Pseudo-Random Sequence Generator Using Cascade Chaotic Maps for DSSS and GNSS

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This study explores using cascade chaotic systems (CCS) to generate complex and unpredictable binary sequences for Direct Sequence Spread Spectrum (DSSS) systems applications, such as those used in GNSS. Experimental results show that the generated sequences exhibit excellent correlation properties and show better randomness and complexity compared to m-sequences, making CCS a promising candidate for advanced DSSS systems.

Introduction

- DSSS system is a key technique used in satellite navigation systems such as GNSS. While traditional pseudo-random noise (PN) sequences enable robust signal spreading, their fixed period imposes constraints on security and flexibility [1].
- Sequences generated by chaotic systems exhibit high complexity and unpredictability, making them a promising alternative to conventional PN codes [2].
- This presentation introduces the CCS [3] and analyzes the unpredictability (via Lyapunov exponent), complexity (via ApEn, PE, and the NIST test), and correlation properties of the generated sequences.
- The analysis compares these sequences with traditional PN codes, such as m-sequences, to evaluate the potential of CCS for DSSS systems.

Main Results

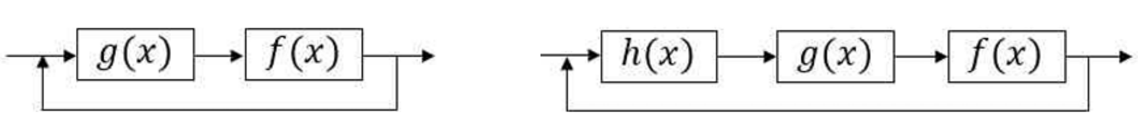

Fig. 1. **Cascade Chaotic System** structure using chaotic maps: two seed maps, three seed maps

