The Global Optimality of the MIMO Cooperative System with Source and Relay Precoders for Capacity Maximization

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***Abstract*—This paper deals with the global optimality of the channel capacity of the multiple-input multiple-output (MIMO) cooperative system which is equipped with precoders at source and relay, and exploits the direct channel between source and destination. Each precoder of the system is individually designed by the Lagrangian method and the final precoders are decided by an iterative structure. To prove the global optimality for the channel capacity of the system with these joint precoders, we show that the channel capacity function of the system is a concave function and the constraints of the system are convex sets.**

***Index Terms*—MIMO system, cooperative system, source precoder, relay precoder, iterative structure, optimal precoder.**

I. Introduction

As the amount of data for communication is rapidly increasing, the demand to transfer high quality data is enlarging. To satisfy this demand, multiple-input multiple-output (MIMO) systems and cooperative systems have been developed [1][2]. MIMO systems provide multiplexing gain, channel capacity improvement, coverage area extension and so on, by using multiple antennas [1]. On the other hand, if the channel condition is poor, performance of communication systems is degraded. To compensate for the degradation, relay systems have been introduced [3]-[5].

The cooperative system is derived from the relay system, but the motivation of the system is somewhat different from that of the general relay system. The general relay system focuses on a solution to cope with the barriers between source and destination, and it is adopted when the channel between source and destination is useless. On the other hand, the cooperative system exploits not only the channel via relay but also the channel between source and destination (SD channel; so called the direct path), and this system focuses on the performance improvement [2][6].

The cooperative system combined with the MIMO system, so called MIMO cooperative system, is developed to have additional system performance improvement, which is classified into three categories: amplifying-and-forward (AF) system, decoding-and-forward (DF) system and non-regenerative (hybrid-and-forward: HF) system [7]-[16]. Among these systems, the MIMO non-regenerative system briskly has been studied to evaluate channel capacity, coverage area, error rate and so on [7][10]-[16]. To specially increase the channel capacity, several MIMO non-regenerative systems with precoders are proposed [7][11]-[15]. The system proposed in [7] tries to achieve the optimal channel capacity using only relay precoder. So, though the system in [7] improves the channel capacity, it cannot obtain the additional channel capacity generated by the source precoder. The system proposed in [11] designs both the source and relay precoders for minimizing the mean square error. The systems proposed in [7] and [11] assume the individual power constraint at source and relay. On the other hand, the system proposed in [12] assumes the sum power constraint of source and relay to improve the channel capacity so that it has more flexibility in designing the precoders than the individual power constraint case. These systems, such as [7], [11] and [12], parallelize the channels and allocate power to each data stream. Although these systems have the common goal of capacity improvement, they consider different channels and suggest different methods to increase the channel capacity. Furthermore, it is not clear whether their precoders are optimal from the viewpoint of the channel capacity or not [7][13].

This paper considers designing source and relay precoders by using a different approach for the effective channel of the system and proves that the proposed precoders are optimal in terms of the channel capacity. To design source and relay precoders, we consider the respective effective channels of source and relay. In each station, each precoder optimizes the system channel capacity by the Lagrangian method using only its own effective channel, and then we apply an iterative structure for global optimization of the precoders. Also, we prove the global optimality of the channel capacity of the proposed MIMO cooperative system to verify that the proposed precoders are optimal from the viewpoint of the channel capacity.

II. MIMO Cooperative System with Source and Relay Precoders

Fig. 1 represents a MIMO cooperative system with source and relay precoders, and this system has M antennas at each station. Most of the systems using the channel state information (CSI) adopt the time division duplexing (TDD) system. The TDD system assumes that the downlink channel is the transpose of the uplink channel, so the station can easily

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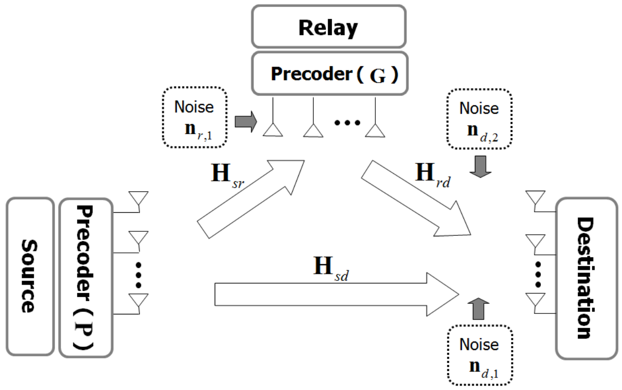

Fig. 1. MIMO cooperative system model with source and relay precoders.

know the CSI if the station receives the signals passing through the channel during uplink process. In the system, the relay and the destination allocate pilots in different positions, and then, the source and the relay can obtain the CSI of the source-destination link and the relay-destination link, respectively, using the pilots allocated by the destination. Also, the source can know the CSI of the source-relay-destination link using the pilots allocated by the destination and the source-relay link using the pilots allocated by the relay. As the source knows the source-relay-destination link and the source-relay link, it can estimate the relay-destination link by a simple inversion operation. However, the relay can not know the source-destination link, as this link does not include the relay. Therefore, we assume that source knows all channels: the channel between the source and the relay (SR channel), the channel between the relay and destination (RD channel), and SD channel. Also, we assume that the relay knows SR channel and RD channel but cannot have SD channel information as the systems proposed in [5], [7] and [11].

