Expanding Generalized Hadamard Matrices over G^m by Substituting Several Generalized Hadamard Matrices over G

Jong-Seon No and Hong-Yeop Song

Abstract: **Over an additive abelian group G of order g and for a given positive integer λ, a generalized Hadamard matrix $GH(g, \lambda)$ is defined as a $g\lambda \times g\lambda$ matrix $[h(i, j)]$, where $1 \leq i \leq g\lambda$ and $1 \leq j \leq g\lambda$, such that every element of G appears exactly λ times in the list $h(i_1, 1) - h(i_2, 1), h(i_1, 2) - h(i_2, 2), \cdots, h(i_1, g\lambda) - h(i_2, g\lambda)$, for any $i_1 \neq i_2$. In this paper, we propose a new method of expanding a $GH(g^m, \lambda_1) = B = [B_{ij}]$ over G^m by replacing each of its m-tuple B_{ij} with $B_{ij} \oplus GH(g, \lambda_2)$ where $m = g\lambda_2$. We may use $g^m\lambda_1$ (not necessarily all distinct) $GH(g, \lambda_2)$'s for the substitution and the resulting matrix is defined over the group of order g.**

Index Terms: **Generalized Hadamard matrices, difference matrices, p^m-ary m-sequences.**

I. INTRODUCTION

A Hadamard matrix, H_λ, of order λ is a $\lambda \times \lambda$ matrix with elements $+1$'s and -1's such that $H_\lambda \cdot H_\lambda^T = \lambda I_\lambda$, where I_λ is the identity matrix of order λ [1]–[3]. This implies that any two distinct rows of H_λ are *orthogonal*. For this reason and many others, Hadamard matrices have been studied in many different but related such areas as wireless communication systems engineering, coding theory, and statistical design theory [3]–[8].

The symbol alphabet can be generalized to a group of order ≥ 2. In this case, the notion of orthogonality should be suitably modified as in the following definition [4], [9]–[13]. We will restrict our discussion to abelian groups written additively in this paper.

Definition 1: Let G be an additive abelian group of order g. Let $u = (u_1, u_2, \cdots, u_{g\lambda})$ and $v = (v_1, v_2, \cdots, v_{g\lambda})$ where $u_i, v_j \in G$. Then u and v are said to be *difference-balanced* if every element of G appears exactly λ times in the list of componentwise differences $u_1 - v_1, u_2 - v_2, \cdots, u_{g\lambda} - v_{g\lambda}$.

Definition 2: Let G be an additive abelian group of order g. For a positive integer λ, a generalized Hadamard matrix $GH(g, \lambda)$ is a $g\lambda \times g\lambda$ matrix over G in which any two distinct rows are difference-balanced.

Remark 1: $GH(2, \lambda/2)$ is a (binary) Hadamard matrix of order λ.

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$$

Fig. 1. A Hadamard matrix of order 8 from Sylvester construction.

For $\lambda = 1$, an easy example of $GH(g, 1)$ is the group table of the cyclic group of order $g \geq 1$. For any other pair of g and λ, see [4], [9]–[12] for specific constructions.

One of the well-known constructions for (binary) Hadamard matrices was originally from Sylvester [1], [2], [4]. It works as follows: If there exist Hadamard matrices H_m and $H_k = [h_{ij}]$ of orders m and k, respectively, then the matrix obtained by replacing each $h_{ij} = \pm 1$ with $\pm H_m$ is a Hadamard matrix of order mk. A Hadamard matrix of order 8 from Sylvester construction is shown in Fig. 1. Observe that H_8 in Fig. 1 has 16 blocks of order 2 and these blocks are either H_2 or $-H_2$, exclusively. Similarly, there is a construction for generalized Hadamard matrices $GH(g, g\lambda_1\lambda_2)$ over G of order g assuming that there exist $B = [B_{ij}] = GH(g, \lambda_1)$ and $C = GH(g, \lambda_2)$, both over G [10]. It replaces each element B_{ij} of B with $B_{ij} + C$, and the blocks are of the form $a + C$ for $a \in G$. We will also call this Sylvester's method for generalized Hadamard matrices.