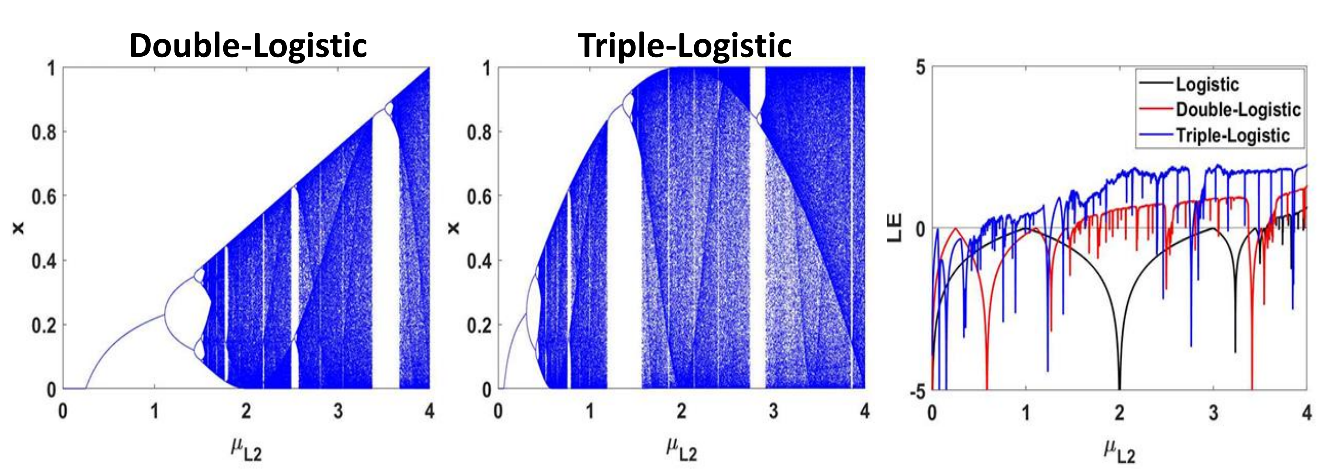

Fig. 2. Bifurcation Diagram and Lyapunov Exponent of Cascade Chaotic Maps

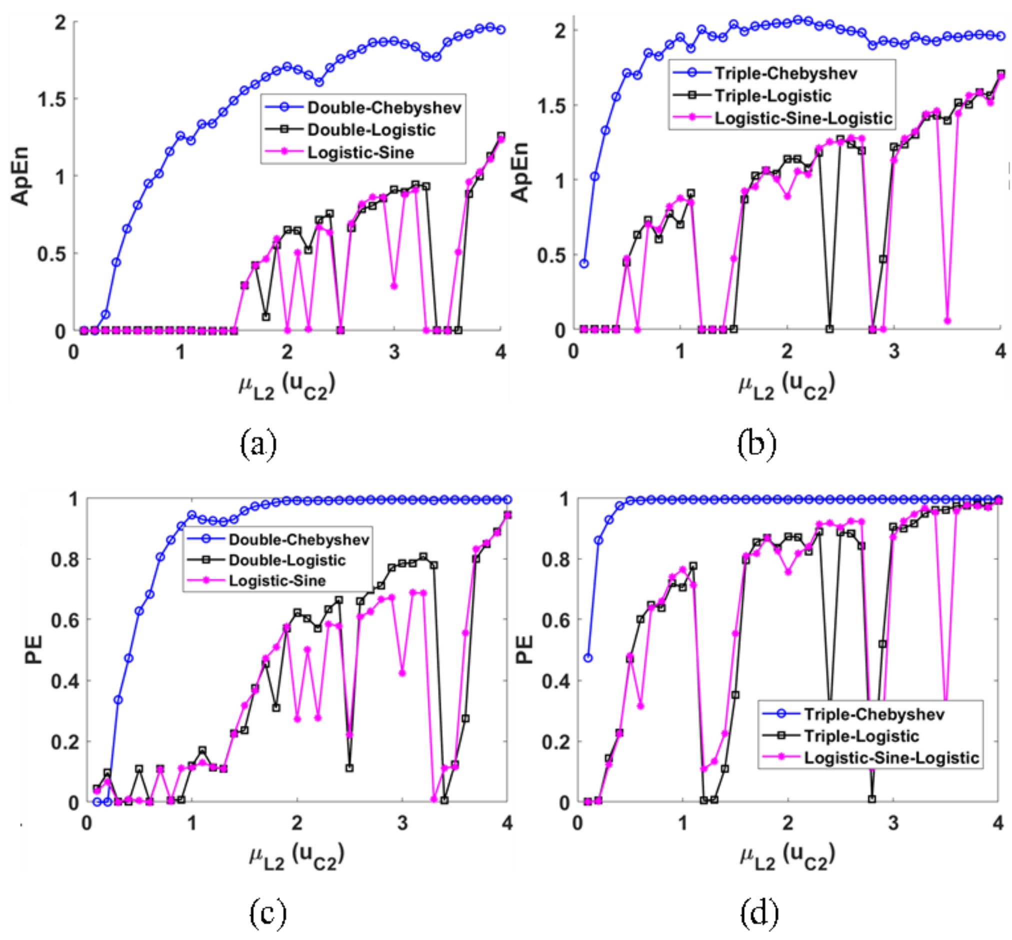

Fig. 3. Approximate entropy and Permutation entropy of Real Number Sequences Generated from Cascade Chaotic Maps

Classification	Frequency	Run	Rank	FFT
Double-Logistic	P	P	P	P
Triple-Logistic	P	P	P	P
Double-Chebyshev	P	P	P	P
Triple-Chebyshev	P	P	P	P
Logistic-Sine	P	P	P	P
Logistic-Sine-Logistic	P	P	P	P
m-sequence	P	P	**F**	**F**

Table I. NIST test results for cascaded chaotic maps and m-sequences

REFERENCES

[1] IS-GPS-800E, Navstar GPS Space Segment/Navigation user segment interfaces. USA: Navstar GPS Joint Program Office, 2018.

[2] F. Liu, S. Jia, X. Xu and M. Tian, "Improved Chaotic Sequence Generation Method Based on Direct Spread Spectrum." Journal of Physics: Conference Series, vol. 1237, no. 4, 2019.

[3] Y. Zhou, Z. Hua, C. M. Pun, and C. L. P. Chen, "Cascade chaotic system with applications," IEEE Trans. Cybern., vol. 45, no. 9, pp. 2001–2012, Sep. 2015.

<table>
<tr><th rowspan="3">Classification</th><th colspan="4">Length: 10000
Initial value: 0.4001 − 0.4100</th><th colspan="4">Length: 100000
Initial value: 0.4001 − 0.4100</th></tr>
<tr><th colspan="2">Normalized Auto-correlation</th><th colspan="2">Normalized Cross-correlation</th><th colspan="2">Normalized Auto-correlation</th><th colspan="2">Normalized Cross-correlation</th></tr>
<tr><th>Average (sidelobe)</th><th>Average (sidelobe max)</th><th>Average</th><th>Max Average</th><th>Average (sidelobe)</th><th>Average (sidelobe max)</th><th>Average</th><th>Max Average</th></tr>
<tr><td>Double-Logistic</td><td rowspan="6">≈ 0.008
≈ −21dB</td><td rowspan="6">≈ 0.04
≈ −14dB</td><td rowspan="6">≈ 0.008
≈ −21dB</td><td rowspan="6">≈ 0.04
≈ −14dB</td><td rowspan="6">≈ 0.002
≈ −27dB</td><td rowspan="6">≈ 0.01
≈ −20dB</td><td rowspan="6">≈ 0.002
≈ −27dB</td><td rowspan="6">≈ 0.01
≈ −20dB</td></tr>
<tr><td>Triple-Logistic</td></tr>
<tr><td>Double-Chebyshev</td></tr>
<tr><td>Triple-Chebyshev</td></tr>
<tr><td>Logisitc-Sine</td></tr>
<tr><td>Logisitc-Sine-Logisitc</td></tr>
<tr><td>m-sequence</td><td>≈ 0.006
≈ −22dB</td><td>≈ 0.02
≈ −16dB</td><td>−</td><td>−</td><td>≈ 0.001
≈ −28dB</td><td>≈ 0.007
≈ −21dB</td><td>−</td><td>−</td></tr>
</table>

TABLE II. CORRELATION PROPERTIES FOR BINARY CHAOTIC SEQUENCES AND M-SEQUENCE