In the first time slot, the source passes an $M \times 1$ complex signal vector $\mathbf{x}$ to a source precoder, $\mathbf{P}$, and the precoded signal is transmitted to the relay and the destination. In this time slot, the received signals at the relay and the destination are given by

$$\begin{aligned} \mathbf{y}_{r,1} &= \mathbf{H}_{sr}\mathbf{P}\mathbf{x} + \mathbf{n}_{r,1} \\ \mathbf{y}_{d,1} &= \mathbf{H}_{sd}\mathbf{P}\mathbf{x} + \mathbf{n}_{d,1}\ , \end{aligned} \tag{1}$$

where $\mathbf{H}_{ij}$ is an $M \times M$ complex channel matrix between i and j ($i \in \{s, r\}$, $j \in \{r, d\}$, where s, r and d represent source, relay, and destination, respectively), and $\mathbf{n}_{r,1}$ and $\mathbf{n}_{d,1}$ are $M \times 1$ complex noise vectors at the relay and the destination, respectively.

In the second time slot, the received signal at the relay passes through a relay precoder, $\mathbf{G}$, and the precoded signal is transmitted to the destination. Then, the received signal at the destination is expressed as

$$\mathbf{y}_{d,2} = \mathbf{H}_{rd}\mathbf{G}\left(\mathbf{H}_{sr}\mathbf{P}\mathbf{x} + \mathbf{n}_{r,1}\right) + \mathbf{n}_{d,2}\ , \tag{2}$$

where $\mathbf{n}_{d,2}$ is an $M \times 1$ complex noise vector at the destination in the second time slot.

We assume that the noise variance of each station is σ^2 and the source transmits a signal with a power of P_s. By using (1) and (2), the mutual information between the source and

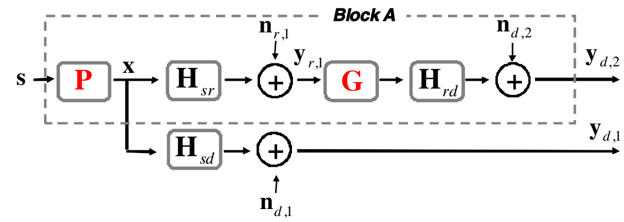

Fig. 2. Block diagram of MIMO cooperaitve system with source and relay precoders.

the destination is given by

$$I\left(\mathbf{x}\,;\,\mathbf{y}\right) = \frac{1}{2}\log_2\left|\mathbf{I}_{2M} + \mathbf{H}\mathbf{P}\mathbf{R}_x\mathbf{P}^H\mathbf{H}^H\mathbf{R}_w^{-1}\right|\ , \tag{3}$$

where

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_{d,1} \\ \mathbf{y}_{d,2} \end{bmatrix},\quad \mathbf{H} = \begin{bmatrix} \mathbf{H}_{sd} \\ \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr} \end{bmatrix}$$
$$\mathbf{R}_x = E\left[\mathbf{x}\mathbf{x}^H\right], \mathbf{R}_w = \begin{bmatrix} \mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{rd}\mathbf{G}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{I}_M \end{bmatrix}\sigma^2\ . \tag{4}$$

Here, $\mathbf{I}_A$ is an $A \times A$ identity matrix, and $|\cdot|$ and $E[\cdot]$ represent determinant and expectation operators, respectively.

The existing system, such as the system proposed in [7], deploys only a relay precoder so that it cannot achieve additional channel capacity. This paper considers both the source and relay precoders of the MIMO cooperative system to maximize the channel capacity. To do so, each precoder is individually designed by using its own effective channel information, and then, an iterative structure is applied for global optimization of the precoders.

A. Precoder Design for the Relay Station

As the relay obtains information only from Block A in Fig. 2, we want to find a relay precoder that maximizes the mutual information of Block A, $I\left(\mathbf{x}\,;\,\mathbf{y}_{d,2}\right)$. The mutual information of Block A is written as

$$\begin{aligned} I\left(\mathbf{x};\mathbf{y}_{d,2}\right) = \tfrac{1}{2}\log_2\Big|\mathbf{I}_M + \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{P}\mathbf{R}_x\mathbf{P}^H\mathbf{H}_{sr}^H \\ \mathbf{G}^H\mathbf{H}_{rd}^H\left(\mathbf{H}_{rd}\mathbf{G}\mathbf{G}^H\mathbf{H}_{rd}^H\sigma^2 + \mathbf{I}_M\sigma^2\right)^{-1}\Big|. \end{aligned} \tag{5}$$

Considering the source precoder, $\mathbf{P}$, the SR effective channel, $\mathbf{H}_{sr,eff}$, is decomposed into

$$\mathbf{H}_{sr,eff} = \mathbf{H}_{sr}\mathbf{P}\ = \mathbf{U}_{sr,eff}\Sigma_{sr,eff}\mathbf{V}_{sr,eff}^H, \tag{6}$$

where $\mathbf{U}_{sr,eff}$, $\Sigma_{sr,eff}$, and $\mathbf{V}_{sr,eff}$ represent left singular matrix of $\mathbf{H}_{sr,eff}$, singular value matrix of $\mathbf{H}_{sr,eff}$, and right singular matrix of $\mathbf{H}_{sr,eff}$, respectively.