In this paper, we propose a new method of expanding a $GH(g^m, \lambda_1) = B = [B_{ij}]$ over G^m by replacing each of its m-tuple B_{ij} with $B_{ij} \oplus GH(g, \lambda_2)$ where $m = g\lambda_2$ and where the operation $\oplus$ will soon be defined in Section II. We may use $g^m\lambda_1$ (not necessarily all distinct) $GH(g, \lambda_2)$'s for the substitution and the resulting matrix is defined over the group of order g.

We will prove the main construction in Section II and give some examples. An example over F_2 is explicitly given using a 4-ary m-sequence of length 15. A brief remark is given in Conclusion.

II. MAIN CONSTRUCTION

Let G be a group of order g. Given a generalized Hadamard matrix $GH(g, \lambda)$ over G, one can transform into another by (i) interchanging any two rows (columns, resp.), and/or (ii) adding

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$a \in G$ to every element of a row (column, resp.). Two generalized Hadamard matrices which differ only by some combination of these four operations are said to be *equivalent*. For a given generalized Hadamard matrix we can find an equivalent one in which the top row and the left-most column consist entirely of 0's, the identity of G. Such a generalized Hadamard matrix is called *normalized*. Clearly the remaining rows (if any) must contain every element of G exactly λ times.

Note that if G is a group of order g, then all the m-tuples of elements of G form a group of order g^m with respect to componentwise addition. We will denote this group by G^m throughout this section. We will use $\oplus$ as the operation of G^m. With slight abuse of notation, we will also use $\oplus$ as in the following definition:

Definition 3: (Operation $\oplus$) Let $X = (x_1, x_2, \cdots, x_m)$ be an m-tuple over G and C be a $k \times m$ matrix over G. Denote the rows of C by $\underline{C_1}, \underline{C_2}, \cdots, \underline{C_k}$. Then

$$X \oplus C = X \oplus \begin{bmatrix} \underline{C_1} \\ \underline{C_2} \\ \vdots \\ \underline{C_k} \end{bmatrix} = \begin{bmatrix} X \oplus \underline{C_1} \\ X \oplus \underline{C_2} \\ \vdots \\ X \oplus \underline{C_k} \end{bmatrix}$$
$$= \begin{bmatrix} x_1 + c_{11}, x_2 + c_{12}, \ldots, x_m + c_{1m} \\ x_1 + c_{21}, x_2 + c_{22}, \ldots, x_m + c_{2m} \\ \vdots \\ x_1 + c_{k1}, x_2 + c_{k2}, \ldots, x_m + c_{km} \end{bmatrix}.$$

The above operation plays the key role in our main construction. As a result of the operation $X \oplus C$, each row $\underline{C_i}$ of C is replaced with $X \oplus \underline{C_i}$. If we put it into another way, in every column of C, for example, in j-th column, the element c_{ij} is replaced with $x_j + c_{ij}$ for $1 \leq i \leq k$, where x_j is the j-th component of X. This proves the following Lemma:

Lemma 1: Let G be a group of order g and let $m = g\lambda$. If X is an m-tuple over G and C is a $GH(g, \lambda)$ over G, then $X \oplus C$ is a $GH(g, \lambda)$ over G.

Theorem 1: (Main) We assume that there exists a generalized Hadamard matrix $B \triangleq [B_{ij}] \triangleq GH(g^m, \lambda_1)$ over G^m where G is a group of order g. We also assume that there exist M generalized Hadamard matrices $C^{(1)}, C^{(2)}, \cdots, C^{(M)}$, not necessarily all distinct, all of which are $GH(g, \lambda_2)$ over G. If $M = g^m \lambda_1$ and $m = g\lambda_2$, then the matrix H over G of size $mM \times mM$ obtained by replacing B_{ij} with $B_{ij} \oplus C^{(i)}$ is a generalized Hadamard matrix $GH(g, g^m \lambda_1 \lambda_2)$ over G.

