B_h sequence [11] if the h-fold sums

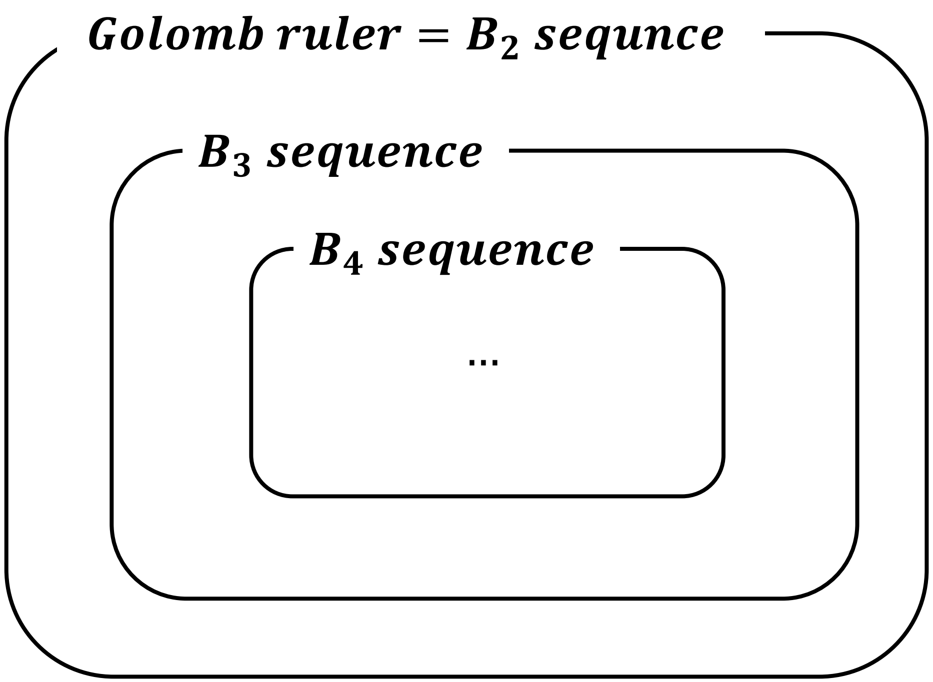
$$a_{j_1} + a_{j_2} + \dots + a_{j_h}$$

  are all distinct for all $j_1 \le j_2 \le \dots \le j_h$.

- The difference $L = a_s - a_1$ is called the length of the B_h sequence.

Relation between Golomb ruler and B_h sequence

- All B_h sequences are special type of Golomb ruler



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Main Construction Platform

- $E = [e(i,j)]$ is a $3 \times s$ exponent matrix of integers
- I is the identity matrix of size $P \times P$
- $I^{(t)}$ is the identity matrix I circularly shifted to the right t times. It is called circular permutation matrix (CPM)

Algorithm 1 Main Construction Platform

Input: A positive integer P and two integer sequences

$$a = (a_1, a_2, a_3) \text{ and } \mathrm{b} = (\mathrm{b}_1, \mathrm{b}_2, \dots, \mathrm{b}_\mathrm{s})$$

Output: Binary $3P \times sP$ matrix H

Step1 : Construct $E = [e(i,j)]$ by $e(i,j) = a_i \cdot b_j$ for all i, j

Step2 : Construct H by replacing each element of E by an appropriate CPM:

$$H = \begin{bmatrix} I^{(a_1 \cdot b_1)} & I^{(a_1 \cdot b_2)} & \dots & I^{(a_1 \cdot b_3)} \\ I^{(a_2 \cdot b_1)} & I^{(a_2 \cdot b_2)} & \dots & I^{(a_2 \cdot b_3)} \\ I^{(a_3 \cdot b_1)} & I^{(a_3 \cdot b_2)} & \dots & I^{(a_3 \cdot b_3)} \end{bmatrix}$$

Construction of the girth-8 codes

Theorem 1 : Assume that the sequence $a = (1,2,3)$, $b = (b_1, b_2, \dots b_s)$ and $P \times P$ CPMs are used in Algorithm 1. Let b be a B_3 sequence of length L. Then, the resulting QC-LDPC code has girth 8 and 8-cycles appear exactly $\binom{s}{2} \times P$ times if

1) $P > 4L$ (in general)

or

2) $P > 3L$ (when P is odd).