As a unitary precoder at transmitter does not affect the channel capacity, the proposed system utilizes $\mathbf{U}_{sr,eff}$ to parallelize the SR effective channel. The channel parallelizing converts the matrix determinant calculation into the summation calculation when we represent the channel capacities of MIMO systems [1]. Then, a precoder of the proposed system is given by

$$\begin{aligned} \mathbf{G}\ &=\ \mathbf{V}_{rd}\,\mathbf{A}\,\mathbf{U}_{sr,eff}^H \\ \mathbf{A}\ &=\ diag\left[a_1,\ a_2,\ \ldots\ ,\ a_M\right], \end{aligned} \tag{7}$$

where $\mathbf{V}_{rd}$ is the right singular matrix of $\mathbf{H}_{rd}$ and a_k is the power allocation factor for the relay precoder. When the source and the relay are assumed to consume powers of P_s and P_r, respectively, the condition to have the maximum value of its own effective channel is following:

$$\text{maximize} \sum_{k=1}^{M} \log_2 \left(\frac{P_s \lambda_{sr,eff,k}^2 \lambda_{rd,k}^2 a_k^2}{M\left(1+\lambda_{rd,k}^2 a_k^2\right)} \right), \tag{8}$$

$$\text{subject to} \sum_{k=1}^{M} \left(\frac{P_s \lambda_{sr,eff,k}^2 a_k^2}{M} + \sigma^2 \right) \le P_r, \tag{9}$$

where $\lambda_{sr,eff,k}$ and $\lambda_{rd,k}$ represent the kth singular values of $\mathbf{H}_{sr,eff}$ and $\mathbf{H}_{rd}$, respectively. By using the Lagrangian method [7][17], we can obtain the power allocation part of the relay precoder satisfying (8) and (9):

$$a_k = \sqrt{\frac{c_k}{\left(P_s/M\right)\lambda_{sr,eff,k}^2 + \sigma^2}}\ , \tag{10}$$

$$c_k = \left[\sqrt{\mu \frac{P_s \lambda_{sr,eff,k}^2}{\lambda_{rd,k}^2} + \left(\frac{P_s \lambda_{sr,eff,k}^2}{2M\lambda_{rd,k}^2} \right)^2} - \frac{P_s \lambda_{sr,eff,k}^2}{2M\lambda_{rd,k}^2} + \frac{\sigma^2}{\lambda_{rd,k}^2} \right]^+, \tag{11}$$

where μ is a constant that makes the allocated power satisfy the relay power constraint.

B. Precoder Design for the Source Station

In the view of system channel capacity, the relay cannot optimize the channel capacity of MIMO cooperative system with direct-path [15]. However, the source can optimize the channel capacity of the system, since the source knows global CSI and is able to control the effective channel of the system.

Considering the noise covariance matrix $\mathbf{R}_w$ in (4) is a colored noise, (3) can be converted into

$$I\left(\mathbf{x};\mathbf{y}\right) = \tfrac{1}{2}\log_2 \left| \mathbf{I}_{2M} + \mathbf{R}_a^{-1/2}\mathbf{H}\mathbf{P}\mathbf{R}_x\mathbf{P}^H\mathbf{H}^H \mathbf{R}_a^{-H/2}\sigma^{-2}\mathbf{I}_{2M} \right|, \tag{12}$$

where

$$\mathbf{R}_a = \begin{bmatrix} \mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{rd}\mathbf{G}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{I}_M \end{bmatrix}. \tag{13}$$

By comparing the mutual information of (3) and (12), an effective channel and an effective noise covariance matrix can be obtained as

$$\begin{aligned} \mathbf{H}_{eff} &= \mathbf{R}_a^{-1/2}\mathbf{H} = \ \mathbf{U}_{eff}\,\Sigma_{eff}\,\mathbf{V}_{eff}^H \\ \mathbf{R}_{w,eff} &= \sigma^2\mathbf{I}_{2M}. \end{aligned} \tag{14}$$

From (12) and (14), the system capacity is then given by

$$I\left(\mathbf{x};\ \mathbf{y}\right) \ = \frac{1}{2}\log_2 \left| \mathbf{I}_{2M} + \mathbf{H}_{eff}\mathbf{P}\mathbf{R}_x\mathbf{P}^H\mathbf{H}_{eff}^H\mathbf{R}_{w,eff}^{-1} \right|. \tag{15}$$

In the proposed system, the source maximizes the channel capacity of (15) by using the Lagrangian method, and then the precoder of the source is designed as

$$\mathbf{P} = \mathbf{V}_{eff}\mathbf{B}, \tag{16}$$

where $\mathbf{B}$ is power allocation part of the source precoder given by

$$\mathbf{B} = diag\left[b_1,\, b_2,\, \ldots,\, b_M\right]. \tag{17}$$

The condition to have the maximum value of (15) is following:

$$\text{maximize} \sum_{k=1}^{M} \log_2 \left(1 + \frac{P_s \lambda_{eff,k}^2}{M\sigma^2} b_k^2 \right) \tag{18}$$

$$\text{subject to} \sum_{k=1}^{M} b_k^2 \le P_s, \tag{19}$$

where $\lambda_{eff,k}$ is the k-th singular value of $\mathbf{H}_{eff}$ in (14). And then, the b_k satisfied (18) and (19) is given by

$$b_k = \sqrt{\left[\mu_s - \frac{M\sigma^2}{P_s\lambda_{eff,k}} \right]^+}, \tag{20}$$

where μ_s is a constant that makes the allocated power satisfy the source power constraint.