Proof: The resulting matrix H looks like the following:

$$H = \begin{bmatrix} B_{11} \oplus C^{(1)} & B_{12} \oplus C^{(1)} & \cdots & B_{1M} \oplus C^{(1)} \\ B_{21} \oplus C^{(2)} & B_{22} \oplus C^{(2)} & \cdots & B_{2M} \oplus C^{(2)} \\ B_{31} \oplus C^{(3)} & B_{32} \oplus C^{(3)} & \cdots & B_{3M} \oplus C^{(3)} \\ \vdots & \vdots & \cdots & \vdots \\ B_{M1} \oplus C^{(M)} & B_{M2} \oplus C^{(M)} & \cdots & B_{MM} \oplus C^{(M)} \end{bmatrix}. \tag{1}$$

The matrix H has a natural partition of M^2 submatrices each of size $m \times m$. Thus, we will call successive m rows of H a row-block. That is, H has M row-blocks and each row-block contains M submatrices of size $m \times m$. All we have to show is that any two distinct rows of H of length L ($\triangleq mM$) over G are difference-balanced in the sense of Definition 1. We will distinguish two cases: (I) two rows from the same row-block and (II) two rows from different row-blocks.

(*CASE I)* By Lemma 1, each of the $m \times m$ submatrices $B_{ij} \oplus C^{(i)}$ for all i and j is a $GH(g, \lambda_2)$ over G. Therefore, any two distinct rows of H from the same row-block are difference-balanced. In this case, note that we used the assumption that each $C^{(i)}$ is a generalized Hadamard matrix.

(CASE II) Now assume $i \neq j$ and consider x-th row of i-th row-block and y-th row of j-th row-block for some x, y with $1 \leq x \leq m$ and $1 \leq y \leq m$. If we denote x-th row of $C^{(i)}$ and y-th row of $C^{(j)}$ by $\underline{C_x^{(i)}}$ and $\underline{C_y^{(j)}}$, respectively, then the two rows look like the following at the bottom of this page. Note that B_{ik}, B_{jk}, $\underline{C_x^{(i)}}$, and $\underline{C_y^{(j)}}$ are m-tuples over G. If we regard two rows in (2) as vectors of length M over G^m, then they are difference-balanced over G^m since i-th row and j-th row of B are difference-balanced. These differences over G^m are, letting $\underline{C_x^{(i)}} \ominus \underline{C_y^{(j)}} \triangleq D$,

$$B_{ik} \ominus B_{jk} \oplus D, \quad \text{for} \quad k = 1, 2, \cdots, M. \tag{3}$$

Note that D is an m-tuple over G and is only a constant bias of the differences $B_{ik} \ominus B_{jk}$, regardless of k. On the other hand, $B_{ik} \ominus B_{jk}$ is computed componentwise, so we may concentrate on the components of the differences in (3). Similarly, we may regard two rows in (2) as vectors of length mM over G, and consider their componentwise differences over G. They are exactly the same as the differences listed in (3) except now that we are looking at the components. They are difference-balanced over G since in the list of M m-tuples over G in which every m-tuple of G^m appears exactly λ_1 times, every element of G appears exactly mM/g times. Note, in this case, that we used the assumption that B is a generalized Hadamard matrix over G^m. □

index	row				(2)
x-th row of i-th row-block	$B_{i1} \oplus \underline{C_x^{(i)}}$	$B_{i2} \oplus \underline{C_x^{(i)}}$	$\cdots$	$B_{iM} \oplus \underline{C_x^{(i)}}$	
y-th row of j-th row-block	$B_{j1} \oplus \underline{C_y^{(j)}}$	$B_{j2} \oplus \underline{C_y^{(j)}}$	$\cdots$	$B_{jM} \oplus \underline{C_y^{(j)}}$	

Remark 2: A generalized Hadamard matrix over a group is a square array. A rectangular array with the same condition on the list of differences of any two distinct rows is called a difference matrix [10]. Our main construction applies easily to the construction of difference matrices, since the proof of Theorem 1 does not use the fact that either B or C is a square array. Therefore, our method directly applies to difference matrices B over G^m and C's over G to construct larger size difference matrix over G.

Remark 3: We may use in the construction all the same $C^{(i)}$'s, some different but equivalent $C^{(i)}$'s, all distinct $C^{(i)}$'s, or all inequivalent $C^{(i)}$'s. We note that the construction method is different from those by Sylvester even if $C^{(i)}$'s are all the same.