$\times$	b_{j_0}	b_{j_1}
1	b_{j_0}	b_{j_1}
2	$2b_{j_0}$	$2b_{j_1}$
3	$3b_{j_0}$	$3b_{j_1}$

$\Longrightarrow$ inevitable 8-cycle appears $\binom{s}{2} \times P$ times

Example

- Using B_3 sequence $\{0,2,11,26,29,45\}$ ($L = 45$)

×	0	2	11	26	29	45
1	0	2	11	26	29	45
2	0	4	22	52	58	90
3	0	6	33	78	78	135

Exponent matrix E

$I^{(0)}$	$I^{(2)}$	$I^{(11)}$	$I^{(26)}$	$I^{(29)}$	$I^{(45)}$
$I^{(0)}$	$I^{(4)}$	$I^{(22)}$	$I^{(52)}$	$I^{(58)}$	$I^{(90)}$
$I^{(0)}$	$I^{(6)}$	$I^{(33)}$	$I^{(78)}$	$I^{(78)}$	$I^{(135)}$

$P = 181 \; or \; 137$

Cycle property

- Comparison of the number of cycles (Golomb ruler vs B_3 sequence)
- Less 8-,10- and 12- cycle for using B_3 sequence

	P=181 (N=1086)	
	Golomb ruler {0,1,8,12,14,17}	B_3 sequence {0,2,11,26,29,45}
4-cycle	0	0
6-cycle	0	0
8-cycle	5249	$2715 = \binom{6}{2} \times 181$
10-cycle	27512	3982
12-cycle	255572	102989

The length of shortest cycle = Girth = 8

Cycle property

- Less 8-,10- and 12- cycle for using B_3 sequence

	P=137 (N=822)	
	Golomb ruler {0,1,8,12,14,17}	B_3 sequence {0,2,11,26,29,45}
4-cycle	0	0
6-cycle	0	0
8-cycle	3973	$2055 = \binom{6}{2} \times 137$
10-cycle	20824	4110
12-cycle	193444	97681