C. Iterative Structure for Optimization

The design of the source and relay precoders exploits individual optimization approach, where the source determines an optimal precoder under the assumption that the relay precoder is fixed, and the relay finds an optimal precoder under the assumption that the source precoder is fixed. However, this approach cannot guarantee the optimization of the system channel capacity and the satisfaction of the transmission power constraints. That is, if the source or relay precoder is changed, the other precoder is not optimal anymore. Moreover, if the source precoder is designed by considering the previous relay precoder, the mismatch between the previous relay percoder and the new source precoder causes over power consumption at the relay. To solve this problem, we exploit an iterative structure: the steps of determining optimal source and relay precoders are repeated until these precoders converge to fixed matrices.

However, the fact that each precoder has fixed values cannot guarantee that the proposed system achieves optimal channel capacity. If the mutual information function of the proposed system has local maximum, the designed precoders may not be optimal. Thus, it is necessary to prove the global optimality of the mutual information function of the proposed system.

III. The Global Optimality of the MIMO Cooperative System with Source and Relay Precoders

If some function is formed by a concave function, this function has one local maximum value and this maximum value is the global maximum value [17]. To confirm that the proposed precoders are globally optimal, we prove that the mutual information function of the proposed system is a concave function about the source and relay precoders. To deal with the general form of the proposed system, we redefine noise covariance matrices $\mathbf{R}_{w,r}$, $\mathbf{R}_{w,d1}$, and $\mathbf{R}_{w,d2}$ instead of $\sigma^2\mathbf{I}$:

$$\begin{aligned} &\mathbf{R}_{w,r} = E\left[\mathbf{n}_r{\mathbf{n}_r}^H\right],\ \mathbf{R}_{w,d1} = E\left[\mathbf{n}_{d,1}{\mathbf{n}_{d,1}}^H\right], \\ &\mathbf{R}_{w,d2} = E\left[\mathbf{n}_{d,2}{\mathbf{n}_{d,2}}^H\right]. \end{aligned} \tag{21}$$

Using the block matrix determinant lemma [7], the channel capacity of the proposed system (3) can be rewritten as

$$I(\mathbf{x};\mathbf{y}) = \tfrac{1}{2}\log_2\Big|\mathbf{I}_M + \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{P}\mathbf{R}_x\mathbf{P}^H\mathbf{H}_{sr}^H\mathbf{G}^H\mathbf{H}_{rd}^H \left(\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}\right)^{-1}\Big| + \tfrac{1}{2}\log_2\Big|\mathbf{I}_M + \mathbf{R}_{w,d1}^{-1/2}\mathbf{H}_{sd}\Big(\mathbf{P}^{-H}\mathbf{R}_x^{-1}\mathbf{P}^{-1} + \mathbf{H}_{sr}^H\mathbf{G}^H\mathbf{H}_{rd}^H \left(\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}\right)^{-1}\mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\Big) \mathbf{H}_{sd}^H\mathbf{R}_{w,d1}^{-H/2}\Big|, \tag{22}$$

where $\mathbf{A}^{-1/2}$ and $\mathbf{A}^{-H/2}$ are the $M\times M$ matrices satisfied $\mathbf{A}^{-1} = \mathbf{A}^{-1/2}\mathbf{A}^{-H/2}$ and $\mathbf{A}^{-H/2} = \left(\mathbf{A}^{-1/2}\right)^H$.

Now, the following proves the optimality of the proposed system.

Theorem: $I(\mathbf{x};\ \mathbf{y})$ in (22) is concave cap with respect to both $\mathbf{P}$ and $\mathbf{G}$.

Proof: In (22), $\mathbf{R}_{w,r}$ is positive definite. If $\mathbf{X}$ is a positive definite matrix, $\mathbf{Y}\mathbf{X}\mathbf{Y}^H$ is semi-positive definite for an arbitrary matrix $\mathbf{Y}$. So, in (22), $\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H$ is semi-positive definite and $\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}$ is a positive definite matrix such that the square root of $\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}$ exists [18][19].

Using the matrix determinant lemma, the determinant in the first term of (22) becomes the following:

$$\Big|\mathbf{I}_M + \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{P}\mathbf{R}_x\mathbf{P}^H\mathbf{H}_{sr}^H\mathbf{G}^H\mathbf{H}_{rd}^H \left(\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}\right)^{-1}\Big| = \Big|\mathbf{I}_M + \left(\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}\right)^{-H/2} \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{P}\mathbf{R}_x\mathbf{P}^H\mathbf{H}_{sr}^H\mathbf{G}^H\mathbf{H}_{rd}^H \left(\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}\right)^{-1/2}\Big|. \tag{23}$$