Example 1: We take G to be the additive group of the finite field F_p of p elements for some prime p. For example, we take $G = \{0, 1, 2\}$ to be the additive group of integers mod 3 so that $p = g = 3$. If we have $B = GH(3^m, \lambda_1)$ over G^m and M (not necessarily distinct) $GH(3, \lambda_2)$'s over G where $m = 3\lambda_2$ and $M = 3^m\lambda_1$, the contruction gives a $GH(3, mM/3)$ over G.

Example 2: If we have $B = GH(p^m, \lambda_1)$ over F_{p^m} and M (not necessarily distinct) $GH(p, \lambda_2)$'s over F_p where $m = p\lambda_2$ and $M = p^m\lambda_1$, the contruction gives a $GH(p, mM/p)$ over F_p. Here, we consider the elements of F_{p^m} as m-tuples over F_p.

For example, consider the case $p = 2$. If $\{b_t \,|\, t = 0, 1, 2, \cdots, N-1\}$ is a 2^m-ary maximal-length linear feedback shift register sequence [14]–[18] of period $N = 2^{mn} - 1$ for some n, then the $(N+1) \times (N+1)$ matrix given by

$$B \triangleq \left[\begin{array}{c|ccccc} 0 & 0 & 0 & 0 & \cdots & 0 \\ \hline 0 & b_0 & b_1 & b_2 & \cdots & b_{N-1} \\ 0 & b_1 & b_2 & b_3 & \cdots & b_0 \\ 0 & b_2 & b_3 & b_4 & \cdots & b_1 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & b_{N-1} & b_0 & b_1 & \cdots & b_{N-2} \end{array}\right] \tag{4}$$

is a generalized Hadamard matrix $GH(g^m, \lambda_1) = GH(2^m, 2^{m(n-1)})$ over the additive group of F_{2^m}. Here, 0 and b_t in (4) can be represented [17] as m-tuples over F_2. Let $M = N + 1 = 2^{mn}$. If we have M (not necessarily all distinct) binary Hadamard matrices $GH(2, m/2)$ over F_2, then we can construct a $GH(2, mM/2) = GH(2, m2^{mn-1})$ over F_2, which is a binary Hadamard matrix of size $m2^{mn} \times m2^{mn}$.

Specifically, let $m = 2$ and $n = 2$. Denote $F_4 = \{0, 1, \beta, \beta^2\}$. Then, b_t in (5) is a 4-ary m-sequence of period 15 at the bottom of this page. This gives a $GH(4, 4)$ over F_4 given as (6) at the bottom of this page.
Denote $0, 1, \beta, \beta^2$ in F_4 by 2-tuples $00, 01, 10, 11$ over F_2, respectively. Suppose we use $C^{(i)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ for $1 \leq i \leq 8$, and its transpose as $C^{(i)}$ for $9 \leq i \leq 16$, then a 32×32 binary Hadamard matrix is obtained if we replace $0, 1, \beta, \beta^2$ in B according to Table 1.

Table 1. Substitution Rule for $GH(2, 16)$ over F_2 where $B = GH(4, 4)$ over F_4.

for the top half of B			for the bottom half of B		
replace 0	with	$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	replace 0	with	$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$
replace 1	with	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	replace 1	with	$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$
replace β	with	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	replace β	with	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
replace β^2	with	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	replace β^2	with	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

III. CONCLUDING REMARKS

A feature of the main construction is that we replace each m-tuple B_{ij} of B with $B_{ij} \oplus C^{(i)}$ for all i. A direction of future research is to investigate equivalence of two generalized

$$\begin{array}{c|ccccccccccccccc} t & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ \hline b_t & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 \end{array} \tag{5}$$

$$B = \left[\begin{array}{c|ccccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
0 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 \\
0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 \\
0 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 \\
0 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 \\
0 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 \\
0 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 \\
0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 \\
0 & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta \\
0 & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta \\
0 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 \\
0 & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta \\
0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 \\
0 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 \\
0 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 \\
0 & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta
\end{array}\right] \tag{6}$$

Hadamard matrices obtained by the proposed construction using some different sets of $C^{(i)}$'s.

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