The length of shortest cycle = Girth = 8

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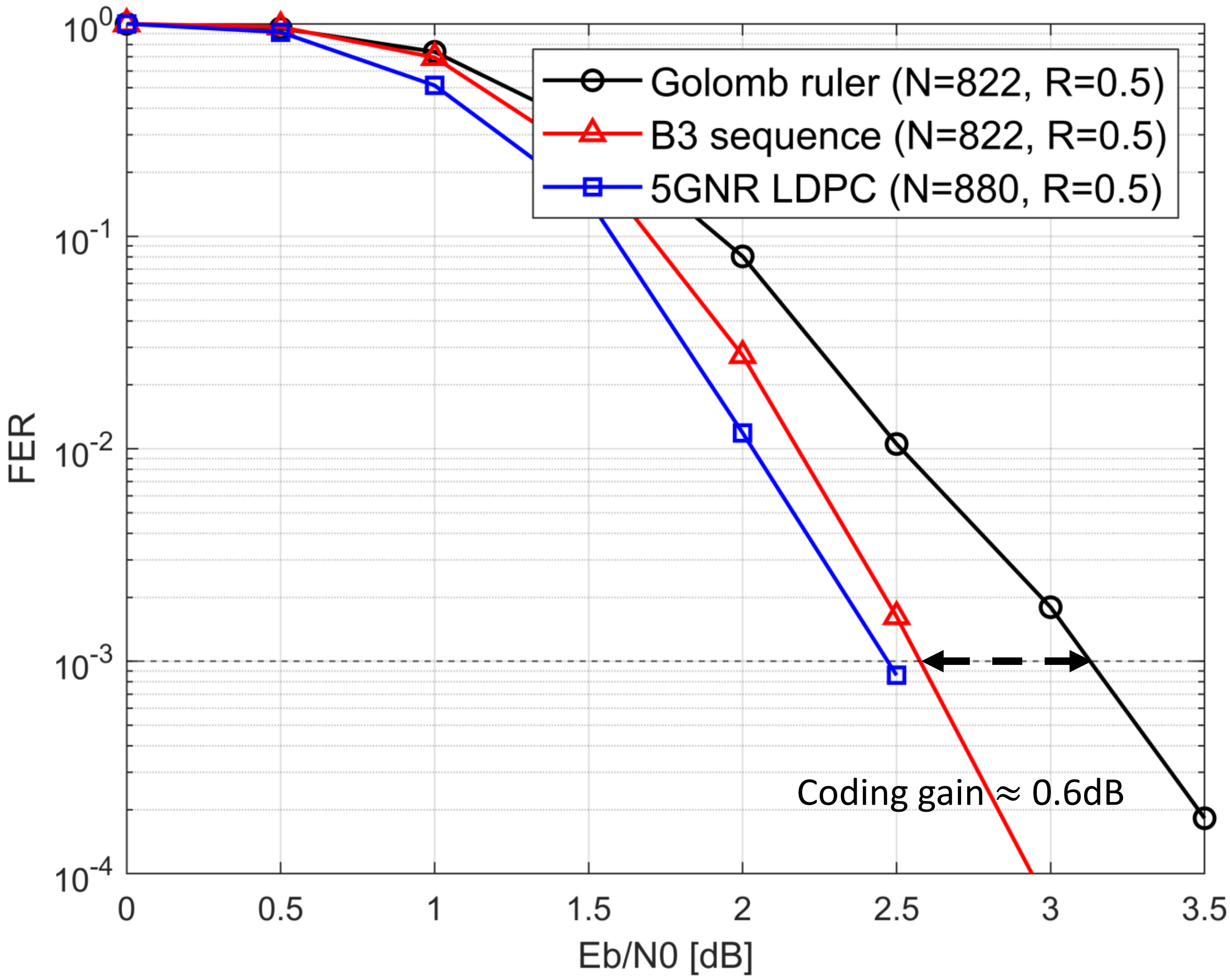
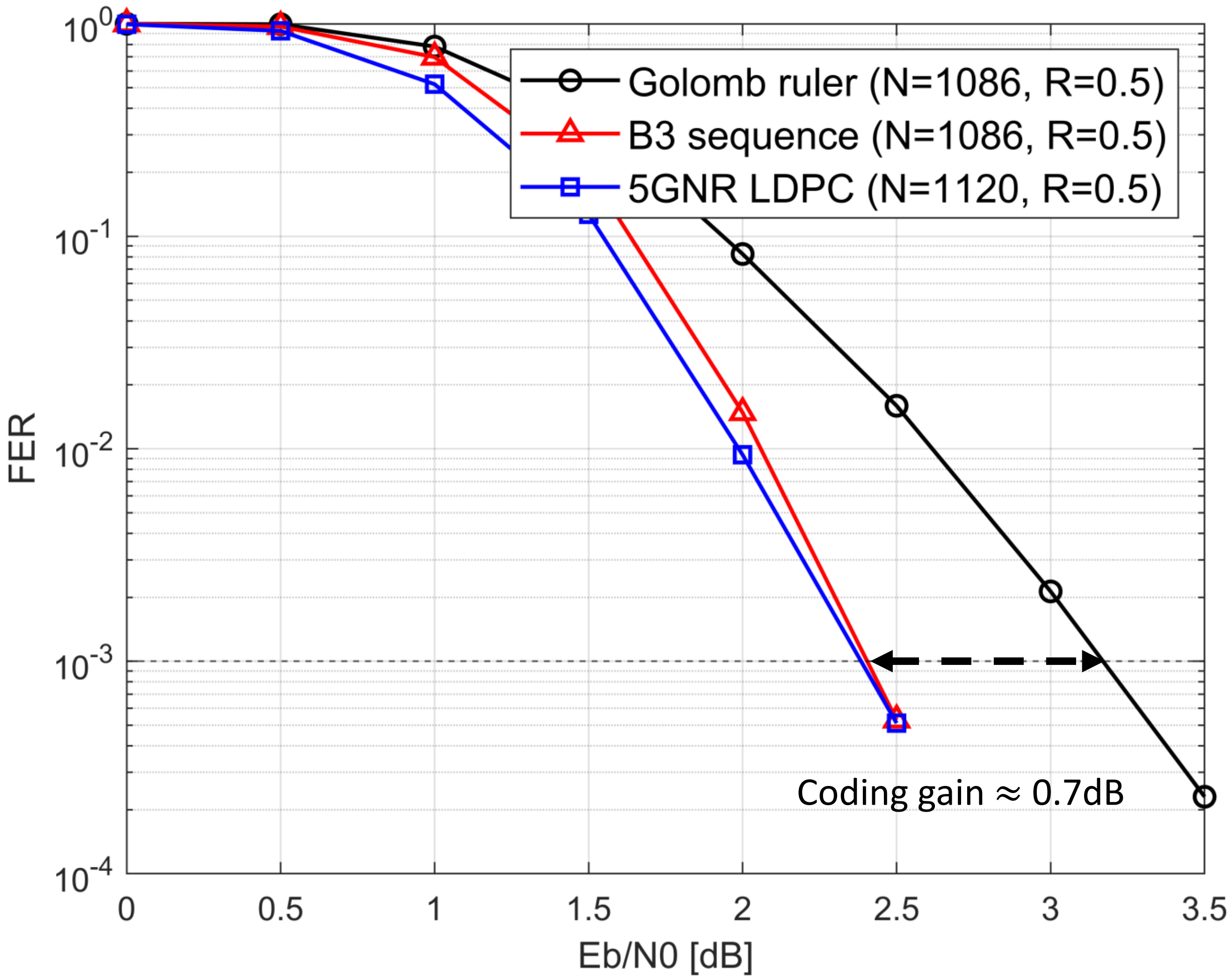
FER performance



FER performance



Conclusion

- In this paper, we constructed QC-LDPC codes from B_3 sequence
- Prove that resulting codes have girth 8 and leaves 8-cycles of some special case only
- Show that resulting codes shows additional coding gain over the code from general Golomb rulers
- Show that that resulting codes have similar performance with the modified LDPC codes from 5GNR basegraph2

Thank for listening