Then, we can rewrite (22) as follows:

$$I(\mathbf{x};\ \mathbf{y}) = \frac{1}{2}\log_2\left(\left|\mathbf{I}_M + \mathbf{C}\mathbf{C}^H\right|\right) + \frac{1}{2}\log_2\left(\left|\mathbf{I}_M + \mathbf{D}\right|\right), \tag{24}$$

where $\mathbf{C}$ and $\mathbf{D}$ are given as

$$\mathbf{C} = \left(\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}\right)^{-H/2}\left(\mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{P}\mathbf{R}_x^{1/2}\right)$$
$$\mathbf{D} = \mathbf{R}_{w,d1}^{-1/2}\mathbf{H}_{sd}\left(\mathbf{P}^{-H}\mathbf{R}_x^{-1}\mathbf{P}^{-1} + \mathbf{H}_{sr}^H\mathbf{G}^H\mathbf{H}_{rd}^H\left(\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}\right)^{-1}\mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\right)^{-1}\mathbf{H}_{sd}^H\mathbf{R}_{w,d1}^{-H/2}. \tag{25}$$

In (25), since $\mathbf{H}_{rd}\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\mathbf{H}_{rd}^H + \mathbf{R}_{w,d2}$ and $\mathbf{P}^{-H}\mathbf{R}_x^{-1}\mathbf{P}^{-1}$ are positive definite matrices, $\mathbf{C}\mathbf{C}^H$ and $\mathbf{D}$ are positive definite matrices, and hence, all the eigenvalues of $\mathbf{C}\mathbf{C}^H$ and $\mathbf{D}$ are non-negative.

Using the matrix determinant lemma again, (24) can be expressed as

$$I(\mathbf{x};\ \mathbf{y}) = \tfrac{1}{2}\log_2\left|\mathbf{I}_M + \Sigma_{\mathbf{C}\mathbf{C}^H}\right| + \tfrac{1}{2}\log_2\left|\mathbf{I}_M + \Sigma_{\mathbf{D}}\right| = \tfrac{1}{2}\log_2\left|\mathbf{I}_M + \Sigma_{\mathbf{C}\mathbf{C}^H} + \Sigma_{\mathbf{D}} + \Sigma_{\mathbf{C}\mathbf{C}^H}\Sigma_{\mathbf{D}}\right|, \tag{26}$$

where $\Sigma_{(\cdot)}$ indicates the eigenvalue matrix of $(\cdot)$. In (26), note that $\mathbf{I}_M + \Sigma_{\mathbf{C}\mathbf{C}^H} + \Sigma_{\mathbf{D}} + \Sigma_{\mathbf{C}\mathbf{C}^H}\Sigma_{\mathbf{D}}$ is satisfied both positive definite and hermitian conditions.

We now redefine the mutual information of the proposed system as

$$I(\mathbf{x},\mathbf{y}) = \frac{1}{2}f\left(\mathbf{J}_{\mathbf{P},\mathbf{G}}\right) = \frac{1}{2}\log_2\left|\mathbf{J}_{\mathbf{P},\mathbf{G}}\right|, \tag{27}$$

where $\mathbf{J}_{\mathbf{P},\mathbf{G}}$ is positive definite and hermitian matrix. It is well-known that, if $\mathbf{L}$ is an $M\times M$ positive definite matrix and $\mathbf{M}$ is an $M\times M$ hermitian matrix, then there exists a nonsingular $M\times M$ matrix $\mathbf{N}$ such that $\mathbf{N}^H\mathbf{L}\mathbf{N}$ is the identity matrix and also $\mathbf{N}^H\mathbf{M}\mathbf{N}$ is a diagonal matrix [18]. We denote $f(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1})$ and $f(\mathbf{J}_{\mathbf{P}_2,\mathbf{G}_2})$ as the functions of precoders $(\mathbf{P}_1,\mathbf{G}_1)$ and $(\mathbf{P}_2,\mathbf{G}_2)$, respectively. Then, there exists a nonsingular matrix $\mathbf{N}$ such that $\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1} = \mathbf{N}\mathbf{I}\mathbf{N}^H$ and $\mathbf{J}_{\mathbf{P}_2,\mathbf{G}_2} = \mathbf{N}\mathbf{\Lambda}\mathbf{N}^H$ for $\mathbf{\Lambda} = diag(\lambda_1,\lambda_2,...,\lambda_M)$ with all $\lambda_i > 0$. Then

$$\begin{aligned} f\left(\alpha\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1} + (1-\alpha)\mathbf{J}_{\mathbf{P}_2,\mathbf{G}_2}\right) &= \log_2\left|\alpha\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1} + (1-\alpha)\mathbf{J}_{\mathbf{P}_2,\mathbf{G}_2}\right| \\ &= \log_2\left|\mathbf{N}\left[\alpha\mathbf{I} + (1-\alpha)\mathbf{\Lambda}\right]\mathbf{N}^H\right| \\ &= \log_2\left|\mathbf{N}\mathbf{N}^H\right| + \log_2\left|\alpha\mathbf{I} + (1-\alpha)\mathbf{\Lambda}\right| \\ &= f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + f\left(\alpha\mathbf{I} + (1-\alpha)\mathbf{\Lambda}\right) \end{aligned} \tag{28}$$

and

$$\begin{aligned} \alpha f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + (1-\alpha) f\left(\mathbf{J}_{\mathbf{P}_2,\mathbf{G}_2}\right) &= \alpha f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + (1-\alpha)\log_2\left|\mathbf{N}\mathbf{\Lambda}\mathbf{N}^H\right| \\ &= \alpha f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + (1-\alpha)\left(\log_2\left|\mathbf{N}\mathbf{N}^H\right| + \log_2\left|\mathbf{\Lambda}\right|\right) \\ &= \alpha f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + (1-\alpha) f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + (1-\alpha) f(\mathbf{\Lambda}) \\ &= f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + (1-\alpha) f(\mathbf{\Lambda}). \end{aligned} \tag{29}$$

Since $\log_2(x)$ is concave cap for all $x > 0$, for $0 \le \alpha \le 1$, we have

$$\log_2\left(\alpha x + (1-\alpha)y\right) \ \ge\ \alpha\log_2 x + (1-\alpha)\log_2 y, \tag{30}$$

for all $x, y > 0$. Therefore,

$$\begin{aligned} f\left(\alpha\mathbf{I} + (1-\alpha)\Lambda\right) &= \sum_{i=1}^{M}\log\left(\alpha + (1-\alpha)\lambda_i\right) \\ &\ge \sum_{i=1}^{M}\left(\alpha\log_2 1 + (1-\alpha)\log_2\lambda_i\right) \\ &= (1-\alpha)\sum_{i=1}^{M}\log_2\lambda_i \\ &= (1-\alpha) f(\Lambda). \end{aligned} \tag{31}$$

From (28), (29) and (31), we finally have

$$f\left(\alpha\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1} + (1-\alpha)\mathbf{J}_{\mathbf{P}_2,\mathbf{G}_2}\right) \ge \alpha f\left(\mathbf{J}_{\mathbf{P}_1,\mathbf{G}_1}\right) + (1-\alpha) f\left(\mathbf{J}_{\mathbf{P}_2,\mathbf{G}_2}\right), \tag{32}$$

which is a sufficient condition for $f(\cdot)$ to be a concave cap function. Therefore, $I(\mathbf{x};\ \mathbf{y})$ in (22) is concave cap.

Now, we need to show that the constraints of the proposed system are convex sets. The proposed system has transmitted power constraint in each station as following.

$$tr\left\{\mathbf{P}\mathbf{P}^H\right\} \le P_s \tag{33}$$

$$tr\left\{\mathbf{G}\mathbf{H}_{sr}\mathbf{H}_{sr}^H\mathbf{G}^H + \mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\right\} \le P_r, \tag{34}$$

where P_r is available power at relay.

To show that the constraints are convex sets, we define $g(\mathbf{A})$ and $h(\mathbf{A})$:

$$g(\mathbf{A}) = tr\left\{\mathbf{A}\mathbf{A}^H\right\} \tag{35}$$

$$h(\mathbf{A}) = tr\left\{\mathbf{A}\mathbf{C}\mathbf{A}^H\right\}, \tag{36}$$

where $\mathbf{C}$ is a positive definite matrix. Then we can easily calculate following equations.

$$\begin{aligned} & g\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right) \\ & = tr\left\{\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right)\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right)^H\right\} \\ & = tr\left\{\alpha^2\mathbf{A}\mathbf{A}^H+\alpha(1-\alpha)\left(\mathbf{A}\mathbf{B}^H+\mathbf{B}\mathbf{A}^H\right) \right. \\ & \quad \left. +(1-\alpha)^2\mathbf{B}\mathbf{B}^H\right\}, \end{aligned} \tag{37}$$

$$\alpha g(\mathbf{A})+(1-\alpha)g(\mathbf{B}) = tr\left\{\alpha\mathbf{A}\mathbf{A}^H+(1-\alpha)\mathbf{B}\mathbf{B}^H\right\}, \tag{38}$$

where $0<\alpha<1$. Then,

$$\begin{aligned} & g\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right)-\alpha g(\mathbf{A})-(1-\alpha)g(\mathbf{B}) \\ & = tr\left\{\alpha^2\mathbf{A}\mathbf{A}^H+\alpha(1-\alpha)\left(\mathbf{A}\mathbf{B}^H+\mathbf{B}\mathbf{A}^H\right)\right. \\ & \quad \left. +(1-\alpha)^2\mathbf{B}\mathbf{B}^H-\alpha\mathbf{A}\mathbf{A}^H-(1-\alpha)\mathbf{B}\mathbf{B}^H\right\} \\ & = tr\left\{\alpha(\alpha-1)\mathbf{A}\mathbf{A}^H-\alpha(\alpha-1)\left(\mathbf{A}\mathbf{B}^H\right.\right. \\ & \quad \left.\left.+\mathbf{B}\mathbf{A}^H\right)+\alpha(\alpha-1)\mathbf{B}\mathbf{B}^H\right\} \\ & = \alpha(\alpha-1)\,tr\left\{(\mathbf{A}-\mathbf{B})(\mathbf{A}-\mathbf{B})^H\right\} \\ & \leq 0. \end{aligned} \tag{39}$$

Thus, $g(\mathbf{A})$ is a convex function and the sets satisfied with $tr\left\{\mathbf{P}\mathbf{P}^H\right\} \leq P_s$ are convex sets.

Now, we consider the convexity of $h(\mathbf{A})$:

$$\begin{aligned} & h\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right) \\ & = tr\left\{\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right)\mathbf{C}\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right)^H\right\} \\ & = tr\left\{\alpha^2\mathbf{A}\mathbf{C}\mathbf{A}^H+\alpha(1-\alpha)\left(\mathbf{A}\mathbf{C}\mathbf{B}^H+\mathbf{B}\mathbf{C}\mathbf{A}^H\right)\right. \\ & \quad \left. +(1-\alpha)^2\mathbf{B}\mathbf{C}\mathbf{B}^H\right\} \end{aligned} \tag{40}$$

and

$$\alpha h(\mathbf{A})+(1-\alpha)h(\mathbf{B}) = tr\left\{\alpha\mathbf{A}\mathbf{C}\mathbf{A}^H+(1-\alpha)\mathbf{B}\mathbf{C}\mathbf{B}^H\right\}. \tag{41}$$

Using (40) and (41),

$$\begin{aligned} & h\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right)-\alpha h(\mathbf{A})-(1-\alpha)h(\mathbf{B}) \\ & = tr\left\{\alpha^2\mathbf{A}\mathbf{C}\mathbf{A}^H+\alpha(1-\alpha)\left(\mathbf{A}\mathbf{C}\mathbf{B}^H+\mathbf{B}\mathbf{C}\mathbf{A}^H\right)\right. \\ & \quad \left. +(1-\alpha)^2\mathbf{B}\mathbf{C}\mathbf{B}^H-\alpha\mathbf{A}\mathbf{C}\mathbf{A}^H-(1-\alpha)\mathbf{B}\mathbf{C}\mathbf{B}^H\right\} \\ & = tr\left\{\alpha(\alpha-1)\mathbf{A}\mathbf{C}\mathbf{A}^H-\alpha(1-\alpha)\left(\mathbf{A}\mathbf{C}\mathbf{B}^H+\mathbf{B}\mathbf{C}\right.\right. \\ & \quad \left.\left.\mathbf{A}^H\right)+\alpha(\alpha-1)\mathbf{B}\mathbf{C}\mathbf{B}^H\right\}. \end{aligned} \tag{42}$$

As $\mathbf{C}$ is a positive definite matrix, $\mathbf{C}_r$ exists such that $\mathbf{C}=\mathbf{C}_r\mathbf{C}_r^H$. So

$$\begin{aligned} & h\left(\alpha\mathbf{A}+(1-\alpha)\mathbf{B}\right)-\alpha h(\mathbf{A})-(1-\alpha)h(\mathbf{B}) \\ & = tr\left\{\alpha(\alpha-1)\mathbf{A}\mathbf{C}_r\mathbf{C}_r^H\mathbf{A}^H-\alpha(1-\alpha)\left(\mathbf{A}\mathbf{C}_r\mathbf{C}_r^H\mathbf{B}^H\right.\right. \\ & \quad \left.\left.+\mathbf{B}\mathbf{C}_r\mathbf{C}_r^H\mathbf{A}^H\right)+\alpha(\alpha-1)\mathbf{B}\mathbf{C}_r\mathbf{C}_r^H\mathbf{B}^H\right\} \\ & = \alpha(\alpha-1)\,tr\left\{(\mathbf{A}\mathbf{C}_r-\mathbf{B}\mathbf{C}_r)(\mathbf{A}\mathbf{C}_r-\mathbf{B}\mathbf{C}_r)^H\right\} \\ & \leq 0. \end{aligned} \tag{43}$$

Thus, $h(\mathbf{A})$ is a convex function and the sets satisfied with $tr\left\{\mathbf{G}\mathbf{H}_{sr}\mathbf{H}_{sr}^H\mathbf{G}^H+\mathbf{G}\mathbf{R}_{w,r}\mathbf{G}^H\right\} \leq P_r$ are convex sets.

As the object function and the constraint sets satisfy concave function and convex sets, respectively, the proposed precoders give an optimal channel capacity. ■

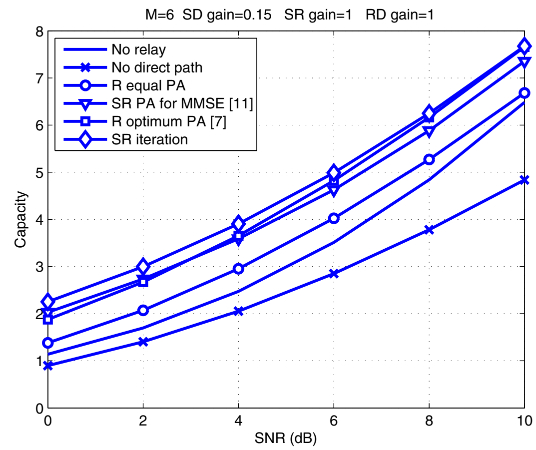

Fig. 3. The channel capacities of the existing systems and the 'SR iteration' system.

IV. Simulation Results

We assume that the SR, RD and SD channels are $M \times M$ MIMO channels with flat Rayleigh fading and there are no correlations among antennas. Also, it is assumed that zero-mean complex Gaussian noise with variance of σ^2 is added to each station.

Fig. 3 depicts channel capacity performances of various MIMO cooperative systems, when the number of antennas is 6 ($M=6$), SR channel gain is 1, RD channel gain is 1, and SD channel gain is 0.15. In this figure, the 'No relay' system indicates a MIMO system without a relay. This system can transmit twice as many data symbols as the cooperative systems. The 'No direct path' system is the general relay system that has a relay but does not use direct path. In these channel environments, the one-hop system ('No relay') cannot give high channel capacity. In the 'R equal PA' system, source and relay transmit data symbols using only power normalization. The 'SR PA for MMSE' system proposed in [11] has the source and relay precoders for minimizing the mean square error (MSE). In the 'R optimum PA' system suggested in [7], the source transmits data symbols without precoding, but the relay allocates an optimum power to each stream. The 'SR iteration' system uses the proposed optimal precoders. When the channel between a transmitter and a receiver is poor, a relay system is adopted. However, in this environment, the relay system that does not use direct path has less channel capacity than the 'No relay' system. Also the 'SR PA for MMSE' system minimizes the MSE, and the criterion that minimizes the MSE is generally effective when all streams have the same modulation order. So, this system has better channel capacity than the 'R optimum PA' in low SNR. However, in high SNR, the adaptive modulation can be adopted, such that the advantage of this system is weak. On the other hand, the 'SR iteration' system has capacity improvement in low SNR and gives similar performance as the 'R optimal PA' system in high SNR region.

Fig. 4 gives a cumulative distribution function about the convergence of the proposed system, when each SD channel gain varies from 0.05 to 0.2. From the result, we can see that the system with low SD channel gain needs more iterations

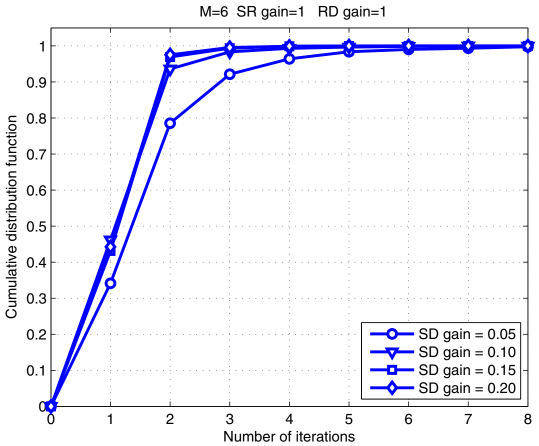

Fig. 4. The convergence CDF of the 'SR iteration' system according to SD channel gain.

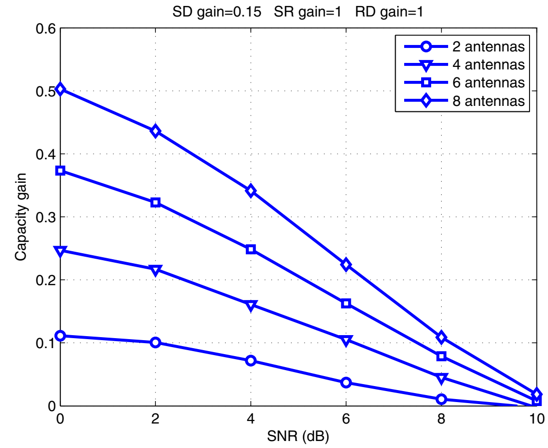

Fig. 6. The channel capacity differences between the 'R optimum PA' and the 'SR iteration' according to the various numbers of antennas.

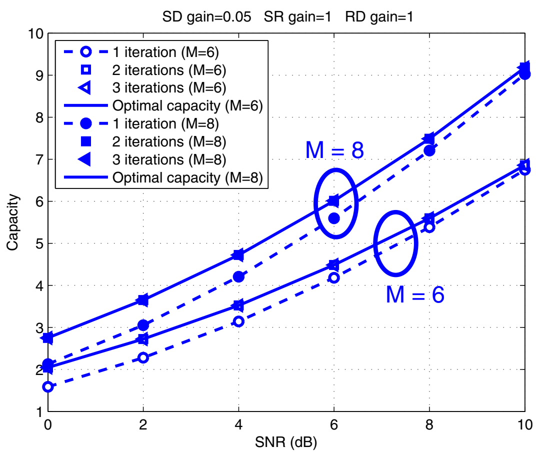

Fig. 5. Channel capcities of the 'SR iteration' system according to the number of iterations.

for achieving optimal precoder than the system with high SD channel gain, and the system with 0.05 SD channel gain converges to final channel capacity region within eight iterations. Also, we can see that as SD channel gain is increasing, the convergence speed of the proposed system is increasing.

Fig. 5 represents the average channel capacities by iterations. In Fig. 4, we confirmed that the system needs more iterations for achieving optimal precoders for the decreasing SD channel gain or the increasing numbers of antennas. In these figures, when the number of antennas is 6 or 8 and SD channel gain is 0.05, the proposed system requires at least eight iterations to converge to optimal precoders. However, Fig. 5 depicts that the channel capacity of the proposed system is close to the optimal channel capacity after two iterations.

Fig. 6 represents the differences between the channel capacities of the proposed system and the 'R optimum PA' system, according to the number of antennas. The channel capacity differences between the proposed system and the 'R optimum PA' system get widen with the increase of the numbers of antennas.

V. CONCLUSIONS

This paper considered source and relay precoders to achieve the optimal channel capacity for the MIMO cooperative system and proved the global optimality of the channel capacity of the system with these precoders. To design the optimal precoders, we use the Lagrangian method at each station and adopt an iterative structure. Also, in order to show that these precoders are optimal from the viewpoint of channel capacity, we proved that the mutual information function of the proposed system is a concave function and the constraints are the convex sets